# The Laws of Motion

CHAPTER

5


5.1 The Concept of Force

5.2 Newton's First Law and Inertial Frames

5.3 Mass

5.4 Newton's Second Law

5.5 The Gravitational Force and Weight

5.6 Newton's Third Law

5.7 Analysis Models Using Newton's Second Law

5.8 Forces of Friction


A person sculls on a calm waterway. The water exerts forces on the oars to accelerate the boat. *(Tetra Images/Getty Images)*

**In Chapters 2 and 4, we *described* the motion of an object in terms of its position,** velocity, and acceleration without considering what might *influence* that motion. Now we consider that influence: Why does the motion of an object change? What might cause one object to remain at rest and another object to accelerate? Why is it generally easier to move a small object than a large object? The two main factors we need to consider are the *forces* acting on an object and the *mass* of the object. In this chapter, we begin our study of *dynamics* by discussing the three basic laws of motion, which deal with forces and masses and were formulated more than three centuries ago by Isaac Newton.

## 5.1 The Concept of Force

Everyone has a basic understanding of the concept of force from everyday experience. When you push your empty dinner plate away, you exert a force on it. Similarly, you exert a force on a ball when you throw or kick it. In these examples, the word *force* refers to an interaction with an object by means of muscular activity and some change in the object's velocity. Forces do not always cause motion, however. For example, when you are sitting, a gravitational force acts on your body and yet you remain stationary. As a second example, you can push (in other words, exert a force) on a large boulder and not be able to move it.

What force (if any) causes the Moon to orbit the Earth? Newton answered this and related questions by stating that forces are what cause any change in the velocity of an object. The Moon's velocity changes in direction as it moves in a nearly circular

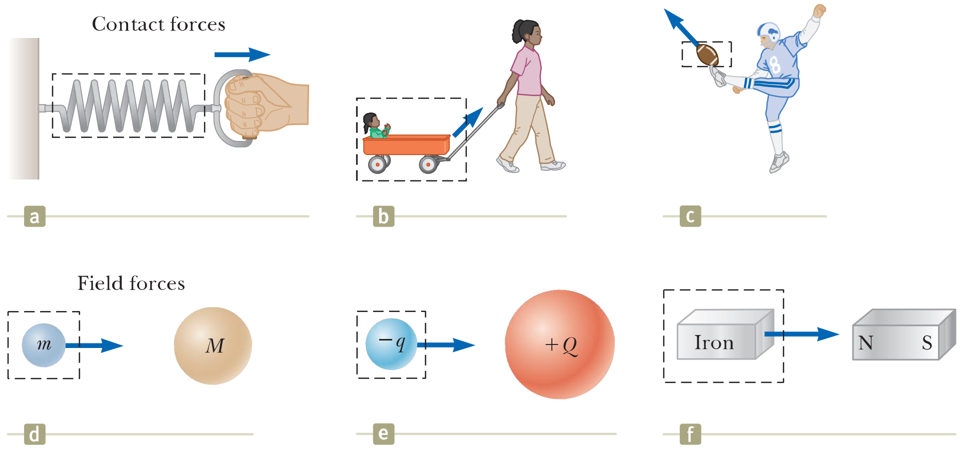



**Figure 5.1** Some examples of applied forces. In each case, a force is exerted on the object within the boxed area. Some agent in the environment external to the boxed area exerts a force on the object.

orbit around the Earth. This change in velocity is caused by the gravitational force exerted by the Earth on the Moon.

When a coiled spring is pulled, as in Figure 5.1a, the spring stretches. When a stationary cart is pulled, as in Figure 5.1b, the cart moves. When a football is kicked, as in Figure 5.1c, it is both deformed and set in motion. These situations are all examples of a class of forces called *contact forces.* That is, they involve physical contact between two objects. Other examples of contact forces are the force exerted by gas molecules on the walls of a container and the force exerted by your feet on the floor.

Another class of forces, known as *field forces,* does not involve physical contact between two objects. These forces act through empty space. The gravitational force of attraction between two objects with mass, illustrated in Figure 5.1d, is an example of this class of force. The gravitational force keeps objects bound to the Earth and the planets in orbit around the Sun. Another common field force is the electric force that one electric charge exerts on another (Fig. 5.1e), such as the attractive electric force between an electron and a proton that form a hydrogen atom. A third example of a field force is the force a bar magnet exerts on a piece of iron (Fig. 5.1f).

The distinction between contact forces and field forces is not as sharp as you may have been led to believe by the previous discussion. When examined at the atomic level, all the forces we classify as contact forces turn out to be caused by electric (field) forces of the type illustrated in Figure 5.1e. Nevertheless, in developing models for macroscopic phenomena, it is convenient to use both classifications of forces. The only known *fundamental* forces in nature are all field forces: (1) *gravitational forces* between objects, (2) *electromagnetic forces* between electric charges, (3) *strong forces* between subatomic particles, and (4) *weak forces* that arise in certain radioactive decay processes. In classical physics, we are concerned only with gravitational and electromagnetic forces. We will discuss strong and weak forces in Chapter 46.

Bridgeman-Giraudon/Art Resource, NY

**Isaac Newton**
*English physicist and mathematician* (1642–1727)
Isaac Newton was one of the most brilliant scientists in history. Before the age of 30, he formulated the basic concepts and laws of mechanics, discovered the law of universal gravitation, and invented the mathematical methods of calculus. As a consequence of his theories, Newton was able to explain the motions of the planets, the ebb and flow of the tides, and many special features of the motions of the Moon and the Earth. He also interpreted many fundamental observations concerning the nature of light. His contributions to physical theories dominated scientific thought for two centuries and remain important today.

## The Vector Nature of Force

It is possible to use the deformation of a spring to measure force. Suppose a vertical force is applied to a spring scale that has a fixed upper end as shown in Figure 5.2a. The spring elongates when the force is applied, and a pointer on the scale reads the extension of the spring. We can calibrate the spring by defining a reference force $\vec{\mathbf{F}}_1$ as the force that produces a pointer reading of 1.00 cm. If we now apply a different downward force $\vec{\mathbf{F}}_2$ whose magnitude is twice that of the reference force $\vec{\mathbf{F}}_1$ as seen in Figure 5.2b, the pointer moves to 2.00 cm. Figure 5.2c shows that the combined effect of the two collinear forces is the sum of the effects of the individual forces.

Now suppose the two forces are applied simultaneously with $\vec{\mathbf{F}}_1$ downward and $\vec{\mathbf{F}}_2$ horizontal as illustrated in Figure 5.2d. In this case, the pointer reads 2.24 cm. The single force $\vec{\mathbf{F}}$ that would produce this same reading is the sum of the two vectors $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ as described in Figure 5.2d. That is, $|\vec{\mathbf{F}}_1| = \sqrt{F_1^{\,2} + F_2^{\,2}} = 2.24$ units,

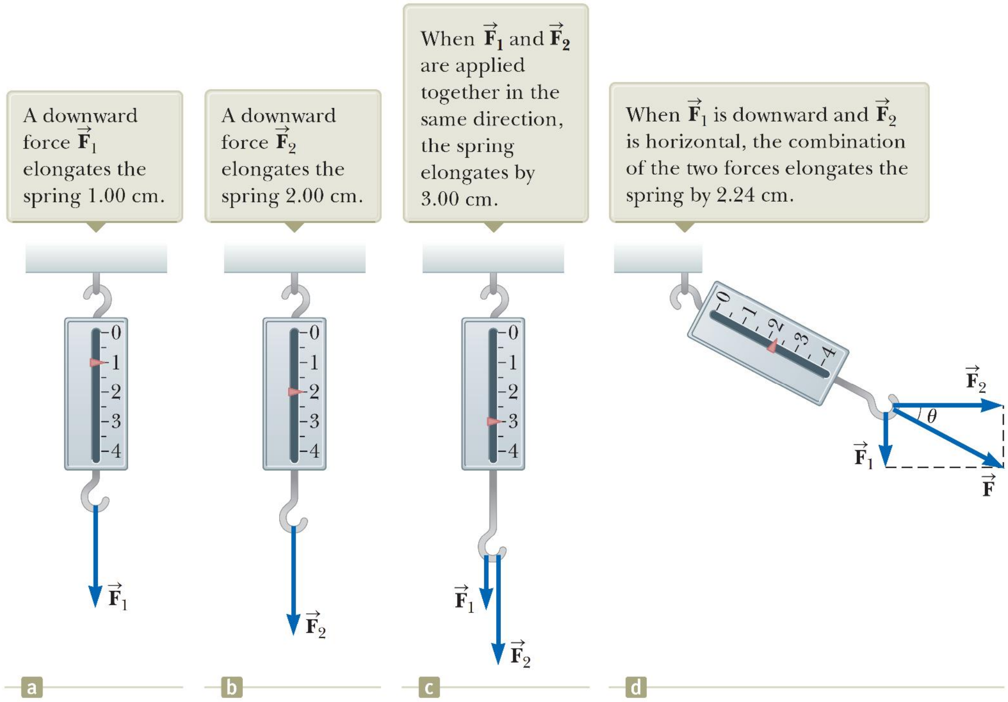



**Figure 5.2** The vector nature of a force is tested with a spring scale.

and its direction is $\theta = \tan^{-1}(-0.500) = -26.6°$. Because forces have been experimentally verified to behave as vectors, you *must* use the rules of vector addition to obtain the net force on an object.

## 5.2 Newton's First Law and Inertial Frames

We begin our study of forces by imagining some physical situations involving a puck on a perfectly level air hockey table (Fig. 5.3). You expect that the puck will remain stationary when it is placed gently at rest on the table. Now imagine your air hockey table is located on a train moving with constant velocity along a perfectly smooth track. If the puck is placed on the table, the puck again remains where it is placed. If the train were to accelerate, however, the puck would start moving along the table opposite the direction of the train's acceleration, just as a set of papers on your dashboard falls onto the floor of your car when you step on the accelerator.

As we saw in Section 4.6, a moving object can be observed from any number of reference frames. **Newton's first law of motion,** sometimes called the *law of inertia,* defines a special set of reference frames called *inertial frames.* This law can be stated as follows:

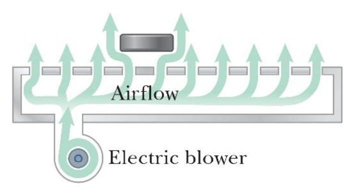



**Figure 5.3** On an air hockey table, air blown through holes in the surface allows the puck to move almost without friction. If the table is not accelerating, a puck placed on the table will remain at rest.

◀ **Newton's first law**

> If an object does not interact with other objects, it is possible to identify a reference frame in which the object has zero acceleration.

◀ **Inertial frame of reference**

Such a reference frame is called an **inertial frame of reference.** When the puck is on the air hockey table located on the ground, you are observing it from an inertial reference frame; there are no horizontal interactions of the puck with any other objects, and you observe it to have zero acceleration in that direction. When you are on the train moving at constant velocity, you are also observing the puck from an inertial reference frame. Any reference frame that moves with constant velocity relative to an inertial frame is itself an inertial frame. When you and the train accelerate, however, you are observing the puck from a **noninertial reference frame** because the train is accelerating relative to the inertial reference frame of the Earth's surface. While the puck appears to be accelerating according to your observations, a reference frame can be identified in which the puck has zero acceleration.

For example, an observer standing outside the train on the ground sees the puck sliding relative to the table but always moving with the same velocity with respect to the ground as the train had before it started to accelerate (because there is almost no friction to "tie" the puck and the train together). Therefore, Newton's first law is still satisfied even though your observations as a rider on the train show an apparent acceleration relative to you.

A reference frame that moves with constant velocity relative to the distant stars is the best approximation of an inertial frame, and for our purposes we can consider the Earth as being such a frame. The Earth is not really an inertial frame because of its orbital motion around the Sun and its rotational motion about its own axis, both of which involve centripetal accelerations. These accelerations are small compared with $g$, however, and can often be neglected. For this reason, we model the Earth as an inertial frame, along with any other frame attached to it.

**Pitfall Prevention 5.1**

**Newton's First Law** Newton's first law does *not* say what happens for an object with *zero net force,* that is, multiple forces that cancel; it says what happens *in the absence of external forces.* This subtle but important difference allows us to define force as that which causes a change in the motion. The description of an object under the effect of forces that balance is covered by Newton's second law.

Let us assume we are observing an object from an inertial reference frame. (We will return to observations made in noninertial reference frames in Section 6.3.) Before about 1600, scientists believed that the natural state of matter was the state of rest. Observations showed that moving objects eventually stopped moving. Galileo was the first to take a different approach to motion and the natural state of matter. He devised thought experiments and concluded that it is not the nature of an object to stop once set in motion: rather, it is its nature to *resist changes in its motion.* In his words, "Any velocity once imparted to a moving body will be rigidly maintained as long as the external causes of retardation are removed." For example, a spacecraft drifting through empty space with its engine turned off will keep moving forever. It would *not* seek a "natural state" of rest.

Given our discussion of observations made from inertial reference frames, we can pose a more practical statement of Newton's first law of motion:

Another statement of Newton's first law ▶

In the absence of external forces and when viewed from an inertial reference frame, an object at rest remains at rest and an object in motion continues in motion with a constant velocity (that is, with a constant speed in a straight line).

In other words, **when no force acts on an object, the acceleration of the object is zero.** From the first law, we conclude that any *isolated object* (one that does not interact with its environment) is either at rest or moving with constant velocity. The tendency of an object to resist any attempt to change its velocity is called **inertia.** Given the statement of the first law above, we can conclude that an object that is accelerating must be experiencing a force. In turn, from the first law, we can define **force** as **that which causes a change in motion of an object.**

Definition of force ▶

**Pitfall Prevention 5.2**

**Force Is the Cause of Changes in Motion** An object can have motion in the absence of forces as described in Newton's first law. Therefore, don't interpret force as the cause of *motion.* Force is the cause of *changes in motion.*

**Quick Quiz 5.1** Which of the following statements is correct? **(a)** It is possible for an object to have motion in the absence of forces on the object. **(b)** It is possible to have forces on an object in the absence of motion of the object. **(c)** Neither statement **(a)** nor statement **(b)** is correct. **(d)** Both statements **(a)** and **(b)** are correct.

## 5.3 Mass

Imagine playing catch with either a basketball or a bowling ball. Which ball is more likely to keep moving when you try to catch it? Which ball requires more effort to throw it? The bowling ball requires more effort. In the language of physics, we say that the bowling ball is more resistant to changes in its velocity than the basketball. How can we quantify this concept?

Definition of mass ▶

**Mass** is that property of an object that specifies how much resistance an object exhibits to changes in its velocity, and as we learned in Section 1.1, the SI unit of mass is the kilogram. Experiments show that the greater the mass of an object, the less that object accelerates under the action of a given applied force.

To describe mass quantitatively, we conduct experiments in which we compare the accelerations a given force produces on different objects. Suppose a force acting on an object of mass $m_1$ produces a change in motion of the object that we can quantify with the object's acceleration $\vec{\mathbf{a}}_1$, and the *same force* acting on an object of mass $m_2$ produces an acceleration $\vec{\mathbf{a}}_2$. The ratio of the two masses is defined as the *inverse* ratio of the magnitudes of the accelerations produced by the force:

$$\frac{m_1}{m_2} \equiv \frac{a_2}{a_1} \tag{5.1}$$

For example, if a given force acting on a 3-kg object produces an acceleration of 4 m/s$^2$, the same force applied to a 6-kg object produces an acceleration of 2 m/s$^2$. According to a huge number of similar observations, we conclude that the magnitude of the acceleration of an object is inversely proportional to its mass when acted on by a given force. If one object has a known mass, the mass of the other object can be obtained from acceleration measurements.

Mass is an inherent property of an object and is independent of the object's surroundings and of the method used to measure it. Also, mass is a scalar quantity and thus obeys the rules of ordinary arithmetic. For example, if you combine a 3-kg mass with a 5-kg mass, the total mass is 8 kg. This result can be verified experimentally by comparing the acceleration that a known force gives to several objects separately with the acceleration that the same force gives to the same objects combined as one unit.

Mass should not be confused with weight. Mass and weight are two different quantities. The weight of an object is equal to the magnitude of the gravitational force exerted on the object and varies with location (see Section 5.5). For example, a person weighing 180 lb on the Earth weighs only about 30 lb on the Moon. On the other hand, the mass of an object is the same everywhere: an object having a mass of 2 kg on the Earth also has a mass of 2 kg on the Moon.

◀ **Mass and weight are different quantities**

## 5.4 Newton's Second Law

Newton's first law explains what happens to an object when no forces act on it: it maintains its original motion; it either remains at rest or moves in a straight line with constant speed. Newton's second law answers the question of what happens to an object when one or more forces act on it.

Imagine performing an experiment in which you push a block of mass $m$ across a frictionless, horizontal surface. When you exert some horizontal force $\vec{\mathbf{F}}$ on the block, it moves with some acceleration $\vec{\mathbf{a}}$. If you apply a force twice as great on the same block, experimental results show that the acceleration of the block doubles; if you increase the applied force to $3\vec{\mathbf{F}}$, the acceleration triples; and so on. From such observations, we conclude that the acceleration of an object is directly proportional to the force acting on it: $\vec{\mathbf{F}} \propto \vec{\mathbf{a}}$. This idea was first introduced in Section 2.4 when we discussed the direction of the acceleration of an object. We also know from the preceding section that the magnitude of the acceleration of an object is inversely proportional to its mass: $|\vec{\mathbf{a}}| \propto 1/m$.

These experimental observations are summarized in **Newton's second law:**

When viewed from an inertial reference frame, the acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass:

$$\vec{\mathbf{a}} \propto \frac{\sum \vec{\mathbf{F}}}{m}$$

**Pitfall Prevention 5.3**

***$m\vec{a}$* Is Not a Force** Equation 5.2 does *not* say that the product $m\vec{\mathbf{a}}$ is a force. All forces on an object are added vectorially to generate the net force on the left side of the equation. This net force is then equated to the product of the mass of the object and the acceleration that results from the net force. Do *not* include an "$m\vec{\mathbf{a}}$ force" in your analysis of the forces on an object.

If we choose a proportionality constant of 1, we can relate mass, acceleration, and force through the following mathematical statement of Newton's second law:[1]

$$\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}} \tag{5.2}$$

◀ **Newton's second law**

[1]Equation 5.2 is valid only when the speed of the object is much less than the speed of light. We treat the relativistic situation in Chapter 39.

In both the textual and mathematical statements of Newton's second law, we have indicated that the acceleration is due to the *net force* $\sum \vec{\mathbf{F}}$ acting on an object. The **net force** on an object is the vector sum of all forces acting on the object. (We sometimes refer to the net force as the *total force,* the *resultant force,* or the *unbalanced force.*) In solving a problem using Newton's second law, it is imperative to determine the correct net force on an object. Many forces may be acting on an object, but there is only one acceleration.

Equation 5.2 is a vector expression and hence is equivalent to three component equations:

Newton's second law: component form

$$\sum F_x = ma_x \qquad \sum F_y = ma_y \qquad \sum F_z = ma_z \tag{5.3}$$

**Quick Quiz 5.2** An object experiences no acceleration. Which of the following *cannot* be true for the object? **(a)** A single force acts on the object. **(b)** No forces act on the object. **(c)** Forces act on the object, but the forces cancel.

**Quick Quiz 5.3** You push an object, initially at rest, across a frictionless floor with a constant force for a time interval $\Delta t$, resulting in a final speed of $v$ for the object. You then repeat the experiment, but with a force that is twice as large. What time interval is now required to reach the same final speed $v$? **(a)** $4\,\Delta t$ **(b)** $2\,\Delta t$ **(c)** $\Delta t$ **(d)** $\Delta t/2$ **(e)** $\Delta t/4$

The SI unit of force is the **newton** (N). A force of 1 N is the force that, when acting on an object of mass 1 kg, produces an acceleration of 1 m/s². From this definition and Newton's second law, we see that the newton can be expressed in terms of the following fundamental units of mass, length, and time:

Definition of the newton

$$1\text{ N} \equiv 1\text{ kg}\cdot\text{m/s}^2 \tag{5.4}$$

In the U.S. customary system, the unit of force is the **pound** (lb). A force of 1 lb is the force that, when acting on a 1-slug mass,[2] produces an acceleration of 1 ft/s²:

$$1\text{ lb} \equiv 1\text{ slug}\cdot\text{ft/s}^2$$

A convenient approximation is $1\text{ N} \approx \frac{1}{4}\text{ lb}$.

## Example 5.1 An Accelerating Hockey Puck AM

A hockey puck having a mass of 0.30 kg slides on the frictionless, horizontal surface of an ice rink. Two hockey sticks strike the puck simultaneously, exerting the forces on the puck shown in Figure 5.4. The force $\vec{\mathbf{F}}_1$ has a magnitude of 5.0 N, and is directed at $\theta = 20°$ below the $x$ axis. The force $\vec{\mathbf{F}}_2$ has a magnitude of 8.0 N and its direction is $\phi = 60°$ above the $x$ axis. Determine both the magnitude and the direction of the puck's acceleration.

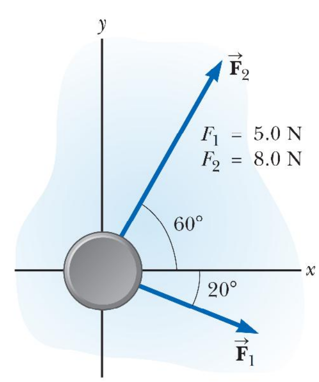



**Figure 5.4** (Example 5.1) A hockey puck moving on a frictionless surface is subject to two forces $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$.

**SOLUTION**

**Conceptualize** Study Figure 5.4. Using your expertise in vector addition from Chapter 3, predict the approximate direction of the net force vector on the puck. The acceleration of the puck will be in the same direction.

**Categorize** Because we can determine a net force and we want an acceleration, this problem is categorized as one that may be solved using Newton's second law. In Section 5.7, we will formally introduce the *particle under a net force* analysis model to describe a situation such as this one.

**Analyze** Find the component of the net force acting on the puck in the $x$ direction:

$$\sum F_x = F_{1x} + F_{2x} = F_1 \cos\theta + F_2 \cos\phi$$

[2]The *slug* is the unit of mass in the U.S. customary system and is that system's counterpart of the SI unit the *kilogram.* Because most of the calculations in our study of classical mechanics are in SI units, the slug is seldom used in this text.

5.1 continued

Find the component of the net force acting on the puck in the $y$ direction:

$$\sum F_y = F_{1y} + F_{2y} = F_1 \sin\theta + F_2 \sin\phi$$

Use Newton's second law in component form (Eq. 5.3) to find the $x$ and $y$ components of the puck's acceleration:

$$a_x = \frac{\sum F_x}{m} = \frac{F_1 \cos\theta + F_2 \cos\phi}{m}$$

$$a_y = \frac{\sum F_y}{m} = \frac{F_1 \sin\theta + F_2 \sin\phi}{m}$$

Substitute numerical values:

$$a_x = \frac{(5.0\text{ N})\cos(-20°) + (8.0\text{ N})\cos(60°)}{0.30\text{ kg}} = 29\text{ m/s}^2$$

$$a_y = \frac{(5.0\text{ N})\sin(-20°) + (8.0\text{ N})\sin(60°)}{0.30\text{ kg}} = 17\text{ m/s}^2$$

Find the magnitude of the acceleration:

$$a = \sqrt{(29\text{ m/s}^2)^2 + (17\text{ m/s}^2)^2} = 34\text{ m/s}^2$$

Find the direction of the acceleration relative to the positive $x$ axis:

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{17}{29}\right) = 31°$$

**Finalize** The vectors in Figure 5.4 can be added graphically to check the reasonableness of our answer. Because the acceleration vector is along the direction of the resultant force, a drawing showing the resultant force vector helps us check the validity of the answer. (Try it!)

**WHAT IF?** Suppose three hockey sticks strike the puck simultaneously, with two of them exerting the forces shown in Figure 5.4. The result of the three forces is that the hockey puck shows *no* acceleration. What must be the components of the third force?

**Answer** If there is zero acceleration, the net force acting on the puck must be zero. Therefore, the three forces must cancel. The components of the third force must be of equal magnitude and opposite sign compared to the components of the net force applied by the first two forces so that all the components add to zero. Therefore, $F_{3x} = -\sum F_x = -(0.30\text{ kg})(29\text{ m/s}^2) = -8.7\text{ N}$ and $F_{3y} = -\sum F_y = -(0.30\text{ kg})(17\text{ m/s}^2) = -5.2\text{ N}$.

## 5.5 The Gravitational Force and Weight

All objects are attracted to the Earth. The attractive force exerted by the Earth on an object is called the **gravitational force** $\vec{\mathbf{F}}_g$. This force is directed toward the center of the Earth,[3] and its magnitude is called the **weight** of the object.

We saw in Section 2.6 that a freely falling object experiences an acceleration $\vec{\mathbf{g}}$ acting toward the center of the Earth. Applying Newton's second law $\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}}$ to a freely falling object of mass $m$, with $\vec{\mathbf{a}} = \vec{\mathbf{g}}$ and $\sum \vec{\mathbf{F}} = \vec{\mathbf{F}}_g$, gives

$$\vec{\mathbf{F}}_g = m\vec{\mathbf{g}} \quad (5.5)$$

Therefore, the weight of an object, being defined as the magnitude of $\vec{\mathbf{F}}_g$, is given by

$$F_g = mg \quad (5.6)$$

Because it depends on $g$, weight varies with geographic location. Because $g$ decreases with increasing distance from the center of the Earth, objects weigh less at higher altitudes than at sea level. For example, a 1 000-kg pallet of bricks used in the construction of the Empire State Building in New York City weighed 9 800 N at street level, but weighed about 1 N less by the time it was lifted from sidewalk level to the top of the building. As another example, suppose a student has a mass

**Pitfall Prevention 5.4**

**"Weight of an Object"** We are familiar with the everyday phrase, the "weight of an object." Weight, however, is not an inherent property of an object; rather, it is a measure of the gravitational force between the object and the Earth (or other planet). Therefore, weight is a property of a *system* of items: the object and the Earth.

**Pitfall Prevention 5.5**

**Kilogram Is Not a Unit of Weight** You may have seen the "conversion" 1 kg = 2.2 lb. Despite popular statements of weights expressed in kilograms, the kilogram is not a unit of *weight*, it is a unit of *mass*. The conversion statement is not an equality; it is an *equivalence* that is valid only on the Earth's surface.

[3]This statement ignores that the mass distribution of the Earth is not perfectly spherical.


The life-support unit strapped to the back of astronaut Harrison Schmitt weighed 300 lb on the Earth and had a mass of 136 kg. During his training, a 50-lb mock-up with a mass of 23 kg was used. Although this strategy effectively simulated the reduced weight the unit would have on the Moon, it did not correctly mimic the unchanging mass. It was more difficult to accelerate the 136-kg unit (perhaps by jumping or twisting suddenly) on the Moon than it was to accelerate the 23-kg unit on the Earth.

of 70.0 kg. The student's weight in a location where $g = 9.80\ \text{m/s}^2$ is 686 N (about 150 lb). At the top of a mountain, however, where $g = 9.77\ \text{m/s}^2$, the student's weight is only 684 N. Therefore, if you want to lose weight without going on a diet, climb a mountain or weigh yourself at 30 000 ft during an airplane flight!

Equation 5.6 quantifies the gravitational force on the object, but notice that this equation does not require the object to be moving. Even for a stationary object or for an object on which several forces act, Equation 5.6 can be used to calculate the magnitude of the gravitational force. The result is a subtle shift in the interpretation of $m$ in the equation. The mass $m$ in Equation 5.6 determines the strength of the gravitational attraction between the object and the Earth. This role is completely different from that previously described for mass, that of measuring the resistance to changes in motion in response to an external force. In that role, mass is also called **inertial mass.** We call $m$ in Equation 5.6 the **gravitational mass.** Even though this quantity is different in behavior from inertial mass, it is one of the experimental conclusions in Newtonian dynamics that gravitational mass and inertial mass have the same value.

Although this discussion has focused on the gravitational force on an object due to the Earth, the concept is generally valid on any planet. The value of $g$ will vary from one planet to the next, but the magnitude of the gravitational force will always be given by the value of $mg$.

**Quick Quiz 5.4** Suppose you are talking by interplanetary telephone to a friend who lives on the Moon. He tells you that he has just won a newton of gold in a contest. Excitedly, you tell him that you entered the Earth version of the same contest and also won a newton of gold! Who is richer? **(a)** You are. **(b)** Your friend is. **(c)** You are equally rich.

### Conceptual Example 5.2 How Much Do You Weigh in an Elevator?

You have most likely been in an elevator that accelerates upward as it moves toward a higher floor. In this case, you feel heavier. In fact, if you are standing on a bathroom scale at the time, the scale measures a force having a magnitude that is greater than your weight. Therefore, you have tactile and measured evidence that leads you to believe you are heavier in this situation. *Are* you heavier?

**SOLUTION**

No; your weight is unchanged. Your experiences are due to your being in a noninertial reference frame. To provide the acceleration upward, the floor or scale must exert on your feet an upward force that is greater in magnitude than your weight. It is this greater force you feel, which you interpret as feeling heavier. The scale reads this upward force, not your weight, and so its reading increases.

## 5.6 Newton's Third Law

If you press against a corner of this textbook with your fingertip, the book pushes back and makes a small dent in your skin. If you push harder, the book does the same and the dent in your skin is a little larger. This simple activity illustrates that forces are *interactions* between two objects: when your finger pushes on the book, the book pushes back on your finger. This important principle is known as **Newton's third law:**

**Newton's third law** ▶

If two objects interact, the force $\vec{\mathbf{F}}_{12}$ exerted by object 1 on object 2 is equal in magnitude and opposite in direction to the force $\vec{\mathbf{F}}_{21}$ exerted by object 2 on object 1:

$$\vec{\mathbf{F}}_{12} = -\vec{\mathbf{F}}_{21} \tag{5.7}$$

When it is important to designate forces as interactions between two objects, we will use this subscript notation, where $\vec{\mathbf{F}}_{ab}$ means "the force exerted *by* a *on* b." The third law is illustrated in Figure 5.5. The force that object 1 exerts on object 2 is popularly called the *action force,* and the force of object 2 on object 1 is called the *reaction force.* These italicized terms are not scientific terms; furthermore, either force can be labeled the action or reaction force. We will use these terms for convenience. In all cases, the action and reaction forces act on *different* objects and must be of the same type (gravitational, electrical, etc.). For example, the force acting on a freely falling projectile is the gravitational force exerted by the Earth on the projectile $\vec{\mathbf{F}}_g = \vec{\mathbf{F}}_{\text{Ep}}$ (E = Earth, p = projectile), and the magnitude of this force is $mg$. The reaction to this force is the gravitational force exerted by the projectile on the Earth $\vec{\mathbf{F}}_{\text{pE}} = -\vec{\mathbf{F}}_{\text{Ep}}$. The reaction force $\vec{\mathbf{F}}_{\text{pE}}$ must accelerate the Earth toward the projectile just as the action force $\vec{\mathbf{F}}_{\text{Ep}}$ accelerates the projectile toward the Earth. Because the Earth has such a large mass, however, its acceleration due to this reaction force is negligibly small.

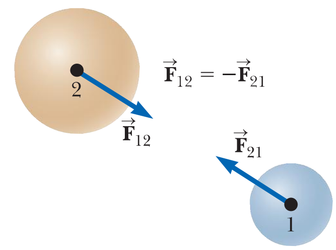



**Figure 5.5** Newton's third law. The force $\vec{\mathbf{F}}_{12}$ exerted by object 1 on object 2 is equal in magnitude and opposite in direction to the force $\vec{\mathbf{F}}_{21}$ exerted by object 2 on object 1.

Consider a computer monitor at rest on a table as in Figure 5.6a. The gravitational force on the monitor is $\vec{\mathbf{F}}_g = \vec{\mathbf{F}}_{\text{Em}}$. The reaction to this force is the force $\vec{\mathbf{F}}_{\text{mE}} = -\vec{\mathbf{F}}_{\text{Em}}$ exerted by the monitor on the Earth. The monitor does not accelerate because it is held up by the table. The table exerts on the monitor an upward force $\vec{\mathbf{n}} = \vec{\mathbf{F}}_{\text{tm}}$, called the **normal force.** (*Normal* in this context means *perpendicular.*) In general, whenever an object is in contact with a surface, the surface exerts a normal force on the object. The normal force on the monitor can have any value needed, up to the point of breaking the table. Because the monitor has zero acceleration, Newton's second law applied to the monitor gives us $\sum \vec{\mathbf{F}} = \vec{\mathbf{n}} + m\vec{\mathbf{g}} = 0$, so $n\hat{\mathbf{j}} - mg\hat{\mathbf{j}} = 0$, or $n = mg$. The normal force balances the gravitational force on the monitor, so the net force on the monitor is zero. The reaction force to $\vec{\mathbf{n}}$ is the force exerted by the monitor downward on the table, $\vec{\mathbf{F}}_{\text{mt}} = -\vec{\mathbf{F}}_{\text{tm}} = -\vec{\mathbf{n}}$.

Notice that the forces acting on the monitor are $\vec{\mathbf{F}}_g$ and $\vec{\mathbf{n}}$ as shown in Figure 5.6b. The two forces $\vec{\mathbf{F}}_{\text{mE}}$ and $\vec{\mathbf{F}}_{\text{mt}}$ are exerted on objects other than the monitor.

Figure 5.6 illustrates an extremely important step in solving problems involving forces. Figure 5.6a shows many of the forces in the situation: those acting on the monitor, one acting on the table, and one acting on the Earth. Figure 5.6b, by contrast, shows only the forces acting on *one object,* the monitor, and is called a **force diagram** or a *diagram showing the forces on the object.* The important pictorial representation in Figure 5.6c is called a **free-body diagram.** In a free-body diagram, the particle model is used by representing the object as a dot and showing the forces that act on the object as being applied to the dot. When analyzing an object subject to forces, we are interested in the net force acting on one object, which we will model as a particle. Therefore, a free-body diagram helps us isolate only those forces on the object and eliminate the other forces from our analysis.

### Pitfall Prevention 5.6

**$n$ Does Not Always Equal $mg$** In the situation shown in Figure 5.6 and in many others, we find that $n = mg$ (the normal force has the same magnitude as the gravitational force). This result, however, is *not* generally true. If an object is on an incline, if there are applied forces with vertical components, or if there is a vertical acceleration of the system, then $n \neq mg$. *Always* apply Newton's second law to find the relationship between $n$ and $mg$.

### Pitfall Prevention 5.7

**Newton's Third Law** Remember that Newton's third-law action and reaction forces act on *different* objects. For example, in Figure 5.6, $\vec{\mathbf{n}} = \vec{\mathbf{F}}_{\text{tm}} = -m\vec{\mathbf{g}} = -\vec{\mathbf{F}}_{\text{Em}}$. The forces $\vec{\mathbf{n}}$ and $m\vec{\mathbf{g}}$ are equal in magnitude and opposite in direction, but they do not represent an action–reaction pair because both forces act on the *same* object, the monitor.

### Pitfall Prevention 5.8

**Free-Body Diagrams** The *most important* step in solving a problem using Newton's laws is to draw a proper sketch, the free-body diagram. Be sure to draw *only* those forces that act on the object you are isolating. Be sure to draw *all* forces acting on the object, including any field forces, such as the gravitational force.

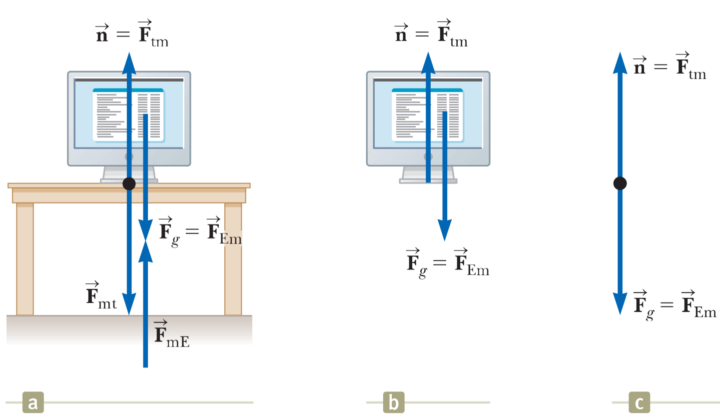



**Figure 5.6** (a) When a computer monitor is at rest on a table, the forces acting on the monitor are the normal force $\vec{\mathbf{n}}$ and the gravitational force $\vec{\mathbf{F}}_g$. The reaction to $\vec{\mathbf{n}}$ is the force $\vec{\mathbf{F}}_{\text{mt}}$ exerted by the monitor on the table. The reaction to $\vec{\mathbf{F}}_g$ is the force $\vec{\mathbf{F}}_{\text{mE}}$ exerted by the monitor on the Earth. (b) A *force diagram* shows the forces on the monitor. (c) A *free-body diagram* shows the monitor as a black dot with the forces acting on it.

**Quick Quiz 5.5** **(i)** If a fly collides with the windshield of a fast-moving bus, which experiences an impact force with a larger magnitude? (a) The fly. (b) The bus. (c) The same force is experienced by both. **(ii)** Which experiences the greater acceleration? (a) The fly. (b) The bus. (c) The same acceleration is experienced by both.

### Conceptual Example 5.3 You Push Me and I'll Push You

A large man and a small boy stand facing each other on frictionless ice. They put their hands together and push against each other so that they move apart.

**(A)** Who moves away with the higher speed?

**SOLUTION**

This situation is similar to what we saw in Quick Quiz 5.5. According to Newton's third law, the force exerted by the man on the boy and the force exerted by the boy on the man are a third-law pair of forces, so they must be equal in magnitude. (A bathroom scale placed between their hands would read the same, regardless of which way it faced.) Therefore, the boy, having the smaller mass, experiences the greater acceleration. Both individuals accelerate for the same amount of time, but the greater acceleration of the boy over this time interval results in his moving away from the interaction with the higher speed.

**(B)** Who moves farther while their hands are in contact?

**SOLUTION**

Because the boy has the greater acceleration and therefore the greater average velocity, he moves farther than the man during the time interval during which their hands are in contact.

## 5.7 Analysis Models Using Newton's Second Law

In this section, we discuss two analysis models for solving problems in which objects are either in equilibrium ($\vec{\mathbf{a}} = 0$) or accelerating under the action of constant external forces. Remember that when Newton's laws are applied to an object, we are interested only in external forces that act on the object. If the objects are modeled as particles, we need not worry about rotational motion. For now, we also neglect the effects of friction in those problems involving motion, which is equivalent to stating that the surfaces are *frictionless.* (The friction force is discussed in Section 5.8.)

We usually neglect the mass of any ropes, strings, or cables involved. In this approximation, the magnitude of the force exerted by any element of the rope on the adjacent element is the same for all elements along the rope. In problem statements, the synonymous terms *light* and *of negligible mass* are used to indicate that a mass is to be ignored when you work the problems. When a rope attached to an object is pulling on the object, the rope exerts a force on the object in a direction away from the object, parallel to the rope. The magnitude $T$ of that force is called the **tension** in the rope. Because it is the magnitude of a vector quantity, tension is a scalar quantity.

### Analysis Model: The Particle in Equilibrium

If the acceleration of an object modeled as a particle is zero, the object is treated with the **particle in equilibrium** model. In this model, the net force on the object is zero:

$$\sum \vec{\mathbf{F}} = 0 \tag{5.8}$$

Consider a lamp suspended from a light chain fastened to the ceiling as in Figure 5.7a. The force diagram for the lamp (Fig. 5.7b) shows that the forces acting on the lamp are the downward gravitational force $\vec{\mathbf{F}}_g$ and the upward force $\vec{\mathbf{T}}$ exerted by the chain. Because there are no forces in the $x$ direction, $\Sigma F_x = 0$ provides no helpful information. The condition $\Sigma F_y = 0$ gives

$$\sum F_y = T - F_g = 0 \text{ or } T = F_g$$

Again, notice that $\vec{\mathbf{T}}$ and $\vec{\mathbf{F}}_g$ are *not* an action–reaction pair because they act on the same object, the lamp. The reaction force to $\vec{\mathbf{T}}$ is a downward force exerted by the lamp on the chain.

Example 5.4 (page 122) shows an application of the particle in equilibrium model.

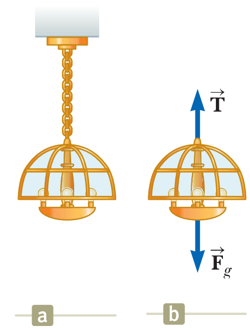



**Figure 5.7** (a) A lamp suspended from a ceiling by a chain of negligible mass. (b) The forces acting on the lamp are the gravitational force $\vec{\mathbf{F}}_g$ and the force $\vec{\mathbf{T}}$ exerted by the chain.

## Analysis Model: The Particle Under a Net Force

If an object experiences an acceleration, its motion can be analyzed with the **particle under a net force** model. The appropriate equation for this model is Newton's second law, Equation 5.2:

$$\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}} \quad (5.2)$$

Consider a crate being pulled to the right on a frictionless, horizontal floor as in Figure 5.8a. Of course, the floor directly under the boy must have friction; otherwise, his feet would simply slip when he tries to pull on the crate! Suppose you wish to find the acceleration of the crate and the force the floor exerts on it. The forces acting on the crate are illustrated in the free-body diagram in Figure 5.8b. Notice that the horizontal force $\vec{\mathbf{T}}$ being applied to the crate acts through the rope. The magnitude of $\vec{\mathbf{T}}$ is equal to the tension in the rope. In addition to the force $\vec{\mathbf{T}}$, the free-body diagram for the crate includes the gravitational force $\vec{\mathbf{F}}_g$ and the normal force $\vec{\mathbf{n}}$ exerted by the floor on the crate.

We can now apply Newton's second law in component form to the crate. The only force acting in the $x$ direction is $\vec{\mathbf{T}}$. Applying $\Sigma F_x = ma_x$ to the horizontal motion gives

$$\sum F_x = T = ma_x \quad \text{or} \quad a_x = \frac{T}{m}$$

No acceleration occurs in the $y$ direction because the crate moves only horizontally. Therefore, we use the particle in equilibrium model in the $y$ direction. Applying the $y$ component of Equation 5.8 yields

$$\sum F_y = n - F_g = 0 \quad \text{or} \quad n = F_g$$

That is, the normal force has the same magnitude as the gravitational force but acts in the opposite direction.

If $\vec{\mathbf{T}}$ is a constant force, the acceleration $a_x = T/m$ also is constant. Hence, the crate is also modeled as a particle under constant acceleration in the $x$ direction, and the equations of kinematics from Chapter 2 can be used to obtain the crate's position $x$ and velocity $v_x$ as functions of time.

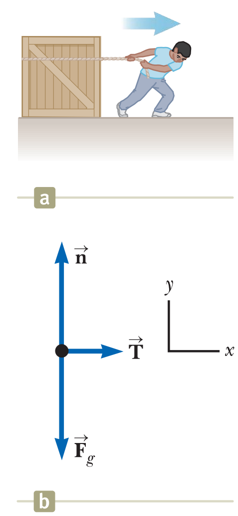



**Figure 5.8** (a) A crate being pulled to the right on a frictionless floor. (b) The free-body diagram representing the external forces acting on the crate.

Notice from this discussion two concepts that will be important in future problem solving: (1) *In a given problem, it is possible to have different analysis models applied in different directions.* The crate in Figure 5.8 is a particle in equilibrium in the vertical direction and a particle under a net force in the horizontal direction. (2) *It is possible to describe an object by multiple analysis models.* The crate is a particle under a net force in the horizontal direction and is also a particle under constant acceleration in the same direction.

In the situation just described, the magnitude of the normal force $\vec{\mathbf{n}}$ is equal to the magnitude of $\vec{\mathbf{F}}_g$, but that is not always the case, as noted in Pitfall Prevention 5.6. For example, suppose a book is lying on a table and you push down on the book with a force $\vec{\mathbf{F}}$ as in Figure 5.9. Because the book is at rest and therefore not accelerating, $\Sigma F_y = 0$, which gives $n - F_g - F = 0$, or $n = F_g + F = mg + F$. In this situation, the normal force is *greater* than the gravitational force. Other examples in which $n \neq F_g$ are presented later.

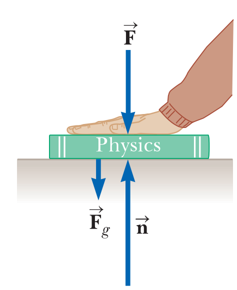


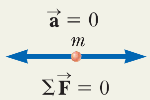

**Figure 5.9** When a force $\vec{\mathbf{F}}$ pushes vertically downward on another object, the normal force $\vec{\mathbf{n}}$ on the object is greater than the gravitational force: $n = F_g + F$.

Several examples below demonstrate the use of the particle under a net force model.

## Analysis Model Particle in Equilibrium

Imagine an object that can be modeled as a particle. If it has several forces acting on it so that the forces all cancel, giving a net force of zero, the object will have an acceleration of zero. This condition is mathematically described as

$$\sum \vec{\mathbf{F}} = 0 \qquad (5.8)$$



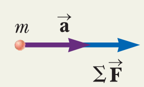

**Examples**

- a chandelier hanging over a dining room table
- an object moving at terminal speed through a viscous medium (Chapter 6)
- a steel beam in the frame of a building (Chapter 12)
- a boat floating on a body of water (Chapter 14)

## Analysis Model Particle Under a Net Force

Imagine an object that can be modeled as a particle. If it has one or more forces acting on it so that there is a net force on the object, it will accelerate in the direction of the net force. The relationship between the net force and the acceleration is

$$\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}} \qquad (5.2)$$




**Examples**

- a crate pushed across a factory floor
- a falling object acted upon by a gravitational force
- a piston in an automobile engine pushed by hot gases (Chapter 22)
- a charged particle in an electric field (Chapter 23)

## Example 5.4 A Traffic Light at Rest AM

A traffic light weighing 122 N hangs from a cable tied to two other cables fastened to a support as in Figure 5.10a. The upper cables make angles of $\theta_1 = 37.0°$ and $\theta_2 = 53.0°$ with the horizontal. These upper cables are not as strong as the vertical cable and will break if the tension in them exceeds 100 N. Does the traffic light remain hanging in this situation, or will one of the cables break?

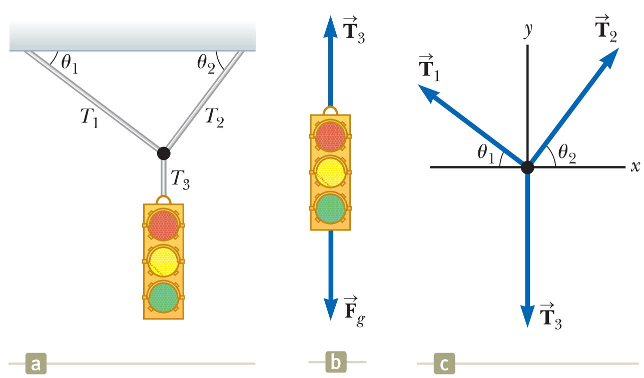



**Figure 5.10** (Example 5.4) (a) A traffic light suspended by cables. (b) The forces acting on the traffic light. (c) The free-body diagram for the knot where the three cables are joined.

**SOLUTION**

**Conceptualize** Inspect the drawing in Figure 5.10a. Let us assume the cables do not break and nothing is moving.

**Categorize** If nothing is moving, no part of the system is accelerating. We can now model the light as a *particle in equilibrium* on which the net force is zero. Similarly, the net force on the knot (Fig. 5.10c) is zero, so it is also modeled as a *particle in equilibrium*.

**Analyze** We construct a diagram of the forces acting on the traffic light, shown in Figure 5.10b, and a free-body diagram for the knot that holds the three cables together, shown in Figure 5.10c. This knot is a convenient object to choose because all the forces of interest act along lines passing through the knot.

From the particle in equilibrium model, apply Equation 5.8 for the traffic light in the $y$ direction:

$$\sum F_y = 0 \ \rightarrow \ T_3 - F_g = 0$$

$$T_3 = F_g$$

5.4 continued

Choose the coordinate axes as shown in Figure 5.10c and resolve the forces acting on the knot into their components:

| Force | x Component | y Component |
|---|---|---|
| $\vec{\mathbf{T}}_1$ | $-T_1 \cos \theta_1$ | $T_1 \sin \theta_1$ |
| $\vec{\mathbf{T}}_2$ | $T_2 \cos \theta_2$ | $T_2 \sin \theta_2$ |
| $\vec{\mathbf{T}}_3$ | 0 | $-F_g$ |

Apply the particle in equilibrium model to the knot:

$$(1)\quad \sum F_x = -T_1 \cos \theta_1 + T_2 \cos \theta_2 = 0$$

$$(2)\quad \sum F_y = T_1 \sin \theta_1 + T_2 \sin \theta_2 + (-F_g) = 0$$

Equation (1) shows that the horizontal components of $\vec{\mathbf{T}}_1$ and $\vec{\mathbf{T}}_2$ must be equal in magnitude, and Equation (2) shows that the sum of the vertical components of $\vec{\mathbf{T}}_1$ and $\vec{\mathbf{T}}_2$ must balance the downward force $\vec{\mathbf{T}}_3$, which is equal in magnitude to the weight of the light.

Solve Equation (1) for $T_2$ in terms of $T_1$:

$$(3)\quad T_2 = T_1\left(\frac{\cos \theta_1}{\cos \theta_2}\right)$$

Substitute this value for $T_2$ into Equation (2):

$$T_1 \sin \theta_1 + T_1\left(\frac{\cos \theta_1}{\cos \theta_2}\right)(\sin \theta_2) - F_g = 0$$

Solve for $T_1$:

$$T_1 = \frac{F_g}{\sin \theta_1 + \cos \theta_1 \tan \theta_2}$$

Substitute numerical values:

$$T_1 = \frac{122 \text{ N}}{\sin 37.0° + \cos 37.0° \tan 53.0°} = 73.4 \text{ N}$$

Using Equation (3), solve for $T_2$:

$$T_2 = (73.4 \text{ N})\left(\frac{\cos 37.0°}{\cos 53.0°}\right) = 97.4 \text{ N}$$

Both values are less than 100 N (just barely for $T_2$), so the cables will not break.

**Finalize** Let us finalize this problem by imagining a change in the system, as in the following What If?

**WHAT IF?** Suppose the two angles in Figure 5.10a are equal. What would be the relationship between $T_1$ and $T_2$?

**Answer** We can argue from the symmetry of the problem that the two tensions $T_1$ and $T_2$ would be equal to each other. Mathematically, if the equal angles are called $\theta$, Equation (3) becomes

$$T_2 = T_1\left(\frac{\cos \theta}{\cos \theta}\right) = T_1$$

which also tells us that the tensions are equal. Without knowing the specific value of $\theta$, we cannot find the values of $T_1$ and $T_2$. The tensions will be equal to each other, however, regardless of the value of $\theta$.

## Conceptual Example 5.5 Forces Between Cars in a Train

Train cars are connected by *couplers,* which are under tension as the locomotive pulls the train. Imagine you are on a train speeding up with a constant acceleration. As you move through the train from the locomotive to the last car, measuring the tension in each set of couplers, does the tension increase, decrease, or stay the same? When the engineer applies the brakes, the couplers are under compression. How does this compression force vary from the locomotive to the last car? (Assume only the brakes on the wheels of the engine are applied.)

**SOLUTION**

While the train is speeding up, tension decreases from the front of the train to the back. The coupler between the locomotive and the first car must apply enough force to accelerate the rest of the cars. As you move back along the

*continued*

5.5 continued

train, each coupler is accelerating less mass behind it. The last coupler has to accelerate only the last car, and so it is under the least tension.

When the brakes are applied, the force again decreases from front to back. The coupler connecting the locomotive to the first car must apply a large force to slow down the rest of the cars, but the final coupler must apply a force large enough to slow down only the last car.

## Example 5.6 The Runaway Car AM

A car of mass $m$ is on an icy driveway inclined at an angle $\theta$ as in Figure 5.11a.

**(A)** Find the acceleration of the car, assuming the driveway is frictionless.

### SOLUTION

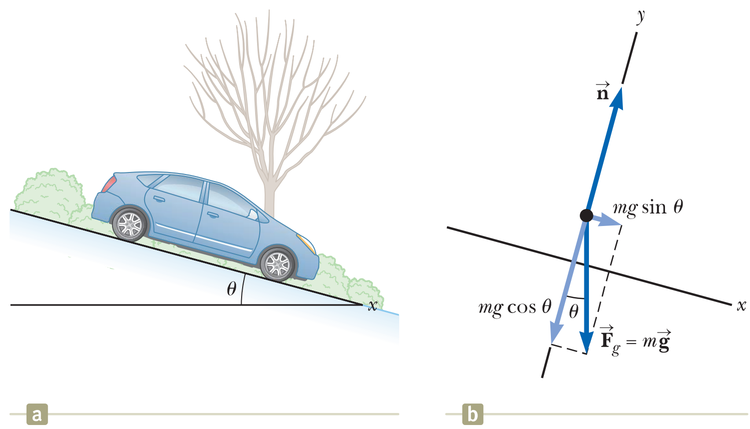



**Figure 5.11** (Example 5.6) (a) A car on a frictionless incline. (b) The free-body diagram for the car. The black dot represents the position of the center of mass of the car. We will learn about center of mass in Chapter 9.

**Conceptualize** Use Figure 5.11a to conceptualize the situation. From everyday experience, we know that a car on an icy incline will accelerate down the incline. (The same thing happens to a car on a hill with its brakes not set.)

**Categorize** We categorize the car as a *particle under a net force* because it accelerates. Furthermore, this example belongs to a very common category of problems in which an object moves under the influence of gravity on an inclined plane.

**Analyze** Figure 5.11b shows the free-body diagram for the car. The only forces acting on the car are the normal force $\vec{n}$ exerted by the inclined plane, which acts perpendicular to the plane, and the gravitational force $\vec{F}_g = m\vec{g}$, which acts vertically downward. For problems involving inclined planes, it is convenient to choose the coordinate axes with $x$ along the incline and $y$ perpendicular to it as in Figure 5.11b. With these axes, we represent the gravitational force by a component of magnitude $mg \sin\theta$ along the positive $x$ axis and one of magnitude $mg \cos\theta$ along the negative $y$ axis. Our choice of axes results in the car being modeled as a particle under a net force in the $x$ direction and a particle in equilibrium in the $y$ direction.

Apply these models to the car:

$$(1)\quad \sum F_x = mg \sin\theta = ma_x$$

$$(2)\quad \sum F_y = n - mg\cos\theta = 0$$

Solve Equation (1) for $a_x$:

$$(3)\quad a_x = g \sin\theta$$

**Finalize** Note that the acceleration component $a_x$ is independent of the mass of the car! It depends only on the angle of inclination and on $g$.

From Equation (2), we conclude that the component of $\vec{F}_g$ perpendicular to the incline is balanced by the normal force; that is, $n = mg\cos\theta$. This situation is a case in which the normal force is *not* equal in magnitude to the weight of the object (as discussed in Pitfall Prevention 5.6 on page 119).

It is possible, although inconvenient, to solve the problem with "standard" horizontal and vertical axes. You may want to try it, just for practice.

**(B)** Suppose the car is released from rest at the top of the incline and the distance from the car's front bumper to the bottom of the incline is $d$. How long does it take the front bumper to reach the bottom of the hill, and what is the car's speed as it arrives there?

5.6 continued

**SOLUTION**

**Conceptualize** Imagine the car is sliding down the hill and you use a stopwatch to measure the entire time interval until it reaches the bottom.

**Categorize** This part of the problem belongs to kinematics rather than to dynamics, and Equation (3) shows that the acceleration $a_x$ is constant. Therefore, you should categorize the car in this part of the problem as a particle under constant acceleration.

**Analyze** Defining the initial position of the front bumper as $x_i = 0$ and its final position as $x_f = d$, and recognizing that $v_{xi} = 0$, choose Equation 2.16 from the particle under constant acceleration model, $x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$:

$$d = \tfrac{1}{2}a_x t^2$$

Solve for $t$:

$$(4)\quad t = \sqrt{\frac{2d}{a_x}} = \sqrt{\frac{2d}{g\sin\theta}}$$

Use Equation 2.17, with $v_{xi} = 0$, to find the final velocity of the car:

$$v_{xf}^{\,2} = 2a_x d$$

$$(5)\quad v_{xf} = \sqrt{2a_x d} = \sqrt{2gd\sin\theta}$$

**Finalize** We see from Equations (4) and (5) that the time $t$ at which the car reaches the bottom and its final speed $v_{xf}$ are independent of the car's mass, as was its acceleration. Notice that we have combined techniques from Chapter 2 with new techniques from this chapter in this example. As we learn more techniques in later chapters, this process of combining analysis models and information from several parts of the book will occur more often. In these cases, use the General Problem-Solving Strategy to help you identify what analysis models you will need.

**WHAT IF?** What previously solved problem does this situation become if $\theta = 90°$?

**Answer** Imagine $\theta$ going to 90° in Figure 5.11. The inclined plane becomes vertical, and the car is an object in free fall! Equation (3) becomes

$$a_x = g\sin\theta = g\sin 90° = g$$

which is indeed the free-fall acceleration. (We find $a_x = g$ rather than $a_x = -g$ because we have chosen positive $x$ to be downward in Fig. 5.11.) Notice also that the condition $n = mg\cos\theta$ gives us $n = mg\cos 90° = 0$. That is consistent with the car falling downward *next to* the vertical plane, in which case there is no contact force between the car and the plane.

## Example 5.7 One Block Pushes Another AM

Two blocks of masses $m_1$ and $m_2$, with $m_1 > m_2$, are placed in contact with each other on a frictionless, horizontal surface as in Figure 5.12a. A constant horizontal force $\vec{\mathbf{F}}$ is applied to $m_1$ as shown.

**(A)** Find the magnitude of the acceleration of the system.

**SOLUTION**

**Conceptualize** Conceptualize the situation by using Figure 5.12a and realize that both blocks must experience the *same* acceleration because they are in contact with each other and remain in contact throughout the motion.

**Categorize** We categorize this problem as one involving a *particle under a net force* because a force is applied to a system of blocks and we are looking for the acceleration of the system.

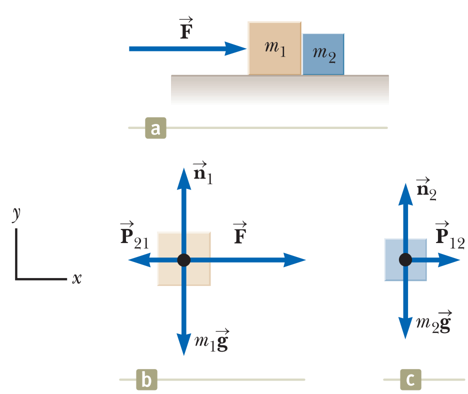



**Figure 5.12** (Example 5.7) (a) A force is applied to a block of mass $m_1$, which pushes on a second block of mass $m_2$. (b) The forces acting on $m_1$. (c) The forces acting on $m_2$.

*continued*

5.7 continued

**Analyze** First model the combination of two blocks as a single particle under a net force. Apply Newton's second law to the combination in the $x$ direction to find the acceleration:

$$\sum F_x = F = (m_1 + m_2)a_x$$

$$(1) \quad a_x = \frac{F}{m_1 + m_2}$$

**Finalize** The acceleration given by Equation (1) is the same as that of a single object of mass $m_1 + m_2$ and subject to the same force.

**(B)** Determine the magnitude of the contact force between the two blocks.

**SOLUTION**

**Conceptualize** The contact force is internal to the system of two blocks. Therefore, we cannot find this force by modeling the whole system (the two blocks) as a single particle.

**Categorize** Now consider each of the two blocks individually by categorizing each as a *particle under a net force.*

**Analyze** We construct a diagram of forces acting on the object for each block as shown in Figures 5.12b and 5.12c, where the contact force is denoted by $\vec{\mathbf{P}}$. From Figure 5.12c, we see that the only horizontal force acting on $m_2$ is the contact force $\vec{\mathbf{P}}_{12}$ (the force exerted by $m_1$ on $m_2$), which is directed to the right.

Apply Newton's second law to $m_2$:

$$(2) \quad \sum F_x = P_{12} = m_2 a_x$$

Substitute the value of the acceleration $a_x$ given by Equation (1) into Equation (2):

$$(3) \quad P_{12} = m_2 a_x = \left(\frac{m_2}{m_1 + m_2}\right)F$$

**Finalize** This result shows that the contact force $P_{12}$ is *less* than the applied force $F$. The force required to accelerate block 2 alone must be less than the force required to produce the same acceleration for the two-block system.

To finalize further, let us check this expression for $P_{12}$ by considering the forces acting on $m_1$, shown in Figure 5.12b. The horizontal forces acting on $m_1$ are the applied force $\vec{\mathbf{F}}$ to the right and the contact force $\vec{\mathbf{P}}_{21}$ to the left (the force exerted by $m_2$ on $m_1$). From Newton's third law, $\vec{\mathbf{P}}_{21}$ is the reaction force to $\vec{\mathbf{P}}_{12}$, so $P_{21} = P_{12}$.

Apply Newton's second law to $m_1$:

$$(4) \quad \sum F_x = F - P_{21} = F - P_{12} = m_1 a_x$$

Solve for $P_{12}$ and substitute the value of $a_x$ from Equation (1):

$$P_{12} = F - m_1 a_x = F - m_1\left(\frac{F}{m_1 + m_2}\right) = \left(\frac{m_2}{m_1 + m_2}\right)F$$

This result agrees with Equation (3), as it must.

**WHAT IF?** Imagine that the force $\vec{\mathbf{F}}$ in Figure 5.12 is applied toward the left on the right-hand block of mass $m_2$. Is the magnitude of the force $\vec{\mathbf{P}}_{12}$ the same as it was when the force was applied toward the right on $m_1$?

**Answer** When the force is applied toward the left on $m_2$, the contact force must accelerate $m_1$. In the original situation, the contact force accelerates $m_2$. Because $m_1 > m_2$, more force is required, so the magnitude of $\vec{\mathbf{P}}_{12}$ is greater than in the original situation. To see this mathematically, modify Equation (4) appropriately and solve for $\vec{\mathbf{P}}_{12}$.

## Example 5.8 Weighing a Fish in an Elevator AM

A person weighs a fish of mass $m$ on a spring scale attached to the ceiling of an elevator as illustrated in Figure 5.13.

**(A)** Show that if the elevator accelerates either upward or downward, the spring scale gives a reading that is different from the weight of the fish.

5.8 continued

**SOLUTION**

**Conceptualize** The reading on the scale is related to the extension of the spring in the scale, which is related to the force on the end of the spring as in Figure 5.2. Imagine that the fish is hanging on a string attached to the end of the spring. In this case, the magnitude of the force exerted on the spring is equal to the tension $T$ in the string. Therefore, we are looking for $T$. The force $\vec{\mathbf{T}}$ pulls down on the string and pulls up on the fish.

**Categorize** We can categorize this problem by identifying the fish as a *particle in equilibrium* if the elevator is not accelerating or as a *particle under a net force* if the elevator is accelerating.

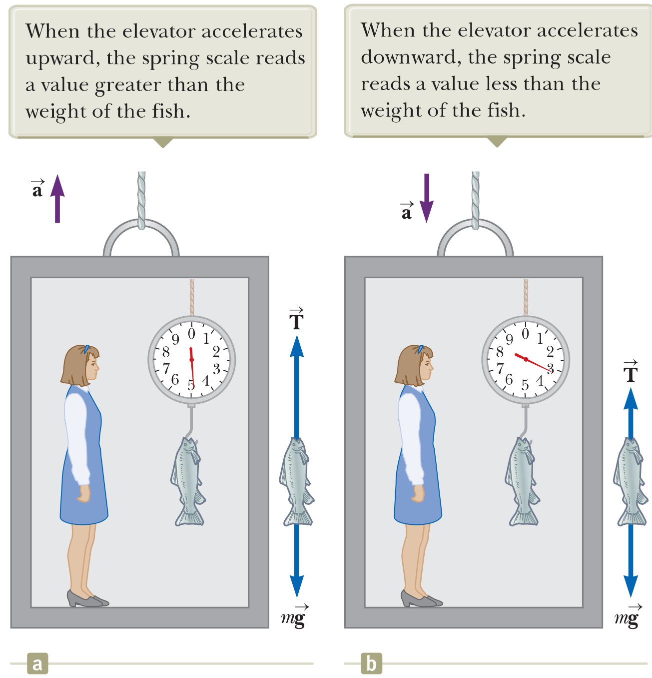



**Figure 5.13** (Example 5.8) A fish is weighed on a spring scale in an accelerating elevator car.

**Analyze** Inspect the diagrams of the forces acting on the fish in Figure 5.13 and notice that the external forces acting on the fish are the downward gravitational force $\vec{\mathbf{F}}_g = m\vec{\mathbf{g}}$ and the force $\vec{\mathbf{T}}$ exerted by the string. If the elevator is either at rest or moving at constant velocity, the fish is a particle in equilibrium, so $\sum F_y = T - F_g = 0$ or $T = F_g = mg$. (Remember that the scalar $mg$ is the weight of the fish.)

Now suppose the elevator is moving with an acceleration $\vec{\mathbf{a}}$ relative to an observer standing outside the elevator in an inertial frame. The fish is now a particle under a net force.

Apply Newton's second law to the fish:

$$\sum F_y = T - mg = ma_y$$

Solve for $T$:

$$(1) \quad T = ma_y + mg = mg\left(\frac{a_y}{g} + 1\right) = F_g\left(\frac{a_y}{g} + 1\right)$$

where we have chosen upward as the positive $y$ direction. We conclude from Equation (1) that the scale reading $T$ is greater than the fish's weight $mg$ if $\vec{\mathbf{a}}$ is upward, so $a_y$ is positive (Fig. 5.13a), and that the reading is less than $mg$ if $\vec{\mathbf{a}}$ is downward, so $a_y$ is negative (Fig. 5.13b).

**(B)** Evaluate the scale readings for a 40.0-N fish if the elevator moves with an acceleration $a_y = \pm 2.00 \text{ m/s}^2$.

**SOLUTION**

Evaluate the scale reading from Equation (1) if $\vec{\mathbf{a}}$ is upward:

$$T = (40.0 \text{ N})\left(\frac{2.00 \text{ m/s}^2}{9.80 \text{ m/s}^2} + 1\right) = 48.2 \text{ N}$$

Evaluate the scale reading from Equation (1) if $\vec{\mathbf{a}}$ is downward:

$$T = (40.0 \text{ N})\left(\frac{-2.00 \text{ m/s}^2}{9.80 \text{ m/s}^2} + 1\right) = 31.8 \text{ N}$$

**Finalize** Take this advice: if you buy a fish in an elevator, make sure the fish is weighed while the elevator is either at rest or accelerating downward! Furthermore, notice that from the information given here, one cannot determine the direction of the velocity of the elevator.

**WHAT IF?** Suppose the elevator cable breaks and the elevator and its contents are in free fall. What happens to the reading on the scale?

**Answer** If the elevator falls freely, the fish's acceleration is $a_y = -g$. We see from Equation (1) that the scale reading $T$ is zero in this case; that is, the fish *appears* to be weightless.

## Example 5.9 The Atwood Machine AM

When two objects of unequal mass are hung vertically over a frictionless pulley of negligible mass as in Figure 5.14a, the arrangement is called an *Atwood machine*. The device is sometimes used in the laboratory to determine the value of $g$. Determine the magnitude of the acceleration of the two objects and the tension in the lightweight string.

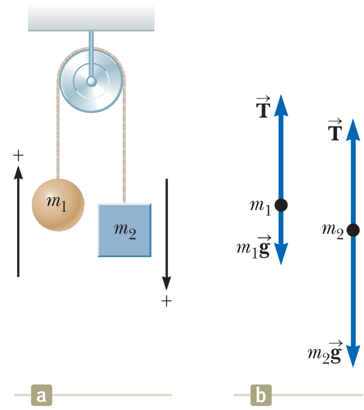



**Figure 5.14** (Example 5.9) The Atwood machine. (a) Two objects connected by a massless inextensible string over a frictionless pulley. (b) The free-body diagrams for the two objects.

### SOLUTION

**Conceptualize** Imagine the situation pictured in Figure 5.14a in action: as one object moves upward, the other object moves downward. Because the objects are connected by an inextensible string, their accelerations must be of equal magnitude.

**Categorize** The objects in the Atwood machine are subject to the gravitational force as well as to the forces exerted by the strings connected to them. Therefore, we can categorize this problem as one involving two *particles under a net force*.

**Analyze** The free-body diagrams for the two objects are shown in Figure 5.14b. Two forces act on each object: the upward force $\vec{\mathbf{T}}$ exerted by the string and the downward gravitational force. In problems such as this one in which the pulley is modeled as massless and frictionless, the tension in the string on both sides of the pulley is the same. If the pulley has mass or is subject to friction, the tensions on either side are not the same and the situation requires techniques we will learn in Chapter 10.

We must be very careful with signs in problems such as this one. In Figure 5.14a, notice that if object 1 accelerates upward, object 2 accelerates downward. Therefore, for consistency with signs, if we define the upward direction as positive for object 1, we must define the downward direction as positive for object 2. With this sign convention, both objects accelerate in the same direction as defined by the choice of sign. Furthermore, according to this sign convention, the $y$ component of the net force exerted on object 1 is $T - m_1g$, and the $y$ component of the net force exerted on object 2 is $m_2g - T$.

From the particle under a net force model, apply Newton's second law to object 1:

$$(1) \quad \sum F_y = T - m_1g = m_1a_y$$

Apply Newton's second law to object 2:

$$(2) \quad \sum F_y = m_2g - T = m_2a_y$$

Add Equation (2) to Equation (1), noticing that $T$ cancels:

$$-m_1g + m_2g = m_1a_y + m_2a_y$$

Solve for the acceleration:

$$(3) \quad a_y = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)g$$

Substitute Equation (3) into Equation (1) to find $T$:

$$(4) \quad T = m_1(g + a_y) = \left(\frac{2m_1m_2}{m_1 + m_2}\right)g$$

**Finalize** The acceleration given by Equation (3) can be interpreted as the ratio of the magnitude of the unbalanced force on the system $(m_2 - m_1)g$ to the total mass of the system $(m_1 + m_2)$, as expected from Newton's second law. Notice that the sign of the acceleration depends on the relative masses of the two objects.

**WHAT IF?** Describe the motion of the system if the objects have equal masses, that is, $m_1 = m_2$.

**Answer** If we have the same mass on both sides, the system is balanced and should not accelerate. Mathematically, we see that if $m_1 = m_2$, Equation (3) gives us $a_y = 0$.

**WHAT IF?** What if one of the masses is much larger than the other: $m_1 >> m_2$?

**Answer** In the case in which one mass is infinitely larger than the other, we can ignore the effect of the smaller mass. Therefore, the larger mass should simply fall as if the smaller mass were not there. We see that if $m_1 >> m_2$, Equation (3) gives us $a_y = -g$.

## Example 5.10 Acceleration of Two Objects Connected by a Cord AM

A ball of mass $m_1$ and a block of mass $m_2$ are attached by a lightweight cord that passes over a frictionless pulley of negligible mass as in Figure 5.15a. The block lies on a frictionless incline of angle $\theta$. Find the magnitude of the acceleration of the two objects and the tension in the cord.

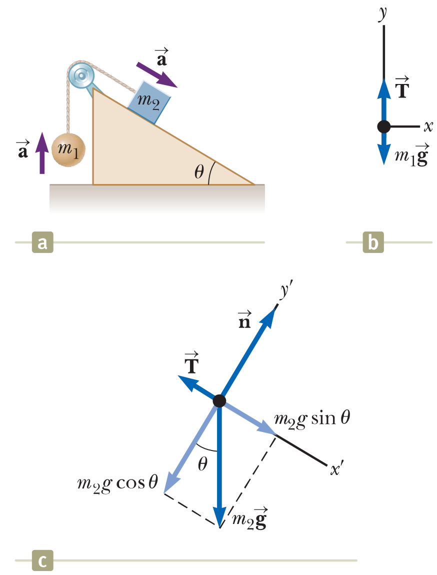



**Figure 5.15** (Example 5.10) (a) Two objects connected by a lightweight cord strung over a frictionless pulley. (b) The free-body diagram for the ball. (c) The free-body diagram for the block. (The incline is frictionless.)

### SOLUTION

**Conceptualize** Imagine the objects in Figure 5.15 in motion. If $m_2$ moves down the incline, then $m_1$ moves upward. Because the objects are connected by a cord (which we assume does not stretch), their accelerations have the same magnitude. Notice the normal coordinate axes in Figure 5.15b for the ball and the "tilted" axes for the block in Figure 5.15c.

**Categorize** We can identify forces on each of the two objects and we are looking for an acceleration, so we categorize the objects as *particles under a net force*. For the block, this model is only valid for the $x'$ direction. In the $y'$ direction, we apply the *particle in equilibrium* model because the block does not accelerate in that direction.

**Analyze** Consider the free-body diagrams shown in Figures 5.15b and 5.15c.

Apply Newton's second law in the $y$ direction to the ball, choosing the upward direction as positive:

$$(1)\quad \sum F_y = T - m_1 g = m_1 a_y = m_1 a$$

For the ball to accelerate upward, it is necessary that $T > m_1 g$. In Equation (1), we replaced $a_y$ with $a$ because the acceleration has only a $y$ component.

For the block, we have chosen the $x'$ axis along the incline as in Figure 5.15c. For consistency with our choice for the ball, we choose the positive $x'$ direction to be down the incline.

Apply the particle under a net force model to the block in the $x'$ direction and the particle in equilibrium model in the $y'$ direction:

$$(2)\quad \sum F_{x'} = m_2 g \sin\theta - T = m_2 a_{x'} = m_2 a$$

$$(3)\quad \sum F_{y'} = n - m_2 g\cos\theta = 0$$

In Equation (2), we replaced $a_{x'}$ with $a$ because the two objects have accelerations of equal magnitude $a$.

Solve Equation (1) for $T$:

$$(4)\quad T = m_1(g + a)$$

Substitute this expression for $T$ into Equation (2):

$$m_2 g\sin\theta - m_1(g + a) = m_2 a$$

Solve for $a$:

$$(5)\quad a = \left(\frac{m_2\sin\theta - m_1}{m_1 + m_2}\right)g$$

Substitute this expression for $a$ into Equation (4) to find $T$:

$$(6)\quad T = \left(\frac{m_1 m_2(\sin\theta + 1)}{m_1 + m_2}\right)g$$

**Finalize** The block accelerates down the incline only if $m_2\sin\theta > m_1$. If $m_1 > m_2\sin\theta$, the acceleration is up the incline for the block and downward for the ball. Also notice that the result for the acceleration, Equation (5), can be interpreted as the magnitude of the net external force acting on the ball–block system divided by the total mass of the system; this result is consistent with Newton's second law.

**WHAT IF?** What happens in this situation if $\theta = 90°$?

*continued*

5.10 continued

**Answer** If $\theta = 90°$, the inclined plane becomes vertical and there is no interaction between its surface and $m_2$. Therefore, this problem becomes the Atwood machine of Example 5.9. Letting $\theta \rightarrow 90°$ in Equations (5) and (6) causes them to reduce to Equations (3) and (4) of Example 5.9!

**WHAT IF?** What if $m_1 = 0$?

**Answer** If $m_1 = 0$, then $m_2$ is simply sliding down an inclined plane without interacting with $m_1$ through the string. Therefore, this problem becomes the sliding car problem in Example 5.6. Letting $m_1 \rightarrow 0$ in Equation (5) causes it to reduce to Equation (3) of Example 5.6!

## 5.8 Forces of Friction

When an object is in motion either on a surface or in a viscous medium such as air or water, there is resistance to the motion because the object interacts with its surroundings. We call such resistance a **force of friction.** Forces of friction are very important in our everyday lives. They allow us to walk or run and are necessary for the motion of wheeled vehicles.

Imagine that you are working in your garden and have filled a trash can with yard clippings. You then try to drag the trash can across the surface of your concrete patio as in Figure 5.16a. This surface is *real,* not an idealized, frictionless surface. If we apply an external horizontal force $\vec{\mathbf{F}}$ to the trash can, acting to the right, the trash can remains stationary when $\vec{\mathbf{F}}$ is small. The force on the trash can that counteracts $\vec{\mathbf{F}}$ and keeps it from moving acts toward the left and is called the

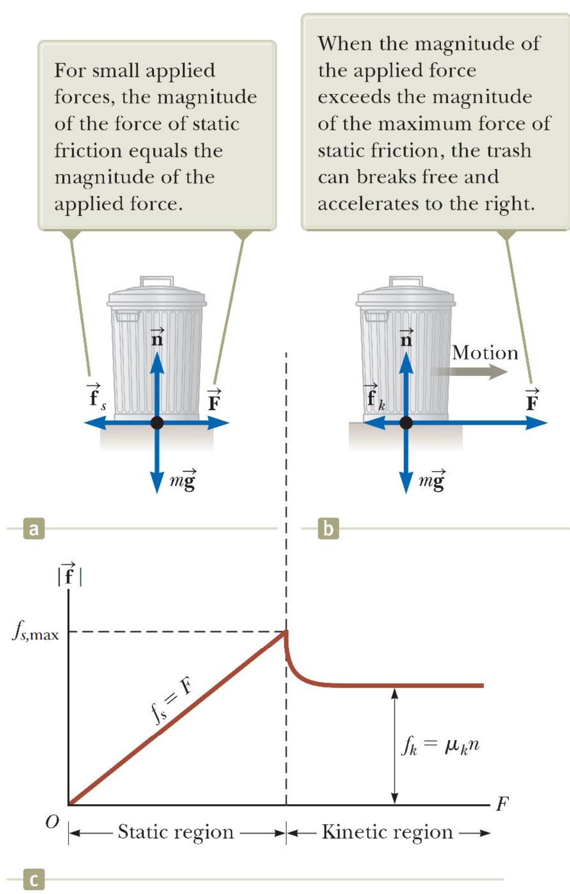



**Figure 5.16** (a) and (b) When pulling on a trash can, the direction of the force of friction $\vec{\mathbf{f}}$ between the can and a rough surface is opposite the direction of the applied force $\vec{\mathbf{F}}$. (c) A graph of friction force versus applied force. Notice that $f_{s,\max} > f_k$.

**force of static friction $\vec{\mathbf{f}}_s$.** As long as the trash can is not moving, $f_s = F$. Therefore, if $\vec{\mathbf{F}}$ is increased, $\vec{\mathbf{f}}_s$ also increases. Likewise, if $\vec{\mathbf{F}}$ decreases, $\vec{\mathbf{f}}_s$ also decreases.

◀ **Force of static friction**

Experiments show that the friction force arises from the nature of the two surfaces: because of their roughness, contact is made only at a few locations where peaks of the material touch. At these locations, the friction force arises in part because one peak physically blocks the motion of a peak from the opposing surface and in part from chemical bonding ("spot welds") of opposing peaks as they come into contact. Although the details of friction are quite complex at the atomic level, this force ultimately involves an electrical interaction between atoms or molecules.

If we increase the magnitude of $\vec{\mathbf{F}}$ as in Figure 5.16b, the trash can eventually slips. When the trash can is on the verge of slipping, $f_s$ has its maximum value $f_{s,\max}$ as shown in Figure 5.16c. When $F$ exceeds $f_{s,\max}$, the trash can moves and accelerates to the right. We call the friction force for an object in motion the **force of kinetic friction $\vec{\mathbf{f}}_k$.** When the trash can is in motion, the force of kinetic friction on the can is less than $f_{s,\max}$ (Fig. 5.16c). The net force $F - f_k$ in the $x$ direction produces an acceleration to the right, according to Newton's second law. If $F = f_k$, the acceleration is zero and the trash can moves to the right with constant speed. If the applied force $\vec{\mathbf{F}}$ is removed from the moving can, the friction force $\vec{\mathbf{f}}_k$ acting to the left provides an acceleration of the trash can in the $-x$ direction and eventually brings it to rest, again consistent with Newton's second law.

◀ **Force of kinetic friction**

Experimentally, we find that, to a good approximation, both $f_{s,\max}$ and $f_k$ are proportional to the magnitude of the normal force exerted on an object by the surface. The following descriptions of the force of friction are based on experimental observations and serve as the simplification model we shall use for forces of friction in problem solving:

- The magnitude of the force of static friction between any two surfaces in contact can have the values

$$f_s \leq \mu_s n \quad \textbf{(5.9)}$$

  where the dimensionless constant $\mu_s$ is called the **coefficient of static friction** and $n$ is the magnitude of the normal force exerted by one surface on the other. The equality in Equation 5.9 holds when the surfaces are on the verge of slipping, that is, when $f_s = f_{s,\max} = \mu_s n$. This situation is called *impending motion*. The inequality holds when the surfaces are not on the verge of slipping.
- The magnitude of the force of kinetic friction acting between two surfaces is

$$f_k = \mu_k n \quad \textbf{(5.10)}$$

  where $\mu_k$ is the **coefficient of kinetic friction.** Although the coefficient of kinetic friction can vary with speed, we shall usually neglect any such variations in this text.
- The values of $\mu_k$ and $\mu_s$ depend on the nature of the surfaces, but $\mu_k$ is generally less than $\mu_s$. Typical values range from around 0.03 to 1.0. Table 5.1 (page 132) lists some reported values.
- The direction of the friction force on an object is parallel to the surface with which the object is in contact and opposite to the actual motion (kinetic friction) or the impending motion (static friction) of the object relative to the surface.
- The coefficients of friction are nearly independent of the area of contact between the surfaces. We might expect that placing an object on the side having the most area might increase the friction force. Although this method provides more points in contact, the weight of the object is spread out over a larger area and the individual points are not pressed together as tightly. Because these effects approximately compensate for each other, the friction force is independent of the area.

**Pitfall Prevention 5.9**

**The Equal Sign Is Used in Limited Situations** In Equation 5.9, the equal sign is used *only* in the case in which the surfaces are just about to break free and begin sliding. Do not fall into the common trap of using $f_s = \mu_s n$ in *any* static situation.

**Pitfall Prevention 5.10**

**Friction Equations** Equations 5.9 and 5.10 are *not* vector equations. They are relationships between the *magnitudes* of the vectors representing the friction and normal forces. Because the friction and normal forces are perpendicular to each other, the vectors cannot be related by a multiplicative constant.

**Pitfall Prevention 5.11**

**The Direction of the Friction Force** Sometimes, an incorrect statement about the friction force between an object and a surface is made—"the friction force on an object is opposite to its motion or impending motion"—rather than the correct phrasing, "the friction force on an object is opposite to its motion or impending motion *relative to the surface*."

**Table 5.1 Coefficients of Friction**

| | $\mu_s$ | $\mu_k$ |
|---|---|---|
| Rubber on concrete | 1.0 | 0.8 |
| Steel on steel | 0.74 | 0.57 |
| Aluminum on steel | 0.61 | 0.47 |
| Glass on glass | 0.94 | 0.4 |
| Copper on steel | 0.53 | 0.36 |
| Wood on wood | 0.25–0.5 | 0.2 |
| Waxed wood on wet snow | 0.14 | 0.1 |
| Waxed wood on dry snow | — | 0.04 |
| Metal on metal (lubricated) | 0.15 | 0.06 |
| Teflon on Teflon | 0.04 | 0.04 |
| Ice on ice | 0.1 | 0.03 |
| Synovial joints in humans | 0.01 | 0.003 |

*Note:* All values are approximate. In some cases, the coefficient of friction can exceed 1.0.

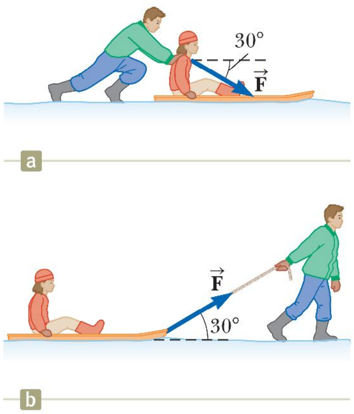



**Figure 5.17** (Quick Quiz 5.7) A father slides his daughter on a sled either by (a) pushing down on her shoulders or (b) pulling up on a rope.

**Quick Quiz 5.6** You press your physics textbook flat against a vertical wall with your hand. What is the direction of the friction force exerted by the wall on the book? **(a)** downward **(b)** upward **(c)** out from the wall **(d)** into the wall

**Quick Quiz 5.7** You are playing with your daughter in the snow. She sits on a sled and asks you to slide her across a flat, horizontal field. You have a choice of **(a)** pushing her from behind by applying a force downward on her shoulders at 30° below the horizontal (Fig. 5.17a) or **(b)** attaching a rope to the front of the sled and pulling with a force at 30° above the horizontal (Fig. 5.17b). Which would be easier for you and why?

## Example 5.11 Experimental Determination of $\mu_s$ and $\mu_k$ AM

The following is a simple method of measuring coefficients of friction. Suppose a block is placed on a rough surface inclined relative to the horizontal as shown in Figure 5.18. The incline angle is increased until the block starts to move. Show that you can obtain $\mu_s$ by measuring the critical angle $\theta_c$ at which this slipping just occurs.

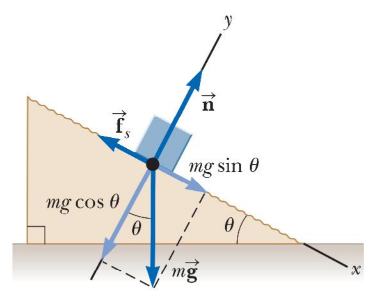



**Figure 5.18** (Example 5.11) The external forces exerted on a block lying on a rough incline are the gravitational force $m\vec{g}$, the normal force $\vec{n}$, and the force of friction $\vec{f}_s$. For convenience, the gravitational force is resolved into a component $mg \sin \theta$ along the incline and a component $mg \cos \theta$ perpendicular to the incline.

**SOLUTION**

**Conceptualize** Consider Figure 5.18 and imagine that the block tends to slide down the incline due to the gravitational force. To simulate the situation, place a coin on this book's cover and tilt the book until the coin begins to slide. Notice how this example differs from Example 5.6. When there is no friction on an incline, *any* angle of the incline will cause a stationary object to begin moving. When there is friction, however, there is no movement of the object for angles less than the critical angle.

**Categorize** The block is subject to various forces. Because we are raising the plane to the angle at which the block is just ready to begin to move but is not moving, we categorize the block as a *particle in equilibrium.*

**Analyze** The diagram in Figure 5.18 shows the forces on the block: the gravitational force $m\vec{g}$, the normal force $\vec{n}$, and the force of static friction $\vec{f}_s$. We choose $x$ to be parallel to the plane and $y$ perpendicular to it.

From the particle in equilibrium model, apply Equation 5.8 to the block in both the $x$ and $y$ directions:

$$(1)\quad \sum F_x = mg \sin \theta - f_s = 0$$

$$(2)\quad \sum F_y = n - mg \cos \theta = 0$$

5.11 continued

Substitute $mg = n/\cos\theta$ from Equation (2) into Equation (1):

$$(3)\quad f_s = mg\sin\theta = \left(\frac{n}{\cos\theta}\right)\sin\theta = n\tan\theta$$

When the incline angle is increased until the block is on the verge of slipping, the force of static friction has reached its maximum value $\mu_s n$. The angle $\theta$ in this situation is the critical angle $\theta_c$. Make these substitutions in Equation (3):

$$\mu_s n = n\tan\theta_c$$

$$\mu_s = \tan\theta_c$$

We have shown, as requested, that the coefficient of static friction is related only to the critical angle. For example, if the block just slips at $\theta_c = 20.0°$, we find that $\mu_s = \tan 20.0° = 0.364$.

**Finalize** Once the block starts to move at $\theta \geq \theta_c$, it accelerates down the incline and the force of friction is $f_k = \mu_k n$. If $\theta$ is reduced to a value less than $\theta_c$, however, it may be possible to find an angle $\theta'_c$ such that the block moves down the incline with constant speed as a particle in equilibrium again ($a_x = 0$). In this case, use Equations (1) and (2) with $f_s$ replaced by $f_k$ to find $\mu_k$: $\mu_k = \tan\theta'_c$, where $\theta'_c < \theta_c$.

## Example 5.12 The Sliding Hockey Puck AM

A hockey puck on a frozen pond is given an initial speed of 20.0 m/s. If the puck always remains on the ice and slides 115 m before coming to rest, determine the coefficient of kinetic friction between the puck and ice.

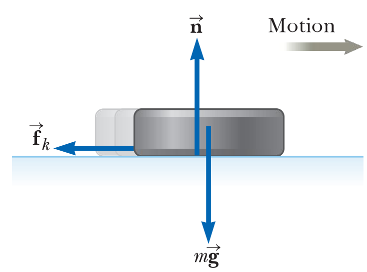



**Figure 5.19** (Example 5.12) After the puck is given an initial velocity to the right, the only external forces acting on it are the gravitational force $m\vec{g}$, the normal force $\vec{n}$, and the force of kinetic friction $\vec{f}_k$.

### SOLUTION

**Conceptualize** Imagine that the puck in Figure 5.19 slides to the right. The kinetic friction force acts to the left and slows the puck, which eventually comes to rest due to that force.

**Categorize** The forces acting on the puck are identified in Figure 5.19, but the text of the problem provides kinematic variables. Therefore, we categorize the problem in several ways. First, it involves modeling the puck as a *particle under a net force* in the horizontal direction: kinetic friction causes the puck to accelerate. There is no acceleration of the puck in the vertical direction, so we use the *particle in equilibrium* model for that direction. Furthermore, because we model the force of kinetic friction as independent of speed, the acceleration of the puck is constant. So, we can also categorize this problem by modeling the puck as a *particle under constant acceleration*.

**Analyze** First, let's find the acceleration algebraically in terms of the coefficient of kinetic friction, using Newton's second law. Once we know the acceleration of the puck and the distance it travels, the equations of kinematics can be used to find the numerical value of the coefficient of kinetic friction. The diagram in Figure 5.19 shows the forces on the puck.

Apply the particle under a net force model in the $x$ direction to the puck:

$$(1)\quad \sum F_x = -f_k = ma_x$$

Apply the particle in equilibrium model in the $y$ direction to the puck:

$$(2)\quad \sum F_y = n - mg = 0$$

Substitute $n = mg$ from Equation (2) and $f_k = \mu_k n$ into Equation (1):

$$-\mu_k n = -\mu_k mg = ma_x$$

$$a_x = -\mu_k g$$

The negative sign means the acceleration is to the left in Figure 5.19. Because the velocity of the puck is to the right, the puck is slowing down. The acceleration is independent of the mass of the puck and is constant because we assume $\mu_k$ remains constant.

*continued*

5.12 continued

Apply the particle under constant acceleration model to the puck, choosing Equation 2.17 from the model, $v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$, with $x_i = 0$ and $v_{xf} = 0$:

$$0 = v_{xi}^2 + 2a_x x_f = v_{xi}^2 - 2\mu_k g x_f$$

Solve for the coefficient of kinetic friction:

$$\mu_k = \frac{v_{xi}^2}{2gx_f}$$

Substitute the numerical values:

$$\mu_k = \frac{(20.0\text{ m/s})^2}{2(9.80\text{ m/s}^2)(115\text{ m})} = 0.177$$

**Finalize** Notice that $\mu_k$ is dimensionless, as it should be, and that it has a low value, consistent with an object sliding on ice.

## Example 5.13 Acceleration of Two Connected Objects When Friction Is Present AM

A block of mass $m_2$ on a rough, horizontal surface is connected to a ball of mass $m_1$ by a lightweight cord over a lightweight, frictionless pulley as shown in Figure 5.20a. A force of magnitude $F$ at an angle $\theta$ with the horizontal is applied to the block as shown, and the block slides to the right. The coefficient of kinetic friction between the block and surface is $\mu_k$. Determine the magnitude of the acceleration of the two objects.

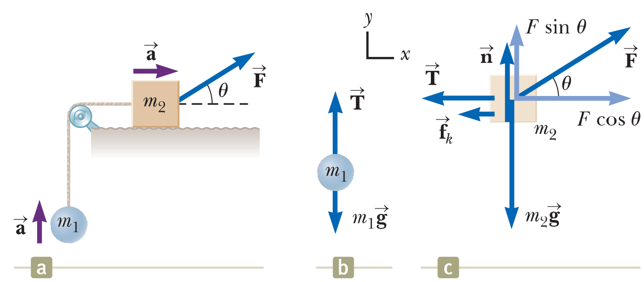



**Figure 5.20** (Example 5.13) (a) The external force $\vec{F}$ applied as shown can cause the block to accelerate to the right. (b, c) Diagrams showing the forces on the two objects, assuming the block accelerates to the right and the ball accelerates upward.

### SOLUTION

**Conceptualize** Imagine what happens as $\vec{F}$ is applied to the block. Assuming $\vec{F}$ is large enough to break the block free from static friction but not large enough to lift the block, the block slides to the right and the ball rises.

**Categorize** We can identify forces and we want an acceleration, so we categorize this problem as one involving two *particles under a net force,* the ball and the block. Because we assume that the block does not rise into the air due to the applied force, we model the block as a *particle in equilibrium* in the vertical direction.

**Analyze** First draw force diagrams for the two objects as shown in Figures 5.20b and 5.20c. Notice that the string exerts a force of magnitude $T$ on both objects. The applied force $\vec{F}$ has $x$ and $y$ components $F\cos\theta$ and $F\sin\theta$, respectively. Because the two objects are connected, we can equate the magnitudes of the $x$ component of the acceleration of the block and the $y$ component of the acceleration of the ball and call them both $a$. Let us assume the motion of the block is to the right.

Apply the particle under a net force model to the block in the horizontal direction:

$$(1)\quad \sum F_x = F\cos\theta - f_k - T = m_2 a_x = m_2 a$$

Because the block moves only horizontally, apply the particle in equilibrium model to the block in the vertical direction:

$$(2)\quad \sum F_y = n + F\sin\theta - m_2 g = 0$$

Apply the particle under a net force model to the ball in the vertical direction:

$$(3)\quad \sum F_y = T - m_1 g = m_1 a_y = m_1 a$$

Solve Equation (2) for $n$:

$$n = m_2 g - F\sin\theta$$

Substitute $n$ into $f_k = \mu_k n$ from Equation 5.10:

$$(4)\quad f_k = \mu_k(m_2 g - F\sin\theta)$$

**5.13** continued

Substitute Equation (4) and the value of $T$ from Equation (3) into Equation (1):

$$F\cos\theta - \mu_k(m_2 g - F\sin\theta) - m_1(a + g) = m_2 a$$

Solve for $a$:

$$(5)\quad a = \frac{F(\cos\theta + \mu_k\sin\theta) - (m_1 + \mu_k m_2)g}{m_1 + m_2}$$

**Finalize** The acceleration of the block can be either to the right or to the left depending on the sign of the numerator in Equation (5). If the velocity is to the left, we must reverse the sign of $f_k$ in Equation (1) because the force of kinetic friction must oppose the motion of the block relative to the surface. In this case, the value of $a$ is the same as in Equation (5), with the two plus signs in the numerator changed to minus signs.

What does Equation (5) reduce to if the force $\vec{\mathbf{F}}$ is removed and the surface becomes frictionless? Call this expression Equation (6). Does this algebraic expression match your intuition about the physical situation in this case? Now go back to Example 5.10 and let angle $\theta$ go to zero in Equation (5) of that example. How does the resulting equation compare with your Equation (6) here in Example 5.13? Should the algebraic expressions compare in this way based on the physical situations?

# Summary

## Definitions

An **inertial frame of reference** is a frame in which an object that does not interact with other objects experiences zero acceleration. Any frame moving with constant velocity relative to an inertial frame is also an inertial frame.

We define **force** as **that which causes a change in motion of an object.**

## Concepts and Principles

**Newton's first law** states that it is possible to find an inertial frame in which an object that does not interact with other objects experiences zero acceleration, or, equivalently, in the absence of an external force, when viewed from an inertial frame, an object at rest remains at rest and an object in uniform motion in a straight line maintains that motion.

**Newton's second law** states that the acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass.

**Newton's third law** states that if two objects interact, the force exerted by object 1 on object 2 is equal in magnitude and opposite in direction to the force exerted by object 2 on object 1.

The **gravitational force** exerted on an object is equal to the product of its mass (a scalar quantity) and the free-fall acceleration:

$$\vec{\mathbf{F}}_g = m\vec{\mathbf{g}} \quad \textbf{(5.5)}$$

The **weight** of an object is the magnitude of the gravitational force acting on the object:

$$F_g = mg \quad \textbf{(5.6)}$$

The maximum **force of static friction** $\vec{\mathbf{f}}_{s,\max}$ between an object and a surface is proportional to the normal force acting on the object. In general, $f_s \le \mu_s n$, where $\mu_s$ is the **coefficient of static friction** and $n$ is the magnitude of the normal force.

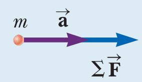
When an object slides over a surface, the magnitude of the **force of kinetic friction** $\vec{\mathbf{f}}_k$ is given by $f_k = \mu_k n$, where $\mu_k$ is the **coefficient of kinetic friction.**

*continued*

## Analysis Models for Problem Solving

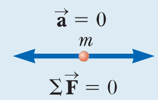
**Particle Under a Net Force** If a particle of mass $m$ experiences a nonzero net force, its acceleration is related to the net force by Newton's second law:

$$\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}} \quad (5.2)$$




**Particle in Equilibrium** If a particle maintains a constant velocity (so that $\vec{\mathbf{a}} = 0$), which could include a velocity of zero, the forces on the particle balance and Newton's second law reduces to

$$\sum \vec{\mathbf{F}} = 0 \quad (5.8)$$




## Objective Questions

**1.** denotes answer available in *Student Solutions Manual/Study Guide*

**1.** The driver of a speeding empty truck slams on the brakes and skids to a stop through a distance $d$. On a second trial, the truck carries a load that doubles its mass. What will now be the truck's "skidding distance"? (a) $4d$ (b) $2d$ (c) $\sqrt{2}d$ (d) $d$ (e) $d/2$

**2.** In Figure OQ5.2, a locomotive has broken through the wall of a train station. During the collision, what can be said about the force exerted by the locomotive on the wall? (a) The force exerted by the locomotive on the wall was larger than the force the wall could exert on the locomotive. (b) The force exerted by the locomotive on the wall was the same in magnitude as the force exerted by the wall on the locomotive. (c) The force exerted by the locomotive on the wall was less than the force exerted by the wall on the locomotive. (d) The wall cannot be said to "exert" a force; after all, it broke.



Figure OQ5.2

**3.** The third graders are on one side of a schoolyard, and the fourth graders are on the other. They are throwing snowballs at each other. Between them, snowballs of various masses are moving with different velocities as shown in Figure OQ5.3. Rank the snowballs (a) through (e) according to the magnitude of the total force exerted on each one. Ignore air resistance. If two snowballs rank together, make that fact clear.

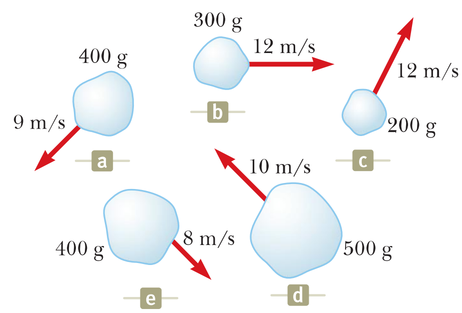



Figure OQ5.3

**4.** The driver of a speeding truck slams on the brakes and skids to a stop through a distance $d$. On another trial, the initial speed of the truck is half as large. What now will be the truck's skidding distance? (a) $2d$ (b) $\sqrt{2}d$ (c) $d$ (d) $d/2$ (e) $d/4$

**5.** An experiment is performed on a puck on a level air hockey table, where friction is negligible. A constant horizontal force is applied to the puck, and the puck's acceleration is measured. Now the same puck is transported far into outer space, where both friction and gravity are negligible. The same constant force is applied to the puck (through a spring scale that stretches the same amount), and the puck's acceleration (relative to the distant stars) is measured. What is the puck's acceleration in outer space? (a) It is somewhat greater than its acceleration on the Earth. (b) It is the same as its acceleration on the Earth. (c) It is less than its acceleration on the Earth. (d) It is infinite because neither friction nor gravity constrains it. (e) It is very large because acceleration is inversely proportional to weight and the puck's weight is very small but not zero.

**6.** The manager of a department store is pushing horizontally with a force of magnitude 200 N on a box of shirts. The box is sliding across the horizontal floor with a forward acceleration. Nothing else touches the box. What must be true about the magnitude of the force of kinetic friction acting on the box (choose one)? (a) It is greater than 200 N. (b) It is less than 200 N. (c) It is equal to 200 N. (d) None of those statements is necessarily true.

**7.** Two objects are connected by a string that passes over a frictionless pulley as in Figure 5.14a, where $m_1 < m_2$ and $a_1$ and $a_2$ are the magnitudes of the respective accelerations. Which mathematical statement is true regarding the magnitude of the acceleration $a_2$ of the mass $m_2$? (a) $a_2 < g$ (b) $a_2 > g$ (c) $a_2 = g$ (d) $a_2 < a_1$ (e) $a_2 > a_1$

**8.** An object of mass $m$ is sliding with speed $v_i$ at some instant across a level tabletop, with which its coefficient of kinetic friction is $\mu$. It then moves through a distance $d$ and comes to rest. Which of the following equations for the speed $v_i$ is reasonable? (a) $v_i = \sqrt{-2\mu mgd}$ (b) $v_i = \sqrt{2\mu mgd}$ (c) $v_i = \sqrt{-2\mu gd}$ (d) $v_i = \sqrt{2\mu gd}$ (e) $v_i = \sqrt{2\mu d}$

**9.** A truck loaded with sand accelerates along a highway. The driving force on the truck remains constant. What happens to the acceleration of the truck if its trailer leaks sand at a constant rate through a hole in its bottom? (a) It decreases at a steady rate. (b) It increases at a steady rate. (c) It increases and then decreases. (d) It decreases and then increases. (e) It remains constant.

**10.** A large crate of mass $m$ is place on the flatbed of a truck but not tied down. As the truck accelerates forward with acceleration $a$, the crate remains at rest relative to the truck. What force causes the crate to accelerate? (a) the normal force (b) the gravitational force (c) the friction force (d) the $ma$ force exerted by the crate (e) No force is required.

**[11.]** If an object is in equilibrium, which of the following statements is *not* true? (a) The speed of the object remains constant. (b) The acceleration of the object is zero. (c) The net force acting on the object is zero. (d) The object must be at rest. (e) There are at least two forces acting on the object.

**12.** A crate remains stationary after it has been placed on a ramp inclined at an angle with the horizontal. Which of the following statements is or are correct about the magnitude of the friction force that acts on the crate? Choose all that are true. (a) It is larger than the weight of the crate. (b) It is equal to $\mu_s n$. (c) It is greater than the component of the gravitational force acting down the ramp. (d) It is equal to the component of the gravitational force acting down the ramp. (e) It is less than the component of the gravitational force acting down the ramp.

**13.** An object of mass $m$ moves with acceleration $\vec{\mathbf{a}}$ down a rough incline. Which of the following forces should appear in a free-body diagram of the object? Choose all correct answers. (a) the gravitational force exerted by the planet (b) $m\vec{\mathbf{a}}$ in the direction of motion (c) the normal force exerted by the incline (d) the friction force exerted by the incline (e) the force exerted by the object on the incline

## Conceptual Questions

[1.] denotes answer available in *Student Solutions Manual/Study Guide*

**1.** If you hold a horizontal metal bar several centimeters above the ground and move it through grass, each leaf of grass bends out of the way. If you increase the speed of the bar, each leaf of grass will bend more quickly. How then does a rotary power lawn mower manage to cut grass? How can it exert enough force on a leaf of grass to shear it off?

**2.** Your hands are wet, and the restroom towel dispenser is empty. What do you do to get drops of water off your hands? How does the motion of the drops exemplify one of Newton's laws? Which one?

**[3.]** In the motion picture *It Happened One Night* (Columbia Pictures, 1934), Clark Gable is standing inside a stationary bus in front of Claudette Colbert, who is seated. The bus suddenly starts moving forward and Clark falls into Claudette's lap. Why did this happen?

**4.** If a car is traveling due westward with a constant speed of 20 m/s, what is the resultant force acting on it?

**5.** A passenger sitting in the rear of a bus claims that she was injured when the driver slammed on the brakes, causing a suitcase to come flying toward her from the front of the bus. If you were the judge in this case, what disposition would you make? Why?

**6.** A child tosses a ball straight up. She says that the ball is moving away from her hand because the ball feels an upward "force of the throw" as well as the gravitational force. (a) Can the "force of the throw" exceed the gravitational force? How would the ball move if it did? (b) Can the "force of the throw" be equal in magnitude to the gravitational force? Explain. (c) What strength can accurately be attributed to the "force of the throw"? Explain. (d) Why does the ball move away from the child's hand?

**7.** A person holds a ball in her hand. (a) Identify all the external forces acting on the ball and the Newton's third-law reaction force to each one. (b) If the ball is dropped, what force is exerted on it while it is falling? Identify the reaction force in this case. (Ignore air resistance.)

**8.** A spherical rubber balloon inflated with air is held stationary, with its opening, on the west side, pinched shut. (a) Describe the forces exerted by the air inside and outside the balloon on sections of the rubber. (b) After the balloon is released, it takes off toward the east, gaining speed rapidly. Explain this motion in terms of the forces now acting on the rubber. (c) Account for the motion of a skyrocket taking off from its launch pad.

**[9.]** A rubber ball is dropped onto the floor. What force causes the ball to bounce?

**10.** Twenty people participate in a tug-of-war. The two teams of ten people are so evenly matched that neither team wins. After the game they notice that a car is stuck in the mud. They attach the tug-of-war rope to the bumper of the car, and all the people pull on the rope. The heavy car has just moved a couple of decimeters when the rope breaks. Why did the rope break in this situation when it did not break when the same twenty people pulled on it in a tug-of-war?

11. Can an object exert a force on itself? Argue for your answer.

12. When you push on a box with a 200-N force instead of a 50-N force, you can feel that you are making a greater effort. When a table exerts a 200-N normal force instead of one of smaller magnitude, is the table really doing anything differently?

13. A weightlifter stands on a bathroom scale. He pumps a barbell up and down. What happens to the reading on the scale as he does so? **What If?** What if he is strong enough to actually *throw* the barbell upward? How does the reading on the scale vary now?

14. An athlete grips a light rope that passes over a low-friction pulley attached to the ceiling of a gym. A sack of sand precisely equal in weight to the athlete is tied to the other end of the rope. Both the sand and the athlete are initially at rest. The athlete climbs the rope, sometimes speeding up and slowing down as he does so. What happens to the sack of sand? Explain.

15. Suppose you are driving a classic car. Why should you avoid slamming on your brakes when you want to stop in the shortest possible distance? (Many modern cars have antilock brakes that avoid this problem.)

16. In Figure CQ5.16, the light, taut, unstretchable cord B joins block 1 and the larger-mass block 2. Cord A exerts a force on block 1 to make it accelerate forward. (a) How does the magnitude of the force exerted by cord A on block 1 compare with the magnitude of the force exerted by cord B on block 2? Is it larger, smaller, or equal? (b) How does the acceleration of block 1 compare with the acceleration (if any) of block 2? (c) Does cord B exert a force on block 1? If so, is it forward or backward? Is it larger, smaller, or equal in magnitude to the force exerted by cord B on block 2?

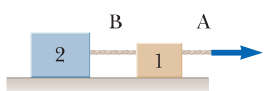



**Figure CQ5.16**

17. Describe two examples in which the force of friction exerted on an object is in the direction of motion of the object.

18. The mayor of a city reprimands some city employees because they will not remove the obvious sags from the cables that support the city traffic lights. What explanation can the employees give? How do you think the case will be settled in mediation?

19. Give reasons for the answers to each of the following questions: (a) Can a normal force be horizontal? (b) Can a normal force be directed vertically downward? (c) Consider a tennis ball in contact with a stationary floor and with nothing else. Can the normal force be different in magnitude from the gravitational force exerted on the ball? (d) Can the force exerted by the floor on the ball be different in magnitude from the force the ball exerts on the floor?

20. Balancing carefully, three boys inch out onto a horizontal tree branch above a pond, each planning to dive in separately. The third boy in line notices that the branch is barely strong enough to support them. He decides to jump straight up and land back on the branch to break it, spilling all three into the pond. When he starts to carry out his plan, at what precise moment does the branch break? Explain. *Suggestion:* Pretend to be the third boy and imitate what he does in slow motion. If you are still unsure, stand on a bathroom scale and repeat the suggestion.

21. Identify action–reaction pairs in the following situations: (a) a man takes a step (b) a snowball hits a girl in the back (c) a baseball player catches a ball (d) a gust of wind strikes a window

22. As shown in Figure CQ5.22, student A, a 55-kg girl, sits on one chair with metal runners, at rest on a classroom floor. Student B, an 80-kg boy, sits on an identical chair. Both students keep their feet off the floor. A rope runs from student A's hands around a light pulley and then over her shoulder to the hands of a teacher standing on the floor behind her. The low-friction axle of the pulley is attached to a second rope held by student B. All ropes run parallel to the chair runners. (a) If student A pulls on her end of the rope, will her chair or will B's chair slide on the floor? Explain why. (b) If instead the teacher pulls on his rope end, which chair slides? Why this one? (c) If student B pulls on his rope, which chair slides? Why? (d) Now the teacher ties his end of the rope to student A's chair. Student A pulls on the end of the rope in her hands. Which chair slides and why?

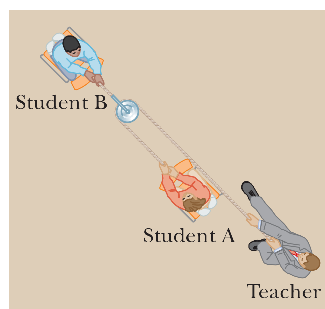



**Figure CQ5.22**

23. A car is moving forward slowly and is speeding up. A student claims that "the car exerts a force on itself" or that "the car's engine exerts a force on the car." (a) Argue that this idea cannot be accurate and that friction exerted by the road is the propulsive force on the car. Make your evidence and reasoning as persuasive as possible. (b) Is it static or kinetic friction? *Suggestions*: Consider a road covered with light gravel. Consider a sharp print of the tire tread on an asphalt road, obtained by coating the tread with dust.

## Problems

The problems found in this chapter may be assigned online in Enhanced WebAssign

**1.** straightforward; **2.** intermediate; **3.** challenging

**1.** full solution available in the *Student Solutions Manual/Study Guide*

AMT Analysis Model tutorial available in Enhanced WebAssign

GP Guided Problem

M Master It tutorial available in Enhanced WebAssign

W Watch It video solution available in Enhanced WebAssign

### Section 5.1 The Concept of Force
### Section 5.2 Newton's First Law and Inertial Frames
### Section 5.3 Mass
### Section 5.4 Newton's Second Law
### Section 5.5 The Gravitational Force and Weight
### Section 5.6 Newton's Third Law

**1.** A woman weighs 120 lb. Determine (a) her weight in newtons and (b) her mass in kilograms.

**2.** If a man weighs 900 N on the Earth, what would he weigh on Jupiter, where the free-fall acceleration is 25.9 m/s$^2$?

**3.** W A 3.00-kg object undergoes an acceleration given by $\vec{\mathbf{a}} = (2.00\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}})$ m/s$^2$. Find (a) the resultant force acting on the object and (b) the magnitude of the resultant force.

**4.** A certain orthodontist uses a wire brace to align a patient's crooked tooth as in Figure P5.4. The tension in the wire is adjusted to have a magnitude of 18.0 N. Find the magnitude of the net force exerted by the wire on the crooked tooth.

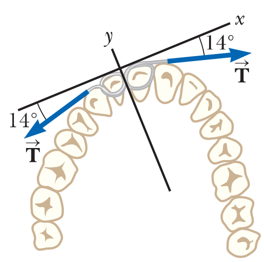


Figure P5.4

**5.** M A toy rocket engine is securely fastened to a large puck that can glide with negligible friction over a horizontal surface, taken as the *xy* plane. The 4.00-kg puck has a velocity of $3.00\hat{\mathbf{i}}$ m/s at one instant. Eight seconds later, its velocity is $(8.00\hat{\mathbf{i}} + 10.00\hat{\mathbf{j}})$ m/s. Assuming the rocket engine exerts a constant horizontal force, find (a) the components of the force and (b) its magnitude.

**6.** The average speed of a nitrogen molecule in air is about $6.70 \times 10^2$ m/s, and its mass is $4.68 \times 10^{-26}$ kg. (a) If it takes $3.00 \times 10^{-13}$ s for a nitrogen molecule to hit a wall and rebound with the same speed but moving in the opposite direction, what is the average acceleration of the molecule during this time interval? (b) What average force does the molecule exert on the wall?

**7.** The distinction between mass and weight was discovered after Jean Richer transported pendulum clocks from Paris, France, to Cayenne, French Guiana, in 1671. He found that they quite systematically ran slower in Cayenne than in Paris. The effect was reversed when the clocks returned to Paris. How much weight would a 90.0 kg person lose in traveling from Paris, where $g = 9.809\,5$ m/s$^2$, to Cayenne, where $g = 9.780\,8$ m/s$^2$? (We will consider how the free-fall acceleration influences the period of a pendulum in Section 15.5.)

**8.** (a) A car with a mass of 850 kg is moving to the right with a constant speed of 1.44 m/s. What is the total force on the car? (b) What is the total force on the car if it is moving to the left?

**9.** **Review.** The gravitational force exerted on a baseball is 2.21 N down. A pitcher throws the ball horizontally with velocity 18.0 m/s by uniformly accelerating it along a straight horizontal line for a time interval of 170 ms. The ball starts from rest. (a) Through what distance does it move before its release? (b) What are the magnitude and direction of the force the pitcher exerts on the ball?

**10.** **Review.** The gravitational force exerted on a baseball is $-F_g\hat{\mathbf{j}}$. A pitcher throws the ball with velocity $v\hat{\mathbf{i}}$ by uniformly accelerating it along a straight horizontal line for a time interval of $\Delta t = t - 0 = t$. (a) Starting from rest, through what distance does the ball move before its release? (b) What force does the pitcher exert on the ball?

**11.** M **Review.** An electron of mass $9.11 \times 10^{-31}$ kg has an initial speed of $3.00 \times 10^5$ m/s. It travels in a straight line, and its speed increases to $7.00 \times 10^5$ m/s in a distance of 5.00 cm. Assuming its acceleration is constant, (a) determine the magnitude of the force exerted on the electron and (b) compare this force with the weight of the electron, which we ignored.

**12.** Besides the gravitational force, a 2.80-kg object is subjected to one other constant force. The object starts from rest and in 1.20 s experiences a displacement of $(4.20\hat{\mathbf{i}} - 3.30\hat{\mathbf{j}})$ m, where the direction of $\hat{\mathbf{j}}$ is the upward vertical direction. Determine the other force.

**13.** One or more external forces, large enough to be easily measured, are exerted on each object enclosed in a dashed box shown in Figure 5.1. Identify the reaction to each of these forces.

**14.** A brick of mass $M$ has been placed on a rubber cushion of mass $m$. Together they are sliding to the right at constant velocity on an ice-covered parking lot. (a) Draw a free-body diagram of the brick and identify each force acting on it. (b) Draw a free-body diagram of the cushion and identify each force acting on it. (c) Identify all of the action–reaction pairs of forces in the brick–cushion–planet system.

**15.** Two forces, $\vec{\mathbf{F}}_1 = (-6.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}})$ N and $\vec{\mathbf{F}}_2 = (-3.00\hat{\mathbf{i}} + 7.00\hat{\mathbf{j}})$ N, act on a particle of mass 2.00 kg that is initially at rest at coordinates (−2.00 m, +4.00 m). (a) What are the components of the particle's velocity at $t = 10.0$ s? (b) In what direction is the particle moving at $t = 10.0$ s? (c) What displacement does the particle undergo during the first 10.0 s? (d) What are the coordinates of the particle at $t = 10.0$ s?

**16.** M The force exerted by the wind on the sails of a sailboat is 390 N north. The water exerts a force of 180 N east. If the boat (including its crew) has a mass of 270 kg, what are the magnitude and direction of its acceleration?

**17.** An object of mass $m$ is dropped at $t = 0$ from the roof of a building of height $h$. While the object is falling, a wind blowing parallel to the face of the building exerts a constant horizontal force $F$ on the object. (a) At what time $t$ does the object strike the ground? Express $t$ in terms of $g$ and $h$. (b) Find an expression in terms of $m$ and $F$ for the acceleration $a_x$ of the object in the horizontal direction (taken as the positive $x$ direction). (c) How far is the object displaced horizontally before hitting the ground? Answer in terms of $m$, $g$, $F$, and $h$. (d) Find the magnitude of the object's acceleration while it is falling, using the variables $F$, $m$, and $g$.

**18.** W A force $\vec{\mathbf{F}}$ applied to an object of mass $m_1$ produces an acceleration of 3.00 m/s$^2$. The same force applied to a second object of mass $m_2$ produces an acceleration of 1.00 m/s$^2$. (a) What is the value of the ratio $m_1/m_2$? (b) If $m_1$ and $m_2$ are combined into one object, find its acceleration under the action of the force $\vec{\mathbf{F}}$.

**19.** M Two forces $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ act on a 5.00-kg object. Taking $F_1 = 20.0$ N and $F_2 = 15.0$ N, find the accelerations of the object for the configurations of forces shown in parts (a) and (b) of Figure P5.19.

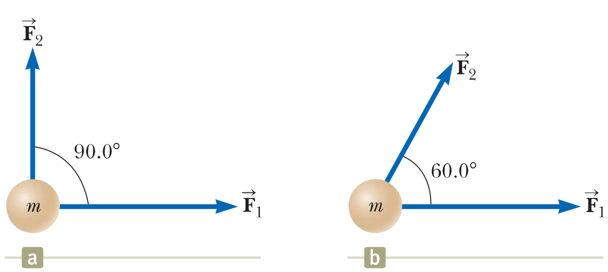



Figure P5.19

**20.** You stand on the seat of a chair and then hop off. (a) During the time interval you are in flight down to the floor, the Earth moves toward you with an acceleration of what order of magnitude? In your solution, explain your logic. Model the Earth as a perfectly solid object. (b) The Earth moves toward you through a distance of what order of magnitude?

**21.** A 15.0-lb block rests on the floor. (a) What force does the floor exert on the block? (b) A rope is tied to the block and is run vertically over a pulley. The other end is attached to a free-hanging 10.0-lb object. What now is the force exerted by the floor on the 15.0-lb block? (c) If the 10.0-lb object in part (b) is replaced with a 20.0-lb object, what is the force exerted by the floor on the 15.0-lb block?

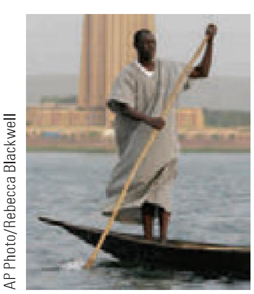
**22.** W **Review.** Three forces acting on an object are given by $\vec{\mathbf{F}}_1 = (-2.00\hat{\mathbf{i}} + 2.00\hat{\mathbf{j}})$ N, $\vec{\mathbf{F}}_2 = (5.00\hat{\mathbf{i}} - 3.00\hat{\mathbf{j}})$ N, and $\vec{\mathbf{F}}_3 = (-45.0\hat{\mathbf{i}})$ N. The object experiences an acceleration of magnitude 3.75 m/s$^2$. (a) What is the direction of the acceleration? (b) What is the mass of the object? (c) If the object is initially at rest, what is its speed after 10.0 s? (d) What are the velocity components of the object after 10.0 s?

**23.** A 1 000-kg car is pulling a 300-kg trailer. Together, the car and trailer move forward with an acceleration of 2.15 m/s$^2$. Ignore any force of air drag on the car and all friction forces on the trailer. Determine (a) the net force on the car, (b) the net force on the trailer, (c) the force exerted by the trailer on the car, and (d) the resultant force exerted by the car on the road.

**24.** If a single constant force acts on an object that moves on a straight line, the object's velocity is a linear function of time. The equation $v = v_i + at$ gives its velocity $v$ as a function of time, where $a$ is its constant acceleration. What if velocity is instead a linear function of position? Assume that as a particular object moves through a resistive medium, its speed decreases as described by the equation $v = v_i - kx$, where $k$ is a constant coefficient and $x$ is the position of the object. Find the law describing the total force acting on this object.

## Section 5.7 Analysis Models Using Newton's Second Law

**25.** **Review.** Figure P5.25 shows a worker poling a boat—a very efficient mode of transportation—across a shallow lake. He pushes parallel to the length of the light pole, exerting a force of magnitude 240 N on the bottom of the lake. Assume the pole lies in the vertical plane containing the keel of the boat. At one moment, the pole makes an angle of 35.0° with the vertical and the water exerts a horizontal drag force of 47.5 N on the boat, opposite to its forward velocity of magnitude 0.857 m/s. The mass of the boat including its cargo and the worker is 370 kg. (a) The water exerts



AP Photo/Rebecca Blackwell

Figure P5.25

a buoyant force vertically upward on the boat. Find the magnitude of this force. (b) Model the forces as constant over a short interval of time to find the velocity of the boat 0.450 s after the moment described.

**26.** An iron bolt of mass 65.0 g hangs from a string 35.7 cm long. The top end of the string is fixed. Without touching it, a magnet attracts the bolt so that it remains stationary, but is displaced horizontally 28.0 cm to the right from the previously vertical line of the string. The magnet is located to the right of the bolt and on the same vertical level as the bolt in the final configuration. (a) Draw a free-body diagram of the bolt. (b) Find the tension in the string. (c) Find the magnetic force on the bolt.

**27.** Figure P5.27 shows the horizontal forces acting on a sailboat moving north at constant velocity, seen from a point straight above its mast. At the particular speed of the sailboat, the water exerts a 220-N drag force on its hull and $\theta$ = 40.0°. For each of the situations (a) and (b) described below, write two component equations representing Newton's second law. Then solve the equations for $P$ (the force exerted by the wind on the sail) and for $n$ (the force exerted by the water on the keel). (a) Choose the $x$ direction as east and the $y$ direction as north. (b) Now choose the $x$ direction as $\theta$ = 40.0° north of east and the $y$ direction as $\theta$ = 40.0° west of north. (c) Compare your solutions to parts (a) and (b). Do the results agree? Is one method significantly easier?

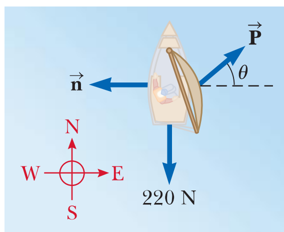



Figure P5.27

**28.** W If the systems shown in Figure P5.28 are in equilibrium. If the spring scales are calibrated in newtons, what do they read? Ignore the masses of the pulleys and strings and assume the pulleys and the incline in Figure P5.28d are frictionless.

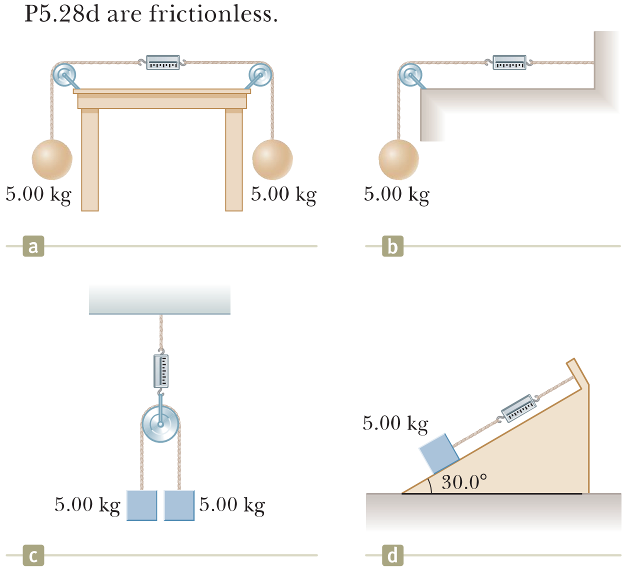



Figure P5.28

**29.** M Assume the three blocks portrayed in Figure P5.29 move on a frictionless surface and a 42-N force acts as shown on the 3.0-kg block. Determine (a) the acceleration given this system, (b) the tension in the cord connecting the 3.0-kg and the 1.0-kg blocks, and (c) the force exerted by the 1.0-kg block on the 2.0-kg block.

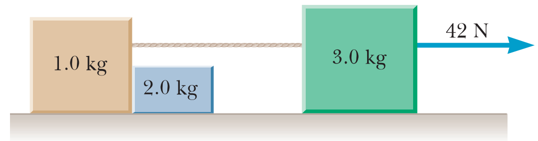



Figure P5.29

**30.** W A block slides down a frictionless plane having an inclination of $\theta$ = 15.0°. The block starts from rest at the top, and the length of the incline is 2.00 m. (a) Draw a free-body diagram of the block. Find (b) the acceleration of the block and (c) its speed when it reaches the bottom of the incline.

**31.** The distance between two telephone poles is 50.0 m. When a 1.00-kg bird lands on the telephone wire midway between the poles, the wire sags 0.200 m. (a) Draw a free-body diagram of the bird. (b) How much tension does the bird produce in the wire? Ignore the weight of the wire.

**32.** W A 3.00-kg object is moving in a plane, with its $x$ and $y$ coordinates given by $x = 5t^2 - 1$ and $y = 3t^3 + 2$, where $x$ and $y$ are in meters and $t$ is in seconds. Find the magnitude of the net force acting on this object at $t$ = 2.00 s.

**33.** AMT W A bag of cement weighing 325 N hangs in equilibrium from three wires as suggested in Figure P5.33. Two of the wires make angles $\theta_1$ = 60.0° and $\theta_2$ = 40.0° with the horizontal. Assuming the system is in equilibrium, find the tensions $T_1$, $T_2$, and $T_3$ in the wires.

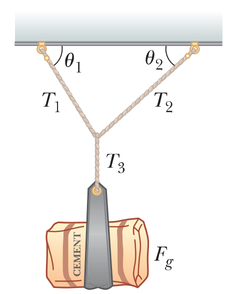



Figure P5.33
Problems 33 and 34.

**34.** A bag of cement whose weight is $F_g$ hangs in equilibrium from three wires as shown in Figure P5.33. Two of the wires make angles $\theta_1$ and $\theta_2$ with the horizontal. Assuming the system is in equilibrium, show that the tension in the left-hand wire is

$$T_1 = \frac{F_g \cos \theta_2}{\sin (\theta_1 + \theta_2)}$$

**35.** Two people pull as hard as they can on horizontal ropes attached to a boat that has a mass of 200 kg. If they pull in the same direction, the boat has an acceleration of 1.52 m/s² to the right. If they pull in opposite directions, the boat has an acceleration of 0.518 m/s² to the left. What is the magnitude of the force each person exerts on the boat? Disregard any other horizontal forces on the boat.

**36.** Figure P5.36 shows loads hanging from the ceiling of an elevator that is moving at constant velocity. Find the tension in each of the three strands of cord supporting each load.

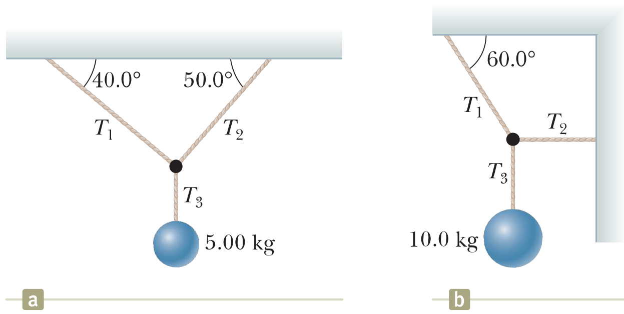



**Figure P5.36**

**37.** An object of mass $m = 1.00$ kg is observed to have an acceleration $\vec{\mathbf{a}}$ with a magnitude of 10.0 m/s$^2$ in a direction 60.0° east of north. Figure P5.37 shows a view of the object from above. The force $\vec{\mathbf{F}}_2$ acting on the object has a magnitude of 5.00 N and is directed north. Determine the magnitude and direction of the one other horizontal force $\vec{\mathbf{F}}_1$ acting on the object.

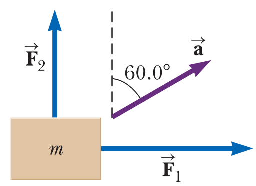



**Figure P5.37**

**38.** A setup similar to the one shown in Figure P5.38 is often used in hospitals to support and apply a horizontal traction force to an injured leg. (a) Determine the force of tension in the rope supporting the leg. (b) What is the traction force exerted to the right on the leg?

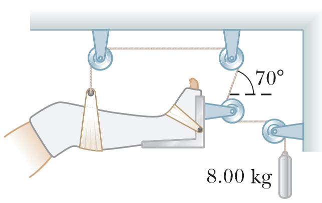



**Figure P5.38**

**39.** A simple accelerometer is constructed inside a car by suspending an object of mass $m$ from a string of length $L$ that is tied to the car's ceiling. As the car accelerates the string–object system makes a constant angle of $\theta$ with the vertical. (a) Assuming that the string mass is negligible compared with $m$, derive an expression for the car's acceleration in terms of $\theta$ and show that it is independent of the mass $m$ and the length $L$. (b) Determine the acceleration of the car when $\theta = 23.0°$.

**40.** AMT W An object of mass $m_1 = 5.00$ kg placed on a frictionless, horizontal table is connected to a string that passes over a pulley and then is fastened to a hanging object of mass $m_2 = 9.00$ kg as shown in Figure P5.40. (a) Draw free-body diagrams of both objects. Find (b) the magnitude of the acceleration of the objects and (c) the tension in the string.

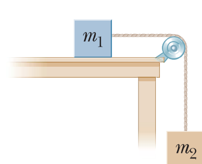



**Figure P5.40** Problems 40, 63, and 87.

**41.** Figure P5.41 shows the speed of a person's body as he does a chin-up. Assume the motion is vertical and the mass of the person's body is 64.0 kg. Determine the force exerted by the chin-up bar on his body at (a) $t = 0$, (b) $t = 0.5$ s, (c) $t = 1.1$ s, and (d) $t = 1.6$ s.

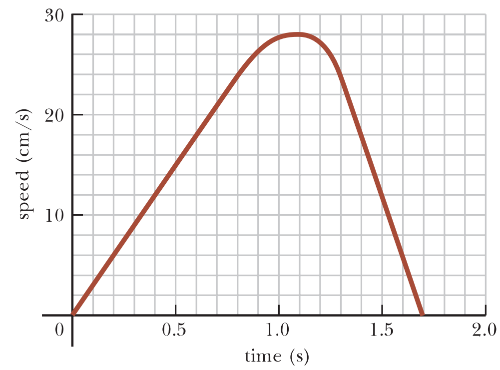



**Figure P5.41**

**42.** Two objects are connected by a light string that passes over a frictionless pulley as shown in Figure P5.42. Assume the incline is frictionless and take $m_1 = 2.00$ kg, $m_2 = 6.00$ kg, and $\theta = 55.0°$. (a) Draw free-body diagrams of both objects. Find (b) the magnitude of the acceleration of the objects, (c) the tension in the string, and (d) the speed of each object 2.00 s after it is released from rest.

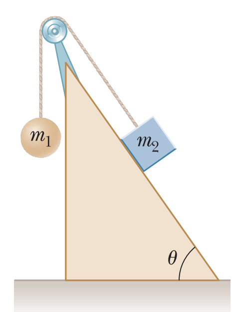



**Figure P5.42**

**43.** Two blocks, each of mass $m = 3.50$ kg, are hung from the ceiling of an elevator as in Figure P5.43. (a) If the elevator moves with an upward acceleration $\vec{\mathbf{a}}$ of magnitude 1.60 m/s$^2$, find the tensions $T_1$ and $T_2$ in the upper and lower strings. (b) If the strings can withstand a maximum tension of 85.0 N, what maximum acceleration can the elevator have before a string breaks?

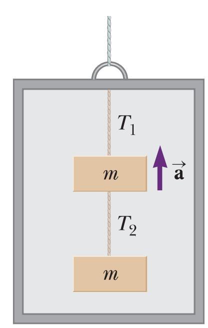



**Figure P5.43** Problems 43 and 44.

**44.** Two blocks, each of mass $m$, are hung from the ceiling of an elevator as in Figure P5.43. The elevator has an upward acceleration $a$. The strings have negligible mass. (a) Find the tensions $T_1$ and $T_2$ in the upper and lower strings in terms of $m$, $a$, and $g$. (b) Compare the two tensions and determine which string would break first if $a$ is made sufficiently large. (c) What are the tensions if the cable supporting the elevator breaks?

**45.** M In the system shown in Figure P5.45, a horizontal force $\vec{\mathbf{F}}_x$ acts on an object of mass $m_2 = 8.00$ kg. The horizontal surface is frictionless. Consider the acceleration of the sliding object as a function of $F_x$. (a) For what values of $F_x$ does the object of mass $m_1 = 2.00$ kg accelerate upward? (b) For what values of $F_x$ is the tension in the cord zero? (c) Plot the acceleration of the $m_2$ object versus $F_x$. Include values of $F_x$ from −100 N to +100 N.

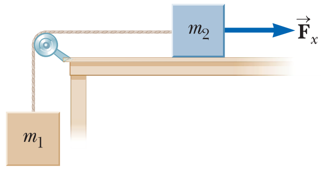



Figure P5.45

**46.** An object of mass $m_1$ hangs from a string that passes over a very light fixed pulley $P_1$ as shown in Figure P5.46. The string connects to a second very light pulley $P_2$. A second string passes around this pulley with one end attached to a wall and the other to an object of mass $m_2$ on a frictionless, horizontal table. (a) If $a_1$ and $a_2$ are the accelerations of $m_1$ and $m_2$, respectively, what is the relation between these accelerations? Find expressions for (b) the tensions in the strings and (c) the accelerations $a_1$ and $a_2$ in terms of the masses $m_1$ and $m_2$, and $g$.

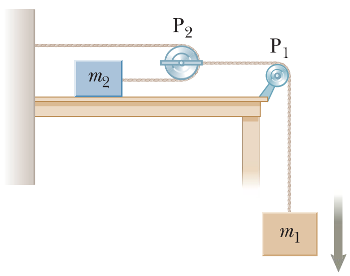



Figure P5.46

**47.** A block is given an initial velocity of 5.00 m/s up a frictionless incline of angle $\theta = 20.0°$ (Fig. P5.47). How far up the incline does the block slide before coming to rest?

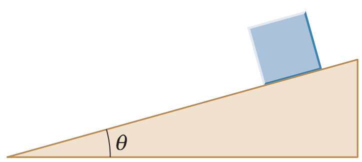



Figure P5.47

**48.** A car is stuck in the mud. A tow truck pulls on the car with the arrangement shown in Fig. P5.48. The tow cable is under a tension of 2 500 N and pulls downward and to the left on the pin at its upper end. The light pin is held in equilibrium by forces exerted by the two bars A and B. Each bar is a *strut*; that is, each is a bar whose weight is small compared to the forces it exerts and which exerts forces only through hinge pins at its ends. Each strut exerts a force directed parallel to its length. Determine the force of tension or compression in each strut. Proceed as follows. Make a guess as to which way (pushing or pulling) each force acts on the top pin. Draw a free-body diagram of the pin. Use the condition for equilibrium of the pin to translate the free-body diagram into equations. From the equations calculate the forces exerted by struts A and B. If you obtain a positive answer, you correctly guessed the direction of the force. A negative answer means that the direction should be reversed, but the absolute value correctly gives the magnitude of the force. If a strut pulls on a pin, it is in tension. If it pushes, the strut is in compression. Identify whether each strut is in tension or in compression.

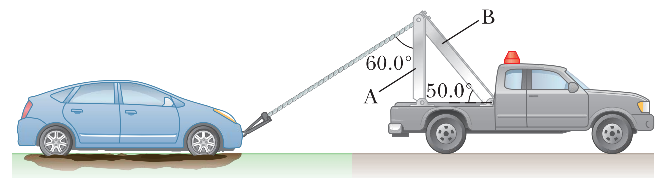



Figure P5.48

**49.** Two blocks of mass 3.50 kg and 8.00 kg are connected by a massless string that passes over a frictionless pulley (Fig. P5.49). The inclines are frictionless. Find (a) the magnitude of the acceleration of each block and (b) the tension in the string.

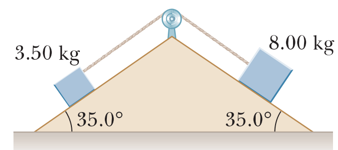



Figure P5.49 Problems 49 and 71.

**50.** In the Atwood machine discussed in Example 5.9 and shown in Figure 5.14a, $m_1 = 2.00$ kg and $m_2 = 7.00$ kg. The masses of the pulley and string are negligible by comparison. The pulley turns without friction, and the string does not stretch. The lighter object is released with a sharp push that sets it into motion at $v_i = 2.40$ m/s downward. (a) How far will $m_1$ descend below its initial level? (b) Find the velocity of $m_1$ after 1.80 s.

**51.** AMT M In Example 5.8, we investigated the apparent weight of a fish in an elevator. Now consider a 72.0-kg man standing on a spring scale in an elevator. Starting from rest, the elevator ascends, attaining its maximum speed of 1.20 m/s in 0.800 s. It travels with this constant speed for the next 5.00 s. The elevator then undergoes a uniform acceleration in the negative $y$ direction for 1.50 s and comes to rest. What does the spring scale register (a) before the elevator starts to move, (b) during the first 0.800 s, (c) while the elevator is traveling at constant speed, and (d) during the time interval it is slowing down?

## Section 5.8 Forces of Friction

**52.** Consider a large truck carrying a heavy load, such as steel beams. A significant hazard for the driver is that the load may slide forward, crushing the cab, if the truck stops suddenly in an accident or even in braking. Assume, for example, that a 10 000-kg load sits on the flatbed of a 20 000-kg truck moving at 12.0 m/s. Assume that the load is not tied down to the truck, but has a coefficient of friction of 0.500 with the flatbed of the truck. (a) Calculate the minimum stopping distance for which the load will not slide forward relative to the truck. (b) Is any piece of data unnecessary for the solution?

**53.** **Review.** A rifle bullet with a mass of 12.0 g traveling toward the right at 260 m/s strikes a large bag of sand and penetrates it to a depth of 23.0 cm. Determine the magnitude and direction of the friction force (assumed constant) that acts on the bullet.

**54.** **Review.** A car is traveling at 50.0 mi/h on a horizontal highway. (a) If the coefficient of static friction between road and tires on a rainy day is 0.100, what is the minimum distance in which the car will stop? (b) What is the stopping distance when the surface is dry and $\mu_s = 0.600$?

**55.** W A 25.0-kg block is initially at rest on a horizontal surface. A horizontal force of 75.0 N is required to set the block in motion, after which a horizontal force of 60.0 N is required to keep the block moving with constant speed. Find (a) the coefficient of static friction and (b) the coefficient of kinetic friction between the block and the surface.

**56.** *Why is the following situation impossible?* Your 3.80-kg physics book is placed next to you on the horizontal seat of your car. The coefficient of static friction between the book and the seat is 0.650, and the coefficient of kinetic friction is 0.550. You are traveling forward at 72.0 km/h and brake to a stop with constant acceleration over a distance of 30.0 m. Your physics book remains on the seat rather than sliding forward onto the floor.

**57.** To determine the coefficients of friction between rubber and various surfaces, a student uses a rubber eraser and an incline. In one experiment, the eraser begins to slip down the incline when the angle of inclination is 36.0° and then moves down the incline with constant speed when the angle is reduced to 30.0°. From these data, determine the coefficients of static and kinetic friction for this experiment.

**58.** Before 1960, people believed that the maximum attainable coefficient of static friction for an automobile tire on a roadway was $\mu_s = 1$. Around 1962, three companies independently developed racing tires with coefficients of 1.6. This problem shows that tires have improved further since then. The shortest time interval in which a piston-engine car initially at rest has covered a distance of one-quarter mile is about 4.43 s. (a) Assume the car's rear wheels lift the front wheels off the pavement as shown in Figure P5.58. What minimum value of $\mu_s$ is necessary to achieve the record time? (b) Suppose the driver were able to increase his or her engine power, keeping other things equal. How would this change affect the elapsed time?

Jamie Squire/Allsport/Getty Images

Figure P5.58

**59.** To meet a U.S. Postal Service requirement, employees' footwear must have a coefficient of static friction of 0.5 or more on a specified tile surface. A typical athletic shoe has a coefficient of static friction of 0.800. In an emergency, what is the minimum time interval in which a person starting from rest can move 3.00 m on the tile surface if she is wearing (a) footwear meeting the Postal Service minimum and (b) a typical athletic shoe?

**60.** W A woman at an airport is towing her 20.0-kg suitcase at constant speed by pulling on a strap at an angle $\theta$ above the horizontal (Fig. P5.60). She pulls on the strap with a 35.0-N force, and the friction force on the suitcase is 20.0 N. (a) Draw a free-body diagram of the suitcase. (b) What angle does the strap make with the horizontal? (c) What is the magnitude of the normal force that the ground exerts on the suitcase?

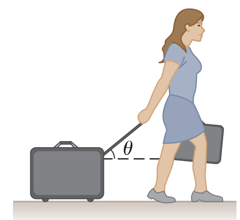



Figure P5.60

**61.** M **Review.** A 3.00-kg block starts from rest at the top of a 30.0° incline and slides a distance of 2.00 m down the incline in 1.50 s. Find (a) the magnitude of the acceleration of the block, (b) the coefficient of kinetic friction between block and plane, (c) the friction force acting on the block, and (d) the speed of the block after it has slid 2.00 m.

**62.** The person in Figure P5.62 weighs 170 lb. As seen from the front, each light crutch makes an angle of 22.0° with the vertical. Half of the person's weight is supported by the crutches. The other half is supported by the vertical forces of the ground on the person's feet. Assuming that the person is moving with constant velocity and the force exerted by the ground on the crutches acts along the crutches, determine (a) the smallest possible coefficient of friction between crutches and ground and (b) the magnitude of the compression force in each crutch.

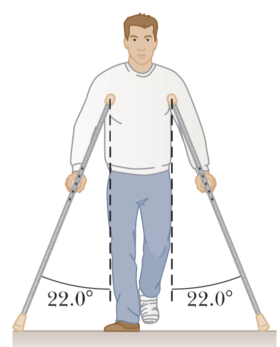



Figure P5.62

**63.** W A 9.00-kg hanging object is connected by a light, inextensible cord over a light, frictionless pulley to a 5.00-kg block that is sliding on a flat table (Fig. P5.40). Taking the coefficient of kinetic friction as 0.200, find the tension in the string.

**64.** Three objects are connected on a table as shown in Figure P5.64. The coefficient of kinetic friction between the block of mass $m_2$ and the table is 0.350. The objects have masses of $m_1$ = 4.00 kg, $m_2$ = 1.00 kg, and $m_3$ =

2.00 kg, and the pulleys are frictionless. (a) Draw a free-body diagram of each object. (b) Determine the acceleration of each object, including its direction. (c) Determine the tensions in the two cords. **What If?** (d) If the tabletop were smooth, would the tensions increase, decrease, or remain the same? Explain.

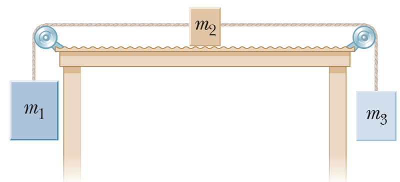



**Figure P5.64**

**65.** AMT M Two blocks connected by a rope of negligible mass are being dragged by a horizontal force (Fig. P5.65). Suppose $F = 68.0$ N, $m_1 = 12.0$ kg, $m_2 = 18.0$ kg, and the coefficient of kinetic friction between each block and the surface is 0.100. (a) Draw a free-body diagram for each block. Determine (b) the acceleration of the system and (c) the tension $T$ in the rope.

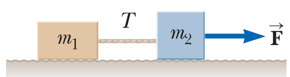



**Figure P5.65**

**66.** A block of mass 3.00 kg is pushed up against a wall by a force $\vec{\mathbf{P}}$ that makes an angle of $\theta = 50.0°$ with the horizontal as shown in Figure P5.66. The coefficient of static friction between the block and the wall is 0.250. (a) Determine the possible values for the magnitude of $\vec{\mathbf{P}}$ that allow the block to remain stationary. (b) Describe what happens if $|\vec{\mathbf{P}}|$ has a larger value and what happens if it is smaller. (c) Repeat parts (a) and (b), assuming the force makes an angle of $\theta = 13.0°$ with the horizontal.

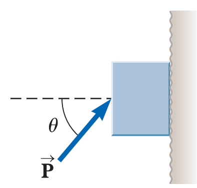



**Figure P5.66**

**67.** **Review.** One side of the roof of a house slopes up at 37.0°. A roofer kicks a round, flat rock that has been thrown onto the roof by a neighborhood child. The rock slides straight up the incline with an initial speed of 15.0 m/s. The coefficient of kinetic friction between the rock and the roof is 0.400. The rock slides 10.0 m up the roof to its peak. It crosses the ridge and goes into free fall, following a parabolic trajectory above the far side of the roof, with negligible air resistance. Determine the maximum height the rock reaches above the point where it was kicked.

**68.** **Review.** A Chinook salmon can swim underwater at 3.58 m/s, and it can also jump vertically upward, leaving the water with a speed of 6.26 m/s. A record salmon has length 1.50 m and mass 61.0 kg. Consider the fish swimming straight upward in the water below the surface of a lake. The gravitational force exerted on it is very nearly canceled out by a buoyant force exerted by the water as we will study in Chapter 14. The fish experiences an upward force $P$ exerted by the water on its threshing tail fin and a downward fluid friction force that we model as acting on its front end. Assume the fluid friction force disappears as soon as the fish's head breaks the water surface and assume the force on its tail is constant. Model the gravitational force as suddenly switching full on when half the length of the fish is out of the water. Find the value of $P$.

**69.** **Review.** A magician pulls a tablecloth from under a 200-g mug located 30.0 cm from the edge of the cloth. The cloth exerts a friction force of 0.100 N on the mug, and the cloth is pulled with a constant acceleration of 3.00 m/s$^2$. How far does the mug move relative to the horizontal tabletop before the cloth is completely out from under it? Note that the cloth must move more than 30 cm relative to the tabletop during the process.

**70.** A 5.00-kg block is placed on top of a 10.0-kg block (Fig. P5.70). A horizontal force of 45.0 N is applied to the 10-kg block, and the 5.00-kg block is tied to the wall. The coefficient of kinetic friction between all moving surfaces is 0.200. (a) Draw a free-body diagram for each block and identify the action–reaction forces between the blocks. (b) Determine the tension in the string and the magnitude of the acceleration of the 10.0-kg block.

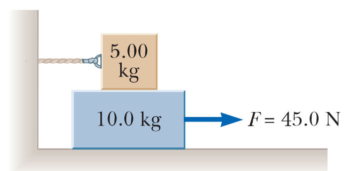



**Figure P5.70**

**71.** The system shown in Figure P5.49 has an acceleration of magnitude 1.50 m/s$^2$. Assume that the coefficient of kinetic friction between block and incline is the same for both inclines. Find (a) the coefficient of kinetic friction and (b) the tension in the string.

## Additional Problems

**72.** A black aluminum glider floats on a film of air above a level aluminum air track. Aluminum feels essentially no force in a magnetic field, and air resistance is negligible. A strong magnet is attached to the top of the glider, forming a total mass of 240 g. A piece of scrap iron attached to one end stop on the track attracts the magnet with a force of 0.823 N when the iron and the magnet are separated by 2.50 cm. (a) Find the acceleration of the glider at this instant. (b) The scrap iron is now attached to another green glider, forming total mass 120 g. Find the acceleration of each glider when the gliders are simultaneously released at 2.50-cm separation.

**73.** A young woman buys an inexpensive used car for stock car racing. It can attain highway speed with an acceleration of 8.40 mi/h · s. By making changes to its engine, she can increase the net horizontal force on the car by 24.0%. With much less expense, she can remove material from the body of the car to decrease its mass by 24.0%. (a) Which of these two changes, if either, will result in the greater increase in the car's acceleration? (b) If she makes both changes, what acceleration can she attain?

**74.** *Why is the following situation impossible?* A book sits on an inclined plane on the surface of the Earth. The angle of the plane with the horizontal is 60.0°. The coefficient of kinetic friction between the book and the plane is 0.300. At time $t = 0$, the book is released from rest. The book then slides through a distance of 1.00 m, measured along the plane, in a time interval of 0.483 s.

**75.** **Review.** A hockey puck struck by a hockey stick is given an initial speed $v_i$ in the positive $x$ direction. The coefficient of kinetic friction between the ice and the puck is $\mu_k$. (a) Obtain an expression for the acceleration of the puck as it slides across the ice. (b) Use the result of part (a) to obtain an expression for the distance $d$ the puck slides. The answer should be in terms of the variables $v_i$, $\mu_k$, and $g$ only.

**76.** A 1.00-kg glider on a horizontal air track is pulled by a string at an angle $\theta$. The taut string runs over a pulley and is attached to a hanging object of mass 0.500 kg as shown in Figure P5.76. (a) Show that the speed $v_x$ of the glider and the speed $v_y$ of the hanging object are related by $v_x = uv_y$, where $u = z(z^2 - h_0^2)^{-1/2}$. (b) The glider is released from rest. Show that at that instant the acceleration $a_x$ of the glider and the acceleration $a_y$ of the hanging object are related by $a_x = ua_y$. (c) Find the tension in the string at the instant the glider is released for $h_0 = 80.0$ cm and $\theta = 30.0°$.

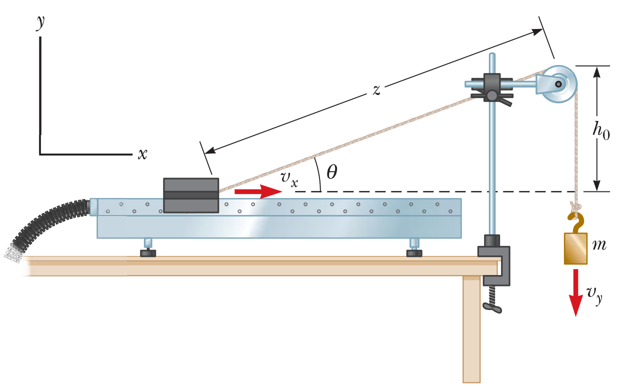



Figure P5.76

**77.** M A frictionless plane is 10.0 m long and inclined at 35.0°. A sled starts at the bottom with an initial speed of 5.00 m/s up the incline. When the sled reaches the point at which it momentarily stops, a second sled is released from the top of the incline with an initial speed $v_i$. Both sleds reach the bottom of the incline at the same moment. (a) Determine the distance that the first sled traveled up the incline. (b) Determine the initial speed of the second sled.

**78.** A rope with mass $m_r$ is attached to a block with mass $m_b$ as in Figure P5.78. The block rests on a frictionless, horizontal surface. The rope does not stretch. The free end of the rope is pulled to the right with a horizontal force $\vec{\mathbf{F}}$. (a) Draw force diagrams for the rope and the block, noting that the tension in the rope is not uniform. (b) Find the acceleration of the system in terms of $m_b$, $m_r$, and $F$. (c) Find the magnitude of the force the rope exerts on the block. (d) What happens to the force on the block as the rope's mass approaches zero? What can you state about the tension in a *light* cord joining a pair of moving objects?

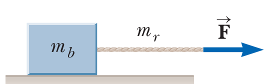



Figure P5.78

**79.** GP Two blocks of masses $m_1$ and $m_2$ are placed on a table in contact with each other as discussed in Example 5.7 and shown in Figure 5.12a. The coefficient of kinetic friction between the block of mass $m_1$ and the table is $\mu_1$, and that between the block of mass $m_2$ and the table is $\mu_2$. A horizontal force of magnitude $F$ is applied to the block of mass $m_1$. We wish to find $P$, the magnitude of the contact force between the blocks. (a) Draw diagrams showing the forces for each block. (b) What is the net force on the system of two blocks? (c) What is the net force acting on $m_1$? (d) What is the net force acting on $m_2$? (e) Write Newton's second law in the $x$ direction for each block. (f) Solve the two equations in two unknowns for the acceleration $a$ of the blocks in terms of the masses, the applied force $F$, the coefficients of friction, and $g$. (g) Find the magnitude $P$ of the contact force between the blocks in terms of the same quantities.

**80.** On a single, light, vertical cable that does not stretch, a crane is lifting a 1 207-kg Ferrari and, below it, a 1 461-kg BMW Z8. The Ferrari is moving upward with speed 3.50 m/s and acceleration 1.25 m/s². (a) How do the velocity and acceleration of the BMW compare with those of the Ferrari? (b) Find the tension in the cable between the BMW and the Ferrari. (c) Find the tension in the cable above the Ferrari.

**81.** An inventive child named Nick wants to reach an apple in a tree without climbing the tree. Sitting in a chair connected to a rope that passes over a frictionless pulley (Fig. P5.81), Nick pulls on the loose end of the rope with such a force that the spring scale reads 250 N. Nick's true weight is 320 N, and the chair weighs 160 N. Nick's feet are not touching the ground. (a) Draw one pair of diagrams showing the forces for Nick and the chair considered as separate systems and another diagram for Nick and the chair considered as one system. (b) Show that the acceleration of the system is *upward* and find its magnitude. (c) Find the force Nick exerts on the chair.

Figure P5.81 Problems 81 and 82.

**82.** In the situation described in Problem 81 and Figure P5.81, the masses of the rope, spring balance, and pulley are negligible. Nick's feet are not touching the ground. (a) Assume Nick is momentarily at rest when he stops pulling down on the rope and passes the end of the rope to another child, of weight 440 N, who is standing on the ground next to him. The rope does not break. Describe the ensuing motion. (b) Instead, assume Nick is momentarily at rest when he ties the end of the rope to a strong hook projecting from the tree trunk. Explain why this action can make the rope break.

**83.** In Example 5.7, we pushed on two blocks on a table. Suppose three blocks are in contact with one another on a frictionless, horizontal surface as shown in Figure P5.83. A horizontal force $\vec{\mathbf{F}}$ is applied to $m_1$. Take $m_1 = 2.00$ kg, $m_2 = 3.00$ kg, $m_3 = 4.00$ kg, and $F = 18.0$ N. (a) Draw a separate free-body diagram for each block. (b) Determine the acceleration of the blocks. (c) Find the *resultant* force on each block. (d) Find the magnitudes of the contact forces between the blocks. (e) You are working on a construction project. A coworker is nailing up plasterboard on one side of a light partition, and you are on the opposite side, providing "backing" by leaning against the wall with your back pushing on it. Every hammer blow makes your back sting. The supervisor helps you put a heavy block of wood between the wall and your back. Using the situation analyzed in parts (a) through (d) as a model, explain how this change works to make your job more comfortable.

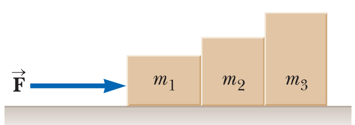



**Figure P5.83**

**84.** An aluminum block of mass $m_1 = 2.00$ kg and a copper block of mass $m_2 = 6.00$ kg are connected by a light string over a frictionless pulley. They sit on a steel surface as shown in Figure P5.84, where $\theta = 30.0°$. (a) When they are released from rest, will they start to move? If they do, determine (b) their acceleration and (c) the tension in the string. If they do not move, determine (d) the sum of the magnitudes of the forces of friction acting on the blocks.

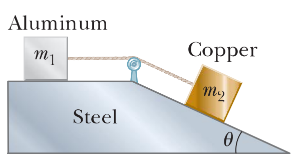



**Figure P5.84**

**85.** An object of mass $M$ is held in place by an applied force $\vec{\mathbf{F}}$ and a pulley system as shown in Figure P5.85. The pulleys are massless and frictionless. (a) Draw diagrams showing the forces on each pulley. Find (b) the tension in each section of rope, $T_1$, $T_2$, $T_3$, $T_4$, and $T_5$ and (c) the magnitude of $\vec{\mathbf{F}}$.

**86.** Any device that allows you to increase the force you exert is a kind of *machine*. Some machines, such as the prybar or the inclined plane, are very simple. Some machines do not even look like machines. For example, your car is stuck in the mud and you can't pull hard enough to get it out. You do, however, have a long cable that you connect taut between your front bumper and the trunk of a stout tree. You now pull sideways on the cable at its midpoint, exerting a force $f$. Each half of the cable is displaced through a small angle $\theta$ from the straight line between the ends of the cable. (a) Deduce an expression for the force acting on the car. (b) Evaluate the cable tension for the case where $\theta = 7.00°$ and $f = 100$ N.

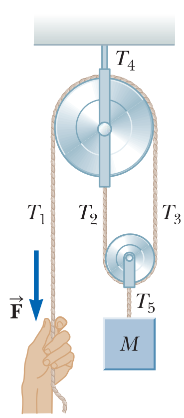



**Figure P5.85**

**87.** M Objects with masses $m_1 = 10.0$ kg and $m_2 = 5.00$ kg are connected by a light string that passes over a frictionless pulley as in Figure P5.40. If, when the system starts from rest, $m_2$ falls 1.00 m in 1.20 s, determine the coefficient of kinetic friction between $m_1$ and the table.

**88.** Consider the three connected objects shown in Figure P5.88. Assume first that the inclined plane is frictionless and that the system is in equilibrium. In terms of $m$, $g$, and $\theta$, find (a) the mass $M$ and (b) the tensions $T_1$ and $T_2$. Now assume that the value of $M$ is double the value found in part (a). Find (c) the acceleration of each object and (d) the tensions $T_1$ and $T_2$. Next, assume that the coefficient of static friction between $m$ and $2m$ and the inclined plane is $m_s$ and that the system is in equilibrium. Find (e) the maximum value of $M$ and (f) the minimum value of $M$. (g) Compare the values of $T_2$ when $M$ has its minimum and maximum values.

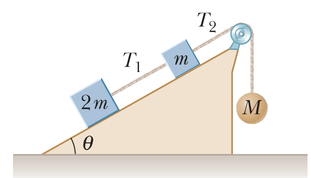



**Figure P5.88**

**89.** A crate of weight $F_g$ is pushed by a force $\vec{\mathbf{P}}$ on a horizontal floor as shown in Figure P5.89. The coefficient of static friction is $\mu_s$, and $\vec{\mathbf{P}}$ is directed at angle $\theta$ below the horizontal. (a) Show that the minimum value of $P$ that will move the crate is given by

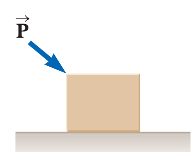



**Figure P5.89**

$$P = \frac{\mu_s F_g \sec\theta}{1 - \mu_s \tan\theta}$$

(b) Find the condition on $\theta$ in terms of $\mu_s$ for which motion of the crate is impossible for any value of $P$.

**90.** A student is asked to measure the acceleration of a glider on a frictionless, inclined plane, using an air track, a stopwatch, and a meterstick. The top of the track is measured to be 1.774 cm higher than the bottom of the track, and the length of the track is $d = 127.1$ cm. The cart is released from rest at the top of the incline, taken as $x = 0$, and its position $x$ along the incline is measured as a function of time. For $x$ values of 10.0 cm, 20.0 cm, 35.0 cm, 50.0 cm, 75.0 cm, and 100 cm, the measured times at which these positions are reached (averaged over five runs) are 1.02 s, 1.53 s, 2.01 s, 2.64 s, 3.30 s, and 3.75 s, respectively. (a) Construct a graph of $x$ versus $t^2$, with a best-fit straight line to describe the data. (b) Determine the acceleration of the cart from the slope of this graph. (c) Explain how your answer to part (b) compares with the theoretical value you calculate using $a = g \sin \theta$ as derived in Example 5.6.

**91.** A flat cushion of mass $m$ is released from rest at the corner of the roof of a building, at height $h$. A wind blowing along the side of the building exerts a constant horizontal force of magnitude $F$ on the cushion as it drops as shown in Figure P5.91. The air exerts no vertical force. (a) Show that the path of the cushion is a straight line. (b) Does the cushion fall with constant velocity? Explain. (c) If $m = 1.20$ kg, $h = 8.00$ m, and $F = 2.40$ N, how far from the building will the cushion hit the level ground? **What If?** (d) If the cushion is thrown downward with a nonzero speed at the top of the building, what will be the shape of its trajectory? Explain.

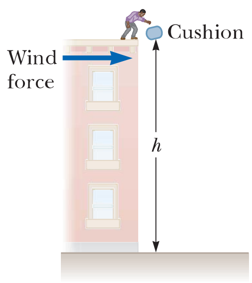


Figure P5.91

**92.** In Figure P5.92, the pulleys and the cord are light, all surfaces are frictionless, and the cord does not stretch. (a) How does the acceleration of block 1 compare with the acceleration of block 2? Explain your reasoning. (b) The mass of block 2 is 1.30 kg. Find its acceleration as it depends on the mass $m_1$ of block 1. (c) **What If?** What does the result of part (b) predict if $m_1$ is very much less than 1.30 kg? (d) What does the result of part (b) predict if $m_1$ approaches infinity? (e) In this last case, what is the tension in the cord? (f) Could you anticipate the answers to parts (c), (d), and (e) without first doing part (b)? Explain.

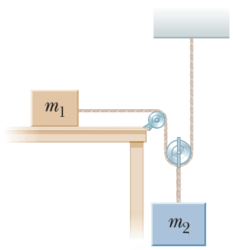


Figure P5.92

**93.** What horizontal force must be applied to a large block of mass $M$ shown in Figure P5.93 so that the tan blocks remain stationary relative to $M$? Assume all surfaces and the pulley are frictionless. Notice that the force exerted by the string accelerates $m_2$.

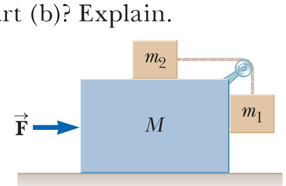


Figure P5.93
Problems 93 and 98.

**94.** An 8.40-kg object slides down a fixed, frictionless, inclined plane. Use a computer to determine and tabulate (a) the normal force exerted on the object and (b) its acceleration for a series of incline angles (measured from the horizontal) ranging from 0° to 90° in 5° increments. (c) Plot a graph of the normal force and the acceleration as functions of the incline angle. (d) In the limiting cases of 0° and 90°, are your results consistent with the known behavior?

**95.** M A car accelerates down a hill (Fig. P5.95), going from rest to 30.0 m/s in 6.00 s. A toy inside the car hangs by a string from the car's ceiling. The ball in the figure represents the toy, of mass 0.100 kg. The acceleration is such that the string remains perpendicular to the ceiling. Determine (a) the angle $\theta$ and (b) the tension in the string.

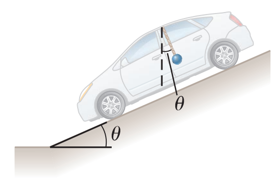


Figure P5.95

## Challenge Problems

**96.** A time-dependent force, $\vec{F} = (8.00\hat{i} - 4.00t\hat{j})$, where $\vec{F}$ is in newtons and $t$ is in seconds, is exerted on a 2.00-kg object initially at rest. (a) At what time will the object be moving with a speed of 15.0 m/s? (b) How far is the object from its initial position when its speed is 15.0 m/s? (c) Through what total displacement has the object traveled at this moment?

**97.** The board sandwiched between two other boards in Figure P5.97 weighs 95.5 N. If the coefficient of static friction between the boards is 0.663, what must be the magnitude of the compression forces (assumed horizontal) acting on both sides of the center board to keep it from slipping?

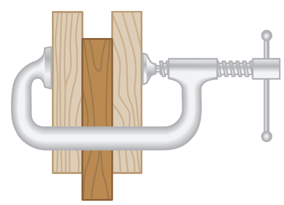


Figure P5.97

**98.** Initially, the system of objects shown in Figure P5.93 is held motionless. The pulley and all surfaces and wheels are frictionless. Let the force $\vec{F}$ be zero and assume that $m_1$ can move only vertically. At the instant after the system of objects is released, find (a) the tension $T$ in the string, (b) the acceleration of $m_2$, (c) the acceleration of $M$, and (d) the acceleration of $m_1$. (*Note:* The pulley accelerates along with the cart.)

**99.** A block of mass 2.20 kg is accelerated across a rough surface by a light cord passing over a small pulley as shown in Figure P5.99. The tension $T$ in the cord is maintained at 10.0 N, and the pulley is 0.100 m above the top of the block. The coefficient of kinetic friction is 0.400. (a) Determine the acceleration of the block when $x = 0.400$ m. (b) Describe the general behavior of the acceleration as the block slides from a location where $x$ is large to $x = 0$. (c) Find the maximum value of the acceleration and the position $x$ for which it occurs. (d) Find the value of $x$ for which the acceleration is zero.

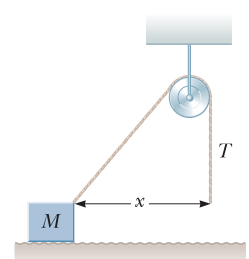


Figure P5.99

**100.** *Why is the following situation impossible?* A 1.30-kg toaster is not plugged in. The coefficient of static friction between the toaster and a horizontal countertop is 0.350. To make the toaster start moving, you carelessly pull on its electric cord. Unfortunately, the cord has become frayed from your previous similar actions and will break if the tension in the cord exceeds 4.00 N. By pulling on the cord at a particular angle, you successfully start the toaster moving without breaking the cord.

**101.** **Review.** A block of mass $m = 2.00$ kg is released from rest at $h = 0.500$ m above the surface of a table, at the top of a $\theta = 30.0°$ incline as shown in Figure P5.101. The frictionless incline is fixed on a table of height $H = 2.00$ m. (a) Determine the acceleration of the block as it slides down the incline. (b) What is the velocity of the block as it leaves the incline? (c) How far from the table will the block hit the floor? (d) What time interval elapses between when the block is released and when it hits the floor? (e) Does the mass of the block affect any of the above calculations?

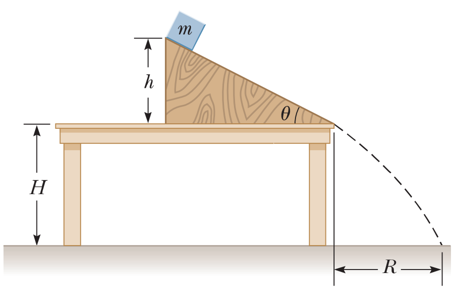



**Figure P5.101** Problems 101 and 102.

**102.** In Figure P5.101, the incline has mass $M$ and is fastened to the stationary horizontal tabletop. The block of mass $m$ is placed near the bottom of the incline and is released with a quick push that sets it sliding upward. The block stops near the top of the incline as shown in the figure and then slides down again, always without friction. Find the force that the tabletop exerts on the incline throughout this motion in terms of $m$, $M$, $g$, and $\theta$.

**103.** A block of mass $m = 2.00$ kg rests on the left edge of a block of mass $M = 8.00$ kg. The coefficient of kinetic friction between the two blocks is 0.300, and the surface on which the 8.00-kg block rests is frictionless. A constant horizontal force of magnitude $F = 10.0$ N is applied to the 2.00-kg block, setting it in motion as shown in Figure P5.103a. If the distance $L$ that the leading edge of the smaller block travels on the larger block is 3.00 m, (a) in what time interval will the smaller block make it to the right side of the 8.00-kg block as shown in Figure P5.103b? (*Note:* Both blocks are set into motion when $\vec{\mathbf{F}}$ is applied.) (b) How far does the 8.00-kg block move in the process?

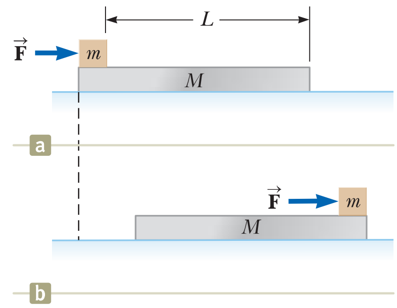



**Figure P5.103**

**104.** A mobile is formed by supporting four metal butterflies of equal mass $m$ from a string of length $L$. The points of support are evenly spaced a distance $\ell$ apart as shown in Figure P5.104. The string forms an angle $\theta_1$ with the ceiling at each endpoint. The center section of string is horizontal. (a) Find the tension in each section of string in terms of $\theta_1$, $m$, and $g$. (b) In terms of $\theta_1$, find the angle $\theta_2$ that the sections of string between the outside butterflies and the inside butterflies form with the horizontal. (c) Show that the distance $D$ between the endpoints of the string is

$$D = \frac{L}{5}\left\{2\cos\theta_1 + 2\cos\left[\tan^{-1}\left(\tfrac{1}{2}\tan\theta_1\right)\right] + 1\right\}$$

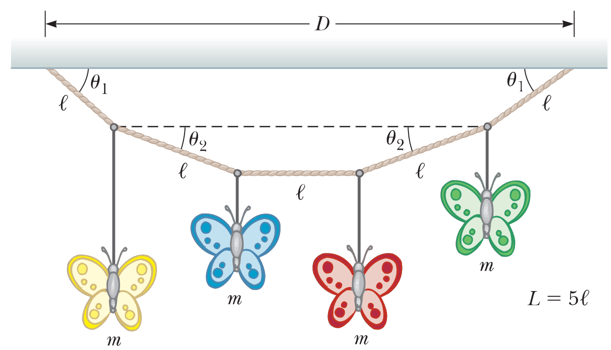



**Figure P5.104**