

chapter 44

Nuclear Structure

- 44.1 Some Properties of Nuclei
- 44.2 Nuclear Binding Energy
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The year 1896 marks the birth of nuclear physics when French physicist Antoine-

Henri Becquerel (1852–1908) discovered radioactivity in uranium compounds. This discovery prompted scientists to investigate the details of radioactivity and, ultimately, the structure of the nucleus. Pioneering work by Ernest Rutherford showed that the radiation emitted from radioactive substances is of three types—alpha, beta, and gamma rays—classified according to the nature of their electric charge and their ability to penetrate matter and ionize air. Later experiments showed that alpha rays are helium nuclei, beta rays are electrons, and gamma rays are high-energy photons.

In 1911, Rutherford, Hans Geiger, and Ernest Marsden performed the alpha-particle scattering experiments described in Section 42.2. These experiments established that the nucleus of an atom can be modeled as a point mass and point charge and that most of the atomic mass is contained in the nucleus. Subsequent studies revealed the pres-



Ötzi the Iceman, a Copper Age man, was discovered by German tourists in the Italian Alps in 1991 when a glacier melted enough to expose his remains. Analysis of his corpse has exposed his last meal, illnesses he suffered, and places he lived. Radioactivity was used to determine that he lived in about 3300 BC. (© Paul Hanny/Gamma Liaison)

ence of a new type of force, the short-range nuclear force, which is predominant at particle separation distances less than approximately 10^{-14} m and is zero for large distances.

In this chapter, we discuss the properties and structure of the atomic nucleus. We start by describing the basic properties of nuclei, followed by a discussion of nuclear forces and binding energy, nuclear models, and the phenomenon of radioactivity. Finally, we explore nuclear reactions and the various processes by which nuclei decay.

44.1 Some Properties of Nuclei

All nuclei are composed of two types of particles: protons and neutrons. The only exception is the ordinary hydrogen nucleus, which is a single proton. We describe the atomic nucleus by the number of protons and neutrons it contains, using the following quantities:

- the **atomic number** Z , which equals the number of protons in the nucleus (sometimes called the *charge number*)
- the **neutron number** N , which equals the number of neutrons in the nucleus
- the **mass number** $A = Z + N$, which equals the number of **nucleons** (neutrons plus protons) in the nucleus

A **nuclide** is a specific combination of atomic number and mass number that represents a nucleus. In representing nuclides, it is convenient to use the symbol A_ZX to convey the numbers of protons and neutrons, where X represents the chemical symbol of the element. For example, ${}^{56}_{26}\text{Fe}$ (iron) has mass number 56 and atomic number 26; therefore, it contains 26 protons and 30 neutrons. When no confusion is likely to arise, we omit the subscript Z because the chemical symbol can always be used to determine Z . Therefore, ${}^{56}_{26}\text{Fe}$ is the same as ${}^{56}\text{Fe}$ and can also be expressed as “iron-56.”

The nuclei of all atoms of a particular element contain the same number of protons but often contain different numbers of neutrons. Nuclei related in this way are called **isotopes**. The isotopes of an element have the same Z value but different N and A values.

The natural abundance of isotopes can differ substantially. For example ${}^{11}_6\text{C}$, ${}^{12}_6\text{C}$, ${}^{13}_6\text{C}$, and ${}^{14}_6\text{C}$ are four isotopes of carbon. The natural abundance of the ${}^{12}_6\text{C}$ isotope is approximately 98.9%, whereas that of the ${}^{13}_6\text{C}$ isotope is only about 1.1%. Some isotopes, such as ${}^{11}_6\text{C}$ and ${}^{14}_6\text{C}$, do not occur naturally but can be produced by nuclear reactions in the laboratory or by cosmic rays.

Even the simplest element, hydrogen, has isotopes: ${}^1_1\text{H}$, the ordinary hydrogen nucleus; ${}^2_1\text{H}$, deuterium; and ${}^3_1\text{H}$, tritium.

Quick Quiz 44.1 For each part of this Quick Quiz, choose from the following answers: (a) protons (b) neutrons (c) nucleons. **(i)** The three nuclei ${}^{12}_6\text{C}$, ${}^{13}_7\text{N}$, and ${}^{14}_8\text{O}$ have the same number of what type of particle? **(ii)** The three nuclei ${}^{12}_7\text{N}$, ${}^{13}_7\text{N}$, and ${}^{14}_7\text{N}$ have the same number of what type of particle? **(iii)** The three nuclei ${}^{14}_6\text{C}$, ${}^{14}_7\text{N}$, and ${}^{14}_8\text{O}$ have the same number of what type of particle?

Charge and Mass

The proton carries a single positive charge e , equal in magnitude to the charge $-e$ on the electron ($e = 1.6 \times 10^{-19}$ C). The neutron is electrically neutral as its name implies. Because the neutron has no charge, it was difficult to detect with early experimental apparatus and techniques. Today, neutrons are easily detected with devices such as plastic scintillators.

Pitfall Prevention 44.1

Mass Number Is Not Atomic Mass

The mass number A should not be confused with the atomic mass. Mass number is an integer specific to an isotope and has no units; it is simply a count of the number of nucleons. Atomic mass has units and is generally not an integer because it is an average of the masses of a given element's naturally occurring isotopes.

TABLE 44.1 Masses of Selected Particles in Various Units

Particle	kg	Mass u	MeV/c ²
Proton	$1.672\,62 \times 10^{-27}$	1.007 276	938.27
Neutron	$1.674\,93 \times 10^{-27}$	1.008 665	939.57
Electron	$9.109\,38 \times 10^{-31}$	$5.485\,79 \times 10^{-4}$	0.510 999
^1_1H atom	$1.673\,53 \times 10^{-27}$	1.007 825	938.783
^4_2He nucleus	$6.644\,66 \times 10^{-27}$	4.001 506	3 727.38
$^{12}_6\text{C}$ atom	$1.992\,65 \times 10^{-27}$	12.000 000	11 177.9

Nuclear masses can be measured with great precision using a mass spectrometer (see Section 29.3) and by the analysis of nuclear reactions. The proton is approximately 1 836 times as massive as the electron, and the masses of the proton and the neutron are almost equal. The **atomic mass unit** u is defined in such a way that the mass of one atom of the isotope ^{12}C is exactly 12 u, where 1 u is equal to $1.660\,539 \times 10^{-27}$ kg. According to this definition, the proton and neutron each have a mass of approximately 1 u and the electron has a mass that is only a small fraction of this value. The masses of these particles and others important to the phenomena discussed in this chapter are given in Table 44.1.

You might wonder how six protons and six neutrons, each having a mass larger than 1 u, can be combined with six electrons to form a carbon-12 atom having a mass of exactly 12 u. The bound system of ^{12}C has a lower rest energy (Section 39.8) than that of six separate protons and six separate neutrons. According to Equation 39.24, $E_R = mc^2$, this lower rest energy corresponds to a smaller mass for the bound system. The difference in mass accounts for the binding energy when the particles are combined to form the nucleus. We shall discuss this point in more detail in Section 44.2.

It is often convenient to express the atomic mass unit in terms of its *rest-energy equivalent*. For one atomic mass unit,

$$E_R = mc^2 = (1.660\,539 \times 10^{-27} \text{ kg})(2.997\,92 \times 10^8 \text{ m/s})^2 = 931.494 \text{ MeV}$$

where we have used the conversion $1 \text{ eV} = 1.602\,176 \times 10^{-19} \text{ J}$.

Based on the rest-energy expression in Equation 39.24, nuclear physicists often express mass in terms of the unit MeV/c^2 .

Example 44.1**The Atomic Mass Unit**

Use Avogadro's number to show that $1 \text{ u} = 1.66 \times 10^{-27} \text{ kg}$.

SOLUTION

Conceptualize From the definition of the mole given in Section 19.5, we know that exactly 12 g (= 1 mol) of ^{12}C contains Avogadro's number of atoms.

Categorize We evaluate the atomic mass unit that was introduced in this section, so we categorize this example as a substitution problem.

Find the mass m of one ^{12}C atom:

$$m = \frac{0.012 \text{ kg}}{6.02 \times 10^{23} \text{ atoms}} = 1.99 \times 10^{-26} \text{ kg}$$

Because one atom of ^{12}C is defined to have a mass of 12.0 u, divide by 12.0 to find the mass equivalent to 1 u:

$$1 \text{ u} = \frac{1.99 \times 10^{-26} \text{ kg}}{12.0} = 1.66 \times 10^{-27} \text{ kg}$$

The Size and Structure of Nuclei

In Rutherford's scattering experiments, positively charged nuclei of helium atoms (alpha particles) were directed at a thin piece of metallic foil. As the alpha particles moved through the foil, they often passed near a metal nucleus. Because of the positive charge on both the incident particles and the nuclei, the particles were deflected from their straight-line paths by the Coulomb repulsive force.

Rutherford used the isolated system (energy) analysis model to find an expression for the separation distance d at which an alpha particle approaching a nucleus head-on is turned around by Coulomb repulsion. In such a head-on collision, the kinetic energy of the incoming particle must be converted completely to electric potential energy of the alpha particle–nucleus system when the particle stops momentarily at the point of closest approach (the final configuration of the system) before moving back along the same path (Active Fig. 44.1). Applying the conservation of energy principle to the system gives

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}mv^2 + 0 = 0 + k_e \frac{q_1q_2}{d}$$

where m is the mass of the alpha particle and v is its initial speed. Solving for d gives

$$d = 2k_e \frac{q_1q_2}{mv^2} = 2k_e \frac{(2e)(Ze)}{mv^2} = 4k_e \frac{Ze^2}{mv^2}$$

where Z is the atomic number of the target nucleus. From this expression, Rutherford found that the alpha particles approached nuclei to within 3.2×10^{-14} m when the foil was made of gold. Therefore, the radius of the gold nucleus must be less than this value. From the results of his scattering experiments, Rutherford concluded that the positive charge in an atom is concentrated in a small sphere, which he called the nucleus, whose radius is no greater than approximately 10^{-14} m.

Because such small lengths are common in nuclear physics, an often-used convenient length unit is the femtometer (fm), which is sometimes called the **fermi** and is defined as

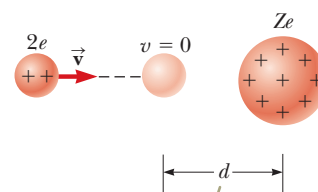
$$1 \text{ fm} \equiv 10^{-15} \text{ m}$$

In the early 1920s, it was known that the nucleus of an atom contains Z protons and has a mass nearly equivalent to that of A protons, where on average $A \approx 2Z$ for lighter nuclei ($Z \leq 20$) and $A > 2Z$ for heavier nuclei. To account for the nuclear mass, Rutherford proposed that each nucleus must also contain $A - Z$ neutral particles that he called neutrons. In 1932, British physicist James Chadwick (1891–1974) discovered the neutron, and he was awarded the Nobel Prize in Physics in 1935 for this important work.

Since the time of Rutherford's scattering experiments, a multitude of other experiments have shown that most nuclei are approximately spherical and have an average radius given by

$$r = aA^{1/3} \quad (44.1)$$

where a is a constant equal to 1.2×10^{-15} m and A is the mass number. Because the volume of a sphere is proportional to the cube of its radius, it follows from Equation 44.1 that the volume of a nucleus (assumed to be spherical) is directly proportional to A , the total number of nucleons. This proportionality suggests that *all nuclei have nearly the same density*. When nucleons combine to form a nucleus, they combine as though they were tightly packed spheres (Fig. 44.2). This fact has led to an analogy between the nucleus and a drop of liquid, in which the density of the drop is independent of its size. We shall discuss the liquid-drop model of the nucleus in Section 44.3.



Because of the Coulomb repulsion between the charges of the same sign, the alpha particle approaches to a distance d from the nucleus, called the distance of closest approach.

ACTIVE FIGURE 44.1

An alpha particle on a head-on collision course with a nucleus of charge Ze .

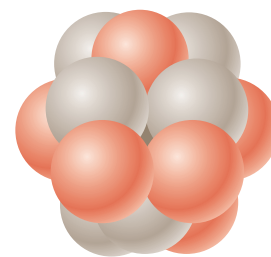


Figure 44.2 A nucleus can be modeled as a cluster of tightly packed spheres, where each sphere is a nucleon.

◀ Nuclear radius

Example 44.2 The Volume and Density of a Nucleus

Consider a nucleus of mass number A .

(A) Find an approximate expression for the mass of the nucleus.

SOLUTION

Conceptualize Imagine the nucleus to be a collection of protons and neutrons as shown in Figure 44.2. The mass number A counts *both* protons and neutrons.

Categorize Let's assume A is large enough that we can imagine the nucleus to be spherical.

Analyze The mass of the proton is approximately equal to that of the neutron. Therefore, if the mass of one of these particles is m , the mass of the nucleus is approximately Am .

(B) Find an expression for the volume of this nucleus in terms of A .

SOLUTION

Assume the nucleus is spherical and use Equation 44.1: $(1) \quad V_{\text{nucleus}} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi a^3 A$

(C) Find a numerical value for the density of this nucleus.

SOLUTION

Use Equation 1.1 and substitute Equation (1):

$$\rho = \frac{m_{\text{nucleus}}}{V_{\text{nucleus}}} = \frac{Am}{\frac{4}{3}\pi a^3 A} = \frac{3m}{4\pi a^3}$$

Substitute numerical values:

$$\rho = \frac{3(1.67 \times 10^{-27} \text{ kg})}{4\pi(1.2 \times 10^{-15} \text{ m})^3} = 2.3 \times 10^{17} \text{ kg/m}^3$$

Finalize The nuclear density is approximately 2.3×10^{14} times the density of water ($\rho_{\text{water}} = 1.0 \times 10^3 \text{ kg/m}^3$).

WHAT IF? What if the Earth could be compressed until it had this density? How large would it be?

Answer Because this density is so large, we predict that an Earth of this density would be very small.

Use Equation 1.1 and the mass of the Earth to find the volume of the compressed Earth:

$$V = \frac{M_E}{\rho} = \frac{5.97 \times 10^{24} \text{ kg}}{2.3 \times 10^{17} \text{ kg/m}^3} = 2.6 \times 10^7 \text{ m}^3$$

From this volume, find the radius:

$$V = \frac{4}{3}\pi r^3 \rightarrow r = \left(\frac{3V}{4\pi}\right)^{1/3} = \left[\frac{3(2.6 \times 10^7 \text{ m}^3)}{4\pi}\right]^{1/3} \\ = 1.8 \times 10^2 \text{ m}$$

An Earth of this radius is indeed a small Earth!

Nuclear Stability

You might expect that the very large repulsive Coulomb forces between the close-packed protons in a nucleus should cause the nucleus to fly apart. Because that does not happen, there must be a counteracting attractive force. The **nuclear force** is a very short range (about 2 fm) attractive force that acts between all nuclear particles. The protons attract each other by means of the nuclear force, and, at the same time, they repel each other through the Coulomb force. The nuclear force also acts between pairs of neutrons and between neutrons and protons. The nuclear force dominates the Coulomb repulsive force within the nucleus (at short ranges), so stable nuclei can exist.

The nuclear force is independent of charge. In other words, the forces associated with the proton–proton, proton–neutron, and neutron–neutron interactions are the same, apart from the additional repulsive Coulomb force for the proton–proton interaction.

Evidence for the limited range of nuclear forces comes from scattering experiments and from studies of nuclear binding energies. The short range of the nuclear force is shown in the neutron–proton (n–p) potential energy plot of Figure 44.3a obtained by scattering neutrons from a target containing hydrogen. The depth of the n–p potential energy well is 40 to 50 MeV, and there is a strong repulsive component that prevents the nucleons from approaching much closer than 0.4 fm.

The nuclear force does not affect electrons, enabling energetic electrons to serve as point-like probes of nuclei. The charge independence of the nuclear force also means that the main difference between the n–p and p–p interactions is that the p–p potential energy consists of a *superposition* of nuclear and Coulomb interactions as shown in Figure 44.3b. At distances less than 2 fm, both p–p and n–p potential energies are nearly identical, but for distances of 2 fm or greater, the p–p potential has a positive energy barrier with a maximum at 4 fm.

The existence of the nuclear force results in approximately 270 stable nuclei; hundreds of other nuclei have been observed, but they are unstable. A plot of neutron number N versus atomic number Z for a number of stable nuclei is given in Figure 44.4. The stable nuclei are represented by the black dots, which lie in a narrow range called the *line of stability*. Notice that the light stable nuclei contain an equal number of protons and neutrons; that is, $N = Z$. Also notice that in heavy stable nuclei, the number of neutrons exceeds the number of protons: above $Z = 20$, the line of stability deviates upward from the line representing $N = Z$. This deviation can be understood by recognizing that as the number of protons increases, the strength of the Coulomb force increases, which tends to break the nucleus apart. As a result, more neutrons are needed to keep the nucleus stable because neutrons experience only the attractive nuclear force. Eventually, the repulsive Coulomb forces between protons cannot be compensated by the addition of more neutrons. This point occurs at $Z = 83$, meaning that elements that contain more than 83 protons do not have stable nuclei.

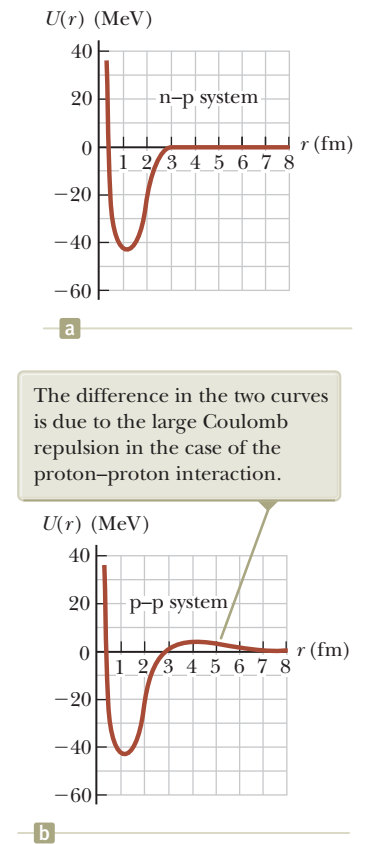
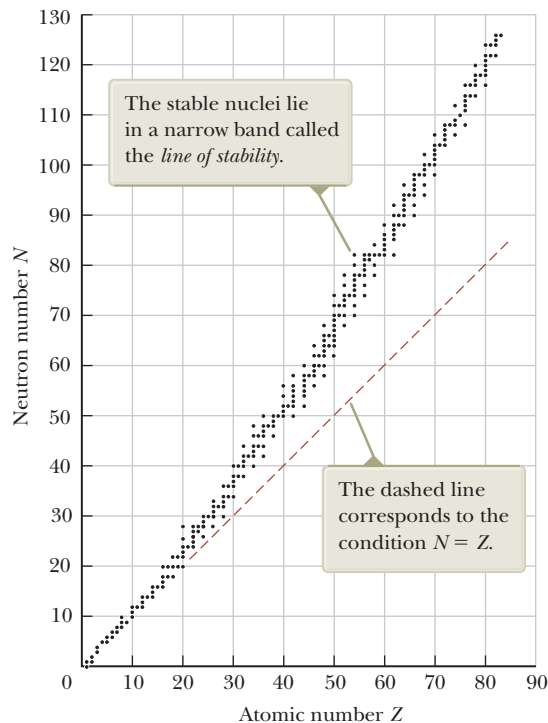


Figure 44.3 (a) Potential energy versus separation distance for a neutron–proton system. (b) Potential energy versus separation distance for a proton–proton system. To display the difference in the curves on this scale, the height of the peak for the proton–proton curve has been exaggerated by a factor of 10.

Figure 44.4 Neutron number N versus atomic number Z for stable nuclei (black dots).

44.2 Nuclear Binding Energy

As mentioned in the discussion of ^{12}C in Section 44.1, the total mass of a nucleus is less than the sum of the masses of its individual nucleons. Therefore, the rest energy of the bound system (the nucleus) is less than the combined rest energy of the separated nucleons. This difference in energy is called the **binding energy** of the nucleus and can be interpreted as the energy that must be added to a nucleus to break it apart into its components. Therefore, to separate a nucleus into protons and neutrons, energy must be delivered to the system.

Conservation of energy and the Einstein mass–energy equivalence relationship show that the binding energy E_b in MeV of any nucleus is

Binding energy of a nucleus ►

$$E_b = [ZM(\text{H}) + Nm_n - M({}^A_Z\text{X})] \times 931.494 \text{ MeV/u} \quad (44.2)$$

where $M(\text{H})$ is the atomic mass of the neutral hydrogen atom, m_n is the mass of the neutron, $M({}^A_Z\text{X})$ represents the atomic mass of an atom of the isotope ${}^A_Z\text{X}$, and the masses are all in atomic mass units. The mass of the Z electrons included in $M(\text{H})$ cancels with the mass of the Z electrons included in the term $M({}^A_Z\text{X})$ within a small difference associated with the atomic binding energy of the electrons. Because atomic binding energies are typically several electron volts and nuclear binding energies are several million electron volts, this difference is negligible.

A plot of binding energy per nucleon E_b/A as a function of mass number A for various stable nuclei is shown in Figure 44.5. Notice that the binding energy in Figure 44.5 peaks in the vicinity of $A = 60$. That is, nuclei having mass numbers either greater or less than 60 are not as strongly bound as those near the middle of the periodic table. The decrease in binding energy per nucleon for $A > 60$ implies that energy is released when a heavy nucleus splits, or *fissions*, into two lighter nuclei. Energy is released in fission because the nucleons in each product nucleus are more tightly bound to one another than are the nucleons in the original nucleus. The important process of fission and a second important process of *fusion*, in which energy is released as light nuclei combine, shall be considered in detail in Chapter 45.

Pitfall Prevention 44.2

Binding Energy

When separate nucleons are combined to form a nucleus, the energy of the system is reduced. Therefore, the change in energy is negative. The absolute value of this change is called the binding energy. This difference in sign may be confusing. For example, an *increase* in binding energy corresponds to a *decrease* in the energy of the system.

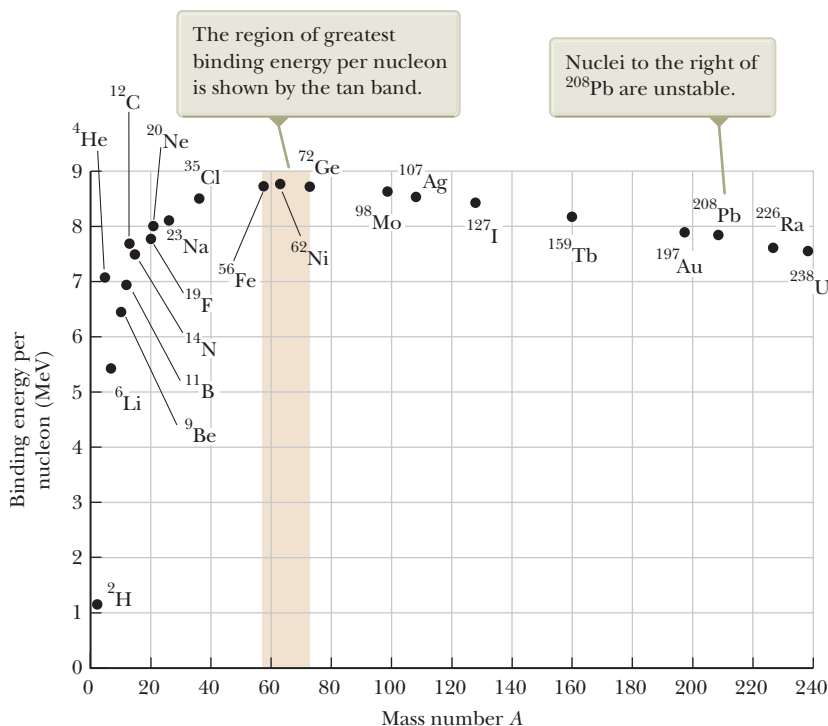


Figure 44.5 Binding energy per nucleon versus mass number for nuclides that lie along the line of stability in Figure 44.4. Some representative nuclides appear as black dots with labels.

Another important feature of Figure 44.5 is that the binding energy per nucleon is approximately constant at around 8 MeV per nucleon for all nuclei with $A > 50$. For these nuclei, the nuclear forces are said to be *saturated*, meaning that in the closely packed structure shown in Figure 44.2, a particular nucleon can form attractive bonds with only a limited number of other nucleons.

Figure 44.5 provides insight into fundamental questions about the origin of the chemical elements. In the early life of the Universe, the only elements that existed were hydrogen and helium. Clouds of cosmic gas coalesced under gravitational forces to form stars. As a star ages, it produces heavier elements from the lighter elements contained within it, beginning by fusing hydrogen atoms to form helium. This process continues as the star becomes older, generating atoms having larger and larger atomic numbers, up to the tan band shown in Figure 44.5.

The nucleus ${}_{28}^{63}\text{Ni}$ has the largest binding energy per nucleon of 8.794 5 MeV. It takes additional energy to create elements with mass numbers larger than 63 because of their lower binding energies per nucleon. This energy comes from the supernova explosion that occurs at the end of some large stars' lives. Therefore, all the heavy atoms in your body were produced from the explosions of ancient stars. You are literally made of stardust!

44.3 Nuclear Models

The details of the nuclear force are still an area of active research. Several nuclear models have been proposed that are useful in understanding general features of nuclear experimental data and the mechanisms responsible for binding energy. Two such models, the liquid-drop model and the shell model, are discussed below.

The Liquid-Drop Model

In 1936, Bohr proposed treating nucleons like molecules in a drop of liquid. In this **liquid-drop model**, the nucleons interact strongly with one another and undergo frequent collisions as they jiggle around within the nucleus. This jiggling motion is analogous to the thermally agitated motion of molecules in a drop of liquid.

Four major effects influence the binding energy of the nucleus in the liquid-drop model:

- **The volume effect.** Figure 44.5 shows that for $A > 50$, the binding energy per nucleon is approximately constant, which indicates that the nuclear force on a given nucleon is due only to a few nearest neighbors and not to all the other nucleons in the nucleus. On average, then, the binding energy associated with the nuclear force for each nucleon is the same in all nuclei: that associated with an interaction with a few neighbors. This property indicates that the total binding energy of the nucleus is proportional to A and therefore proportional to the nuclear volume. The contribution to the binding energy of the entire nucleus is C_1A , where C_1 is an adjustable constant that can be determined by fitting the prediction of the model to experimental results.
- **The surface effect.** Because nucleons on the surface of the drop have fewer neighbors than those in the interior, surface nucleons reduce the binding energy by an amount proportional to their number. Because the number of surface nucleons is proportional to the surface area $4\pi r^2$ of the nucleus (modeled as a sphere) and because $r^2 \propto A^{2/3}$ (Eq. 44.1), the surface term can be expressed as $-C_2A^{2/3}$, where C_2 is a second adjustable constant.
- **The Coulomb repulsion effect.** Each proton repels every other proton in the nucleus. The corresponding potential energy per pair of interacting protons is $k_e e^2/r$, where k_e is the Coulomb constant. The total electric potential energy is equivalent to the work required to assemble Z protons, initially infinitely far apart, into a sphere of volume V . This energy is proportional to the number of proton pairs $Z(Z - 1)/2$ and inversely proportional to the nuclear radius.

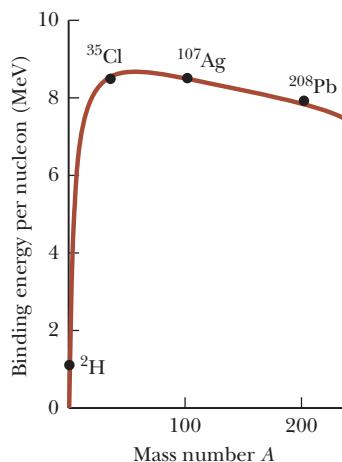


Figure 44.6 The binding-energy curve plotted by using the semiempirical binding-energy formula (red-brown). For comparison to the theoretical curve, experimental values for four sample nuclei are shown.

Consequently, the reduction in binding energy that results from the Coulomb effect is $-C_3Z(Z-1)/A^{1/3}$, where C_3 is yet another adjustable constant.

- **The symmetry effect.** Another effect that lowers the binding energy is related to the symmetry of the nucleus in terms of values of N and Z . For small values of A , stable nuclei tend to have $N \approx Z$. Any large asymmetry between N and Z for light nuclei reduces the binding energy and makes the nucleus less stable. For larger A , the value of N for stable nuclei is naturally larger than Z . This effect can be described by a binding-energy term of the form $-C_4(N-Z)^2/A$, where C_4 is another adjustable constant.¹ For small A , any large asymmetry between values of N and Z makes this term relatively large and reduces the binding energy. For large A , this term is small and has little effect on the overall binding energy.

Adding these contributions gives the following expression for the total binding energy:

$$E_b = C_1A - C_2A^{2/3} - C_3\frac{Z(Z-1)}{A^{1/3}} - C_4\frac{(N-Z)^2}{A} \quad (44.3)$$

This equation, often referred to as the **semiempirical binding-energy formula**, contains four constants that are adjusted to fit the theoretical expression to experimental data. For nuclei having $A \geq 15$, the constants have the values

$$\begin{aligned} C_1 &= 15.7 \text{ MeV} & C_2 &= 17.8 \text{ MeV} \\ C_3 &= 0.71 \text{ MeV} & C_4 &= 23.6 \text{ MeV} \end{aligned}$$

Equation 44.3, together with these constants, fits the known nuclear mass values very well as shown by the theoretical curve and sample experimental values in Figure 44.6. The liquid-drop model does not, however, account for some finer details of nuclear structure, such as stability rules and angular momentum. Equation 44.3 is a *theoretical* equation for the binding energy, based on the liquid-drop model, whereas binding energies calculated from Equation 44.2 are *experimental* values based on mass measurements.

Example 44.3

Applying the Semiempirical Binding-Energy Formula

The nucleus ^{64}Zn has a tabulated binding energy of 559.09 MeV. Use the semiempirical binding-energy formula to generate a theoretical estimate of the binding energy for this nucleus.

SOLUTION

Conceptualize Imagine bringing the separate protons and neutrons together to form a ^{64}Zn nucleus. The rest energy of the nucleus is smaller than the rest energy of the individual particles. The difference in rest energy is the binding energy.

Categorize From the text of the problem, we know to apply the liquid-drop model. This example is a substitution problem.

For the ^{64}Zn nucleus, $Z = 30$, $N = 34$, and $A = 64$. Evaluate the four terms of the semiempirical binding-energy formula:

$$\begin{aligned} C_1A &= (15.7 \text{ MeV})(64) = 1005 \text{ MeV} \\ C_2A^{2/3} &= (17.8 \text{ MeV})(64)^{2/3} = 285 \text{ MeV} \\ C_3\frac{Z(Z-1)}{A^{1/3}} &= (0.71 \text{ MeV})\frac{(30)(29)}{(64)^{1/3}} = 154 \text{ MeV} \\ C_4\frac{(N-Z)^2}{A} &= (23.6 \text{ MeV})\frac{(34-30)^2}{64} = 5.90 \text{ MeV} \end{aligned}$$

¹The liquid-drop model *describes* that heavy nuclei have $N > Z$. The shell model, as we shall see shortly, *explains* why that is true with a physical argument.

44.3 cont.

Substitute these values into Equation 44.3:

$$E_b = 1\,005\text{ MeV} - 285\text{ MeV} - 154\text{ MeV} - 5.90\text{ MeV} = 560\text{ MeV}$$

This value differs from the tabulated value by less than 0.2%. Notice how the sizes of the terms decrease from the first to the fourth term. The fourth term is particularly small for this nucleus, which does not have an excessive number of neutrons.

The Shell Model

The liquid-drop model describes the general behavior of nuclear binding energies relatively well. When the binding energies are studied more closely, however, we find the following features:

- Most stable nuclei have an even value of A . Furthermore, only eight stable nuclei have odd values for both Z and N .
- Figure 44.7 shows a graph of the difference between the binding energy per nucleon calculated by Equation 44.3 and the measured binding energy. There is evidence for regularly spaced peaks in the data that are not described by the semiempirical binding-energy formula. The peaks occur at values of N or Z that have become known as **magic numbers**:

$$Z \text{ or } N = 2, 8, 20, 28, 50, 82 \quad (44.4) \quad \blacktriangleleft \text{ Magic numbers}$$

- High-precision studies of nuclear radii show deviations from the simple expression in Equation 44.1. Graphs of experimental data show peaks in the curve of radius versus N at values of N equal to the magic numbers.
- A group of *isotones* is a collection of nuclei having the same value of N and varying values of Z . When the number of stable isotones is graphed as function of N , there are peaks in the graph, again at the magic numbers in Equation 44.4.
- Several other nuclear measurements show anomalous behavior at the magic numbers.²

These peaks in graphs of experimental data are reminiscent of the peaks in Figure 42.20 for the ionization energy of atoms, which arose because of the shell structure

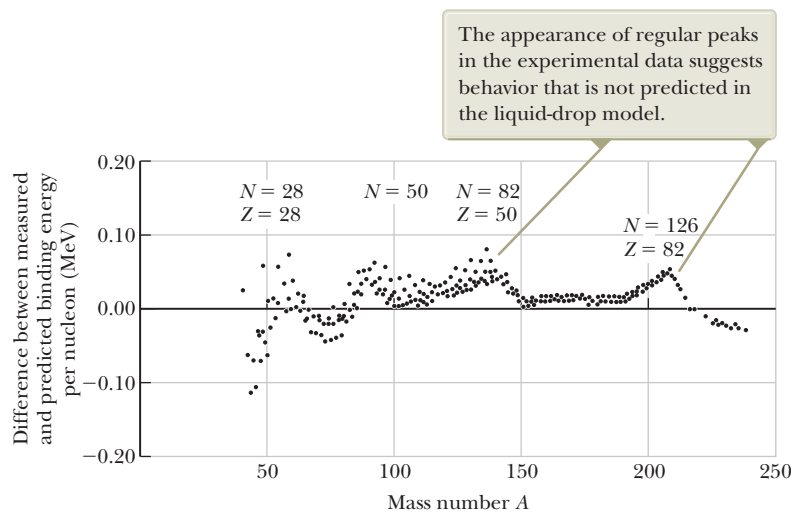


Figure 44.7 The difference between measured binding energies and those calculated from the liquid-drop model is a function of A . (Adapted from R. A. Dunlap, *The Physics of Nuclei and Particles*, Brooks/Cole, Belmont, CA, 2004.)

²For further details, see chapter 5 of R. A. Dunlap, *The Physics of Nuclei and Particles*, Brooks/Cole, Belmont, CA, 2004.

Courtesy of Louise Barker/AIP Niels Bohr Library



Maria Goeppert-Mayer German Scientist (1906–1972)

Goeppert-Mayer was born and educated in Germany. She is best known for her development of the shell model (independent-particle model) of the nucleus, published in 1950. A similar model was simultaneously developed by Hans Jensen, another German scientist. Goeppert-Mayer and Jensen were awarded the Nobel Prize in Physics in 1963 for their extraordinary work in understanding the structure of the nucleus.

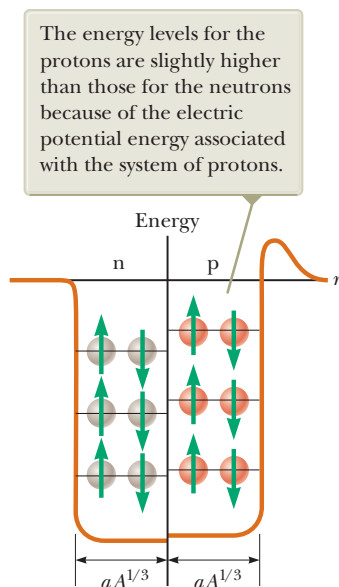


Figure 44.8 A square potential well containing 12 nucleons. The red spheres represent protons, and the gray spheres represent neutrons.

of the atom. The **shell model** of the nucleus, also called the **independent-particle model**, was developed independently by two German scientists: Maria Goeppert-Mayer in 1949 and Hans Jensen (1907–1973) in 1950. Goeppert-Mayer and Jensen shared the 1963 Nobel Prize in Physics for their work. In this model, each nucleon is assumed to exist in a shell, similar to an atomic shell for an electron. The nucleons exist in quantized energy states, and there are few collisions between nucleons. Obviously, the assumptions of this model differ greatly from those made in the liquid-drop model.

The quantized states occupied by the nucleons can be described by a set of quantum numbers. Because both the proton and the neutron have spin $\frac{1}{2}$, the exclusion principle can be applied to describe the allowed states (as it was for electrons in Chapter 42). That is, each state can contain only two protons (or two neutrons) having *opposite* spins (Fig. 44.8). The proton states differ from those of the neutrons because the two species move in different potential wells. The proton energy levels are farther apart than the neutron levels because the protons experience a superposition of the Coulomb force and the nuclear force, whereas the neutrons experience only the nuclear force.

One factor influencing the observed characteristics of nuclear ground states is *nuclear spin-orbit* effects. The atomic spin-orbit interaction between the spin of an electron and its orbital motion in an atom gives rise to the sodium doublet discussed in Section 42.6 and is magnetic in origin. In contrast, the nuclear spin-orbit effect for nucleons is due to the nuclear force. It is much stronger than in the atomic case, and it has opposite sign. When these effects are taken into account, the shell model is able to account for the observed magic numbers.

The shell model helps us understand why nuclei containing an even number of protons and neutrons are more stable than other nuclei. (There are 160 stable even-even isotopes.) Any particular state is filled when it contains two protons (or two neutrons) having opposite spins. An extra proton or neutron can be added to the nucleus only at the expense of increasing the energy of the nucleus. This increase in energy leads to a nucleus that is less stable than the original nucleus. A careful inspection of the stable nuclei shows that the majority have a special stability when their nucleons combine in pairs, which results in a total angular momentum of zero.

The shell model also helps us understand why nuclei tend to have more neutrons than protons. As in Figure 44.8, the proton energy levels are higher than those for neutrons due to the extra energy associated with Coulomb repulsion. This effect becomes more pronounced as Z increases. Consequently, as Z increases and higher states are filled, a proton level for a given quantum number will be much higher in energy than the neutron level for the same quantum number. In fact, it will be even higher in energy than neutron levels for higher quantum numbers. Hence, it is more energetically favorable for the nucleus to form with neutrons in the lower energy levels rather than protons in the higher energy levels, so the number of neutrons is greater than the number of protons.

More sophisticated models of the nucleus have been and continue to be developed. For example, the *collective model* combines features of the liquid-drop and shell models. The development of theoretical models of the nucleus continues to be an active area of research.

44.4 Radioactivity

In 1896, Becquerel accidentally discovered that uranyl potassium sulfate crystals emit an invisible radiation that can darken a photographic plate even though the plate is covered to exclude light. After a series of experiments, he concluded that the radiation emitted by the crystals was of a new type, one that requires no external stimulation and was so penetrating that it could darken protected photographic plates and ionize gases. This process of spontaneous emission of radiation by uranium was soon to be called **radioactivity**.

Subsequent experiments by other scientists showed that other substances were more powerfully radioactive. The most significant early investigations of this type were conducted by Marie and Pierre Curie (1859–1906). After several years of careful and laborious chemical separation processes on tons of pitchblende, a radioactive ore, the Curies reported the discovery of two previously unknown elements, both radioactive, named polonium and radium. Additional experiments, including Rutherford's famous work on alpha-particle scattering, suggested that radioactivity is the result of the *decay*, or disintegration, of unstable nuclei.

Three types of radioactive decay occur in radioactive substances: alpha (α) decay, in which the emitted particles are ${}^4\text{He}$ nuclei; beta (β) decay, in which the emitted particles are either electrons or positrons; and gamma (γ) decay, in which the emitted particles are high-energy photons. A **positron** is a particle like the electron in all respects except that the positron has a charge of $+e$. (The positron is the *antiparticle* of the electron; see Section 46.2.) The symbol e^- is used to designate an electron, and e^+ designates a positron.

We can distinguish among these three forms of radiation by using the scheme described in Figure 44.9. The radiation from radioactive samples that emit all three types of particles is directed into a region in which there is a magnetic field. The radiation beam splits into three components, two bending in opposite directions and the third experiencing no change in direction. This simple observation shows that the radiation of the undeflected beam carries no charge (the gamma ray), the component deflected upward corresponds to positively charged particles (alpha particles), and the component deflected downward corresponds to negatively charged particles (e^-). If the beam includes a positron (e^+), it is deflected upward like the alpha particle, but it follows a different trajectory due to its smaller mass.

The three types of radiation have quite different penetrating powers. Alpha particles barely penetrate a sheet of paper, beta particles can penetrate a few millimeters of aluminum, and gamma rays can penetrate several centimeters of lead.

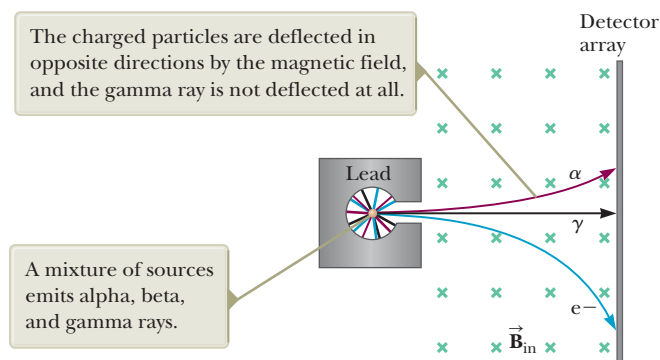
The decay process is probabilistic in nature and can be described with statistical calculations for a radioactive substance of macroscopic size containing a large number of radioactive nuclei. For such large numbers, the rate at which a particular decay process occurs in a sample is proportional to the number of radioactive nuclei present (that is, the number of nuclei that have not yet decayed). If N is the number of undecayed radioactive nuclei present at some instant, the rate of change of N with time is

$$\frac{dN}{dt} = -\lambda N \quad (44.5)$$

where λ , called the **decay constant**, is the probability of decay per nucleus per second. The negative sign indicates that dN/dt is negative; that is, N decreases in time.

Equation 44.5 can be written in the form

$$\frac{dN}{N} = -\lambda dt$$



Time & Life Pictures/Getty Images

Marie Curie

Polish Scientist (1867–1934)

In 1903, Marie Curie shared the Nobel Prize in Physics with her husband, Pierre, and with Becquerel for their studies of radioactive substances. In 1911, she was awarded a Nobel Prize in Chemistry for the discovery of radium and polonium.

Pitfall Prevention 44.3

Rays or Particles?

Early in the history of nuclear physics, the term *radiation* was used to describe the emanations from radioactive nuclei. We now know that alpha radiation and beta radiation involve the emission of particles with nonzero rest energy. Even though they are not examples of electromagnetic radiation, the use of the term *radiation* for all three types of emission is deeply entrenched in our language and in the physics community.

Pitfall Prevention 44.4

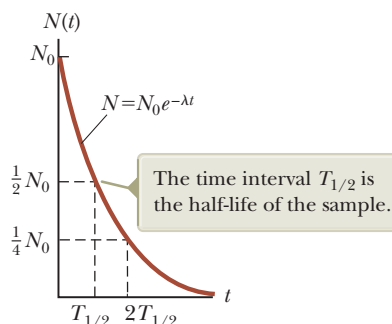
Notation Warning

In Section 44.1, we introduced the symbol N as an integer representing the number of neutrons in a nucleus. In this discussion, the symbol N represents the number of undecayed nuclei in a radioactive sample remaining after some time interval. As you read further, be sure to consider the context to determine the appropriate meaning for the symbol N .

Figure 44.9 The radiation from radioactive sources can be separated into three components by using a magnetic field to deflect the charged particles. The detector array at the right records the events.

ACTIVE FIGURE 44.10

Plot of the exponential decay of radioactive nuclei. The vertical axis represents the number of undecayed radioactive nuclei present at any time t , and the horizontal axis is time.

**Exponential behavior of the number of undecayed nuclei**

which, upon integration, gives

$$N = N_0 e^{-\lambda t} \quad (44.6)$$

where the constant N_0 represents the number of undecayed radioactive nuclei at $t = 0$. Equation 44.6 shows that the number of undecayed radioactive nuclei in a sample decreases exponentially with time. The plot of N versus t shown in Active Figure 44.10 illustrates the exponential nature of the decay.

The **decay rate** R , which is the number of decays per second, can be obtained by combining Equations 44.5 and 44.6:

Exponential behavior of the decay rate

$$R = \left| \frac{dN}{dt} \right| = \lambda N = \lambda N_0 e^{-\lambda t} = R_0 e^{-\lambda t} \quad (44.7)$$

where $R_0 = \lambda N_0$ is the decay rate at $t = 0$. The decay rate R of a sample is often referred to as its **activity**. Note that both N and R decrease exponentially with time.

Another parameter useful in characterizing nuclear decay is the **half-life** $T_{1/2}$:

The **half-life** of a radioactive substance is the time interval during which half of a given number of radioactive nuclei decay.

To find an expression for the half-life, we first set $N = N_0/2$ and $t = T_{1/2}$ in Equation 44.6 to give

$$\frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}}$$

Canceling the N_0 factors and then taking the reciprocal of both sides, we obtain $e^{\lambda T_{1/2}} = 2$. Taking the natural logarithm of both sides gives

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda} \quad (44.8)$$

After a time interval equal to one half-life, there are $N_0/2$ radioactive nuclei remaining (by definition); after two half-lives, half of these remaining nuclei have decayed and $N_0/4$ radioactive nuclei are left; after three half-lives, $N_0/8$ are left; and so on. In general, after n half-lives, the number of undecayed radioactive nuclei remaining is

$$N = N_0 \left(\frac{1}{2}\right)^n \quad (44.9)$$

where n can be an integer or a noninteger.

A frequently used unit of activity is the **curie** (Ci), defined as

$$1 \text{ Ci} \equiv 3.7 \times 10^{10} \text{ decays/s}$$

This value was originally selected because it is the approximate activity of 1 g of radium. The SI unit of activity is the **becquerel** (Bq):

$$1 \text{ Bq} \equiv 1 \text{ decay/s}$$

Therefore, $1 \text{ Ci} = 3.7 \times 10^{10} \text{ Bq}$. The curie is a rather large unit, and the more frequently used activity units are the millicurie and the microcurie.

Pitfall Prevention 44.5**Half-life**

It is *not* true that all the original nuclei have decayed after two half-lives! In one half-life, half of the original nuclei will decay. In the second half-life, half of those remaining will decay, leaving $\frac{1}{4}$ of the original number.

Half-life**The curie****The becquerel**

Quick Quiz 44.2 On your birthday, you measure the activity of a sample of ^{210}Bi , which has a half-life of 5.01 days. The activity you measure is $1.000\ \mu\text{Ci}$. What is the activity of this sample on your next birthday? (a) $1.000\ \mu\text{Ci}$ (b) 0 (c) $\sim 0.2\ \mu\text{Ci}$ (d) $\sim 0.01\ \mu\text{Ci}$ (e) $\sim 10^{-22}\ \mu\text{Ci}$

Example 44.4 How Many Nuclei Are Left?

The isotope carbon-14, $^{14}_6\text{C}$, is radioactive and has a half-life of 5 730 years. If you start with a sample of 1 000 carbon-14 nuclei, how many nuclei will still be undecayed in 25 000 years?

SOLUTION

Conceptualize The time interval of 25 000 years is much longer than the half-life, so only a small fraction of the originally undecayed nuclei will remain.

Categorize The text of the problem allows us to categorize this example as a radioactive decay problem.

Analyze Divide the time interval by the half-life to determine the number of half-lives:

$$n = \frac{25\,000\ \text{yr}}{5\,730\ \text{yr}} = 4.363$$

Determine how many undecayed nuclei are left after this many half-lives using Equation 44.9:

$$N = N_0 \left(\frac{1}{2}\right)^n = 1\,000 \left(\frac{1}{2}\right)^{4.363} = 49$$

Finalize As we have mentioned, radioactive decay is a probabilistic process and accurate statistical predictions are possible only with a very large number of atoms. The original sample in this example contains only 1 000 nuclei, which is certainly not a very large number. Therefore, if you counted the number of undecayed nuclei remaining after 25 000 years, it might not be exactly 49.

Example 44.5 The Activity of Carbon

At time $t = 0$, a radioactive sample contains $3.50\ \mu\text{g}$ of pure $^{11}_6\text{C}$, which has a half-life of 20.4 min.

(A) Determine the number N_0 of nuclei in the sample at $t = 0$.

SOLUTION

Conceptualize The half-life is relatively short, so the number of undecayed nuclei drops rapidly. The molar mass of $^{11}_6\text{C}$ is approximately 11.0 g/mol.

Categorize We evaluate results using equations developed in this section, so we categorize this example as a substitution problem.

Find the number of moles in $3.50\ \mu\text{g}$ of pure $^{11}_6\text{C}$:

$$n = \frac{3.50 \times 10^{-6}\ \text{g}}{11.0\ \text{g/mol}} = 3.18 \times 10^{-7}\ \text{mol}$$

Find the number of undecayed nuclei in this amount of pure $^{11}_6\text{C}$:

$$N_0 = (3.18 \times 10^{-7}\ \text{mol})(6.02 \times 10^{23}\ \text{nuclei/mol}) = 1.92 \times 10^{17}\ \text{nuclei}$$

(B) What is the activity of the sample initially and after 8.00 h?

SOLUTION

Find the initial activity of the sample using Equation 44.7:

$$\begin{aligned} R_0 &= \lambda N_0 = \frac{0.693}{T_{1/2}} N_0 = \frac{0.693}{20.4\ \text{min}} \left(\frac{1\ \text{min}}{60\ \text{s}}\right) (1.92 \times 10^{17}) \\ &= (5.66 \times 10^{-4}\ \text{s}^{-1})(1.92 \times 10^{17}) = 1.09 \times 10^{14}\ \text{Bq} \end{aligned}$$

Use Equation 44.7 to find the activity at $t = 8.00\ \text{h} = 2.88 \times 10^4\ \text{s}$:

$$R = R_0 e^{-\lambda t} = (1.09 \times 10^{14}\ \text{Bq}) e^{-(5.66 \times 10^{-4}\ \text{s}^{-1})(2.88 \times 10^4\ \text{s})} = 8.96 \times 10^6\ \text{Bq}$$

Example 44.6 A Radioactive Isotope of Iodine

A sample of the isotope ^{131}I , which has a half-life of 8.04 days, has an activity of 5.0 mCi at the time of shipment. Upon receipt of the sample at a medical laboratory, the activity is 2.1 mCi. How much time has elapsed between the two measurements?

SOLUTION

Conceptualize The sample is continuously decaying as it is in transit. The decrease in the activity is 58% during the time interval between shipment and receipt, so we expect the elapsed time to be greater than the half-life of 8.04 d.

Categorize The stated activity corresponds to many decays per second, so N is large and we can categorize this problem as one in which we can use our statistical analysis of radioactivity.

Analyze Solve Equation 44.7 for the ratio of the final activity to the initial activity:

$$\frac{R}{R_0} = e^{-\lambda t}$$

Take the natural logarithm of both sides:

$$\ln\left(\frac{R}{R_0}\right) = -\lambda t$$

Solve for the time t :

$$(1) \quad t = -\frac{1}{\lambda} \ln\left(\frac{R}{R_0}\right)$$

Use Equation 44.8 to substitute for λ :

$$t = -\frac{T_{1/2}}{\ln 2} \ln\left(\frac{R}{R_0}\right)$$

Substitute numerical values:

$$t = -\frac{8.04 \text{ d}}{0.693} \ln\left(\frac{2.1 \text{ mCi}}{5.0 \text{ mCi}}\right) = 10 \text{ d}$$

Finalize This result is indeed greater than the half-life, as expected. This example demonstrates the difficulty in shipping radioactive samples with short half-lives. If the shipment is delayed by several days, only a small fraction of the sample might remain upon receipt. This difficulty can be addressed by shipping a combination of isotopes in which the desired isotope is the product of a decay occurring within the sample. It is possible for the desired isotope to be in *equilibrium*, in which case it is created at the same rate as it decays. Therefore, the amount of the desired isotope remains constant during the shipping process and subsequent storage. When needed, the desired isotope can be separated from the rest of the sample; its decay from the initial activity begins at this point rather than upon shipment.

44.5 The Decay Processes

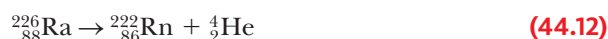
As we stated in Section 44.4, a radioactive nucleus spontaneously decays by one of three processes: alpha decay, beta decay, or gamma decay. Active Figure 44.11 shows a close-up view of a portion of Figure 44.4 from $Z = 65$ to $Z = 80$. The black circles are the stable nuclei seen in Figure 44.4. In addition, unstable nuclei above and below the line of stability for each value of Z are shown. Above the line of stability, the blue circles show unstable nuclei that are neutron-rich and undergo a beta decay process in which an electron is emitted. Below the black circles are red circles corresponding to proton-rich unstable nuclei that primarily undergo a beta-decay process in which a positron is emitted or a competing process called electron capture. Beta decay and electron capture are described in more detail below. Further below the line of stability (with a few exceptions) are tan circles that represent very proton-rich nuclei for which the primary decay mechanism is alpha decay, which we discuss first.

Alpha Decay

A nucleus emitting an alpha particle (${}^4_2\text{He}$) loses two protons and two neutrons. Therefore, the atomic number Z decreases by 2, the mass number A decreases by 4, and the neutron number decreases by 2. The decay can be written



where X is called the **parent nucleus** and Y the **daughter nucleus**. As a general rule in any decay expression such as this one, (1) the sum of the mass numbers A must be the same on both sides of the decay and (2) the sum of the atomic numbers Z must be the same on both sides of the decay. As examples, ${}^{238}\text{U}$ and ${}^{226}\text{Ra}$ are both alpha emitters and decay according to the schemes



The decay of ${}^{226}\text{Ra}$ is shown in Active Figure 44.12.

When the nucleus of one element changes into the nucleus of another as happens in alpha decay, the process is called **spontaneous decay**. In any spontaneous decay, relativistic energy and momentum of the isolated parent nucleus must be conserved. If we call M_X the mass of the parent nucleus, M_Y the mass of the daughter nucleus, and M_α the mass of the alpha particle, we can define the **disintegration energy** Q of the system as

$$Q = (M_X - M_Y - M_\alpha)c^2 \quad (44.13)$$

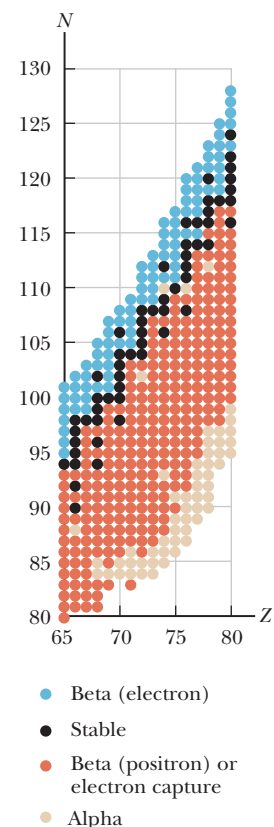
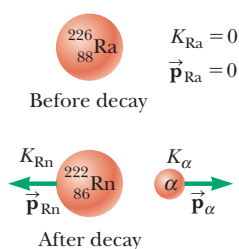
The energy Q is in joules when the masses are in kilograms and c is the speed of light, 3.00×10^8 m/s. When the masses are expressed in atomic mass units u , however, Q can be calculated in MeV using the expression

$$Q = (M_X - M_Y - M_\alpha) \times 931.494 \text{ MeV}/u \quad (44.14)$$

Table 44.2 (page 1352) contains information on selected isotopes, including masses of neutral atoms that can be used in Equation 44.14 and similar equations.

The disintegration energy Q appears in the form of kinetic energy in the daughter nucleus and the alpha particle and is sometimes referred to as the Q value of the nuclear decay. Consider the case of the ${}^{226}\text{Ra}$ decay described in Active Figure 44.12. If the parent nucleus is at rest before the decay, the total kinetic energy of the products is 4.87 MeV. (See Example 44.7.) Most of this kinetic energy is associated with the alpha particle because this particle is much less massive than the daughter nucleus. That is, because momentum must be conserved, the lighter alpha particle recoils with a much higher speed than does the daughter nucleus. Generally, less massive particles carry off most of the energy in nuclear decays.

Experimental observations of alpha-particle energies show a number of discrete energies rather than a single energy because the daughter nucleus may be left in an excited quantum state after the decay. As a result, not all the disintegration energy is available as kinetic energy of the alpha particle and daughter nucleus. The emission of an alpha particle is followed by one or more gamma-ray photons (discussed shortly) as the excited nucleus decays to the ground state. The observed discrete alpha-particle energies represent evidence of the quantized nature of the nucleus and allow a determination of the energies of the quantum states.



ACTIVE FIGURE 44.11

A close-up view of the line of stability in Figure 44.4 from $Z = 65$ to $Z = 80$. The black dots represent stable nuclei as in Figure 44.4. The other colored dots represent unstable isotopes above and below the line of stability, with the color of the dot indicating the primary means of decay.

Pitfall Prevention 44.6

Another Q

We have seen the symbol Q before, but this use is a brand-new meaning for this symbol: the disintegration energy. In this context, it is not heat, charge, or quality factor for a resonance, for which we have used Q before.

ACTIVE FIGURE 44.12

The alpha decay of radium-226. The radium nucleus is initially at rest. After the decay, the radon nucleus has kinetic energy K_{Rn} and momentum \vec{p}_{Rn} and the alpha particle has kinetic energy K_α and momentum \vec{p}_α .

TABLE 44.2 Chemical and Nuclear Information for Selected Isotopes

Atomic Number Z	Element	Chemical Symbol	Mass Number A (* means radioactive)	Mass of Neutral Atom (u)	Percent Abundance	Half-life, if Radioactive $T_{1/2}$
-1	electron	e^-	0	0.000 549		
0	neutron	n	1*	1.008 665		614 s
1	hydrogen	$^1\text{H} = \text{p}$	1	1.007 825	99.988 5	
	[deuterium	$^2\text{H} = \text{D}$]	2	2.014 102	0.011 5	
	[tritium	$^3\text{H} = \text{T}$]	3*	3.016 049		12.33 yr
2	helium	He	3	3.016 029	0.000 137	
	[alpha particle	$\alpha = ^4\text{He}$]	4	4.002 603	99.999 863	
			6*	6.018 889		0.81 s
3	lithium	Li	6	6.015 123	7.5	
			7	7.016 005	92.5	
4	beryllium	Be	7*	7.016 930		53.3 d
			8*	8.005 305		10^{-17} s
			9	9.012 182	100	
5	boron	B	10	10.012 937	19.9	
			11	11.009 305	80.1	
6	carbon	C	11*	11.011 434		20.4 min
			12	12.000 000	98.93	
			13	13.003 355	1.07	
			14*	14.003 242		5 730 yr
7	nitrogen	N	13*	13.005 739		9.96 min
			14	14.003 074	99.632	
			15	15.000 109	0.368	
8	oxygen	O	14*	14.008 596		70.6 s
			15*	15.003 066		122 s
			16	15.994 915	99.757	
			17	16.999 132	0.038	
			18	17.999 161	0.205	
9	fluorine	F	18*	18.000 938		109.8 min
			19	18.998 403	100	
10	neon	Ne	20	19.992 440	90.48	
11	sodium	Na	23	22.989 769	100	
12	magnesium	Mg	23*	22.994 124		11.3 s
			24	23.985 042	78.99	
13	aluminum	Al	27	26.981 539	100	
14	silicon	Si	27*	26.986 705		4.2 s
15	phosphorus	P	30*	29.978 314		2.50 min
			31	30.973 762	100	
			32*	31.973 907		14.26 d
16	sulfur	S	32	31.972 071	94.93	
19	potassium	K	39	38.963 707	93.258 1	
			40*	39.963 998	0.011 7	1.28×10^9 yr
20	calcium	Ca	40	39.962 591	96.941	
			42	41.958 618	0.647	
			43	42.958 767	0.135	
25	manganese	Mn	55	54.938 045	100	
26	iron	Fe	56	55.934 938	91.754	
			57	56.935 394	2.119	

continued

TABLE 44.2 Chemical and Nuclear Information for Selected Isotopes (continued)

Atomic Number Z	Element	Chemical Symbol	Mass Number A (* means radioactive)	Mass of Neutral Atom (u)	Percent Abundance	Half-life, if Radioactive $T_{1/2}$
27	cobalt	Co	57*	56.936 291	100	272 d
			59	58.933 195		5.27 yr
			60*	59.933 817		
28	nickel	Ni	58	57.935 343	68.076 9	
			60	59.930 786	26.223 1	
29	copper	Cu	63	62.929 598	69.17	12.7 h
			64*	63.929 764	30.83	
			65	64.927 789		
30	zinc	Zn	64	63.929 142	48.63	
37	rubidium	Rb	87*	86.909 181	27.83	
38	strontium	Sr	87	86.908 877	7.00	29.1 yr
			88	87.905 612	82.58	
			90*	89.907 738		
			93	92.906 378	100	
42	molybdenum	Mo	94	93.905 088	9.25	
44	ruthenium	Ru	98	97.905 287	1.87	
54	xenon	Xe	136*	135.907 219		2.4×10^{21} yr
55	cesium	Cs	137*	136.907 090		30 yr
56	barium	Ba	137	136.905 827	11.232	
58	cerium	Ce	140	139.905 439	88.450	
59	praseodymium	Pr	141	140.907 653	100	
60	neodymium	Nd	144*	143.910 087	23.8	2.3×10^{15} yr
61	promethium	Pm	145*	144.912 749		17.7 yr
79	gold	Au	197	196.966 569	100	
80	mercury	Hg	198	197.966 769	9.97	
			202	201.970 643	29.86	
			206	205.974 465	24.1	
			207	206.975 897	22.1	
			208	207.976 652	52.4	
82	lead	Pb	214*	213.999 805		26.8 min
83	bismuth	Bi	209	208.980 399	100	
84	polonium	Po	210*	209.982 874		138.38 d
			216*	216.001 915		0.145 s
			218*	218.008 973		3.10 min
86	radon	Rn	220*	220.011 394		55.6 s
			222*	222.017 578		3.823 d
88	radium	Ra	226*	226.025 410		1 600 yr
90	thorium	Th	232*	232.038 055	100	1.40×10^{10} yr
			234*	234.043 601		24.1 d
			234*	234.040 952		2.45×10^5 yr
92	uranium	U	235*	235.043 930	0.720 0	7.04×10^8 yr
			236*	236.045 568		2.34×10^7 yr
			238*	238.050 788	99.274 5	4.47×10^9 yr
			236*	236.046 570		1.15×10^5 yr
93	neptunium	Np	237*	237.048 173		2.14×10^6 yr
94	plutonium	Pu	239*	239.052 163		24 120 yr

Source: G. Audi, A. H. Wapstra, and C. Thibault, "The AME2003 Atomic Mass Evaluation," *Nuclear Physics A* **729**: 337–676, 2003.

If one assumes ^{238}U (or any other alpha emitter) decays by emitting either a proton or a neutron, the mass of the decay products would exceed that of the parent nucleus, corresponding to a negative Q value. A negative Q value indicates that such a proposed decay does not occur spontaneously.

Quick Quiz 44.3 Which of the following is the correct daughter nucleus associated with the alpha decay of $^{157}_{72}\text{Hf}$? (a) $^{153}_{72}\text{Hf}$ (b) $^{153}_{70}\text{Yb}$ (c) $^{157}_{70}\text{Yb}$

Example 44.7

The Energy Liberated When Radium Decays

The ^{226}Ra nucleus undergoes alpha decay according to Equation 44.12. Calculate the Q value for this process. From Table 44.2, the masses are 226.025 410 u for ^{226}Ra , 222.017 578 u for ^{222}Rn , and 4.002 603 u for ^4_2He .

SOLUTION

Conceptualize Study Active Figure 44.12 to understand the process of alpha decay in this nucleus.

Categorize We use an equation developed in this section, so we categorize this example as a substitution problem.

Evaluate Q using Equation 44.14:

$$\begin{aligned} Q &= (M_X - M_Y - M_\alpha) \times 931.494 \text{ MeV/u} \\ &= (226.025 \text{ 410 u} - 222.017 \text{ 578 u} - 4.002 \text{ 603 u}) \times 931.494 \text{ MeV/u} \\ &= (0.005 \text{ 229 u}) \times 931.494 \text{ MeV/u} = 4.87 \text{ MeV} \end{aligned}$$

WHAT IF? Suppose you measured the kinetic energy of the alpha particle from this decay. Would you measure 4.87 MeV?

Answer The value of 4.87 MeV is the disintegration energy for the decay. It includes the kinetic energy of both the alpha particle and the daughter nucleus after the decay. Therefore, the kinetic energy of the alpha particle would be *less* than 4.87 MeV.

Let's determine this kinetic energy mathematically. The parent nucleus is an isolated system that decays into an alpha particle and a daughter nucleus. Therefore, momentum must be conserved for the system.

Set up a conservation of momentum equation, noting that the initial momentum of the system is zero:

$$(1) \quad 0 = M_Y v_Y - M_\alpha v_\alpha$$

Set the disintegration energy equal to the sum of the kinetic energies of the alpha particle and the daughter nucleus (assuming the daughter nucleus is left in the ground state):

$$(2) \quad Q = \frac{1}{2} M_\alpha v_\alpha^2 + \frac{1}{2} M_Y v_Y^2$$

Solve Equation (1) for v_Y and substitute into Equation (2):

$$Q = \frac{1}{2} M_\alpha v_\alpha^2 + \frac{1}{2} M_Y \left(\frac{M_\alpha v_\alpha}{M_Y} \right)^2 = \frac{1}{2} M_\alpha v_\alpha^2 \left(1 + \frac{M_\alpha}{M_Y} \right)$$

$$Q = K_\alpha \left(\frac{M_Y + M_\alpha}{M_Y} \right)$$

Solve for the kinetic energy of the alpha particle:

$$K_\alpha = Q \left(\frac{M_Y}{M_Y + M_\alpha} \right)$$

Evaluate this kinetic energy for the specific decay of ^{226}Ra that we are exploring in this example:

$$K_\alpha = (4.87 \text{ MeV}) \left(\frac{222}{222 + 4} \right) = 4.78 \text{ MeV}$$

To understand the mechanism of alpha decay, let's model the parent nucleus as a system consisting of (1) the alpha particle, already formed as an entity within the nucleus, and (2) the daughter nucleus that will result when the alpha particle is emitted. Figure 44.13 shows a plot of potential energy versus separation distance r between the alpha particle and the daughter nucleus, where the distance marked R is the range of the nuclear force. The curve represents the combined effects of (1) the repulsive Coulomb force, which gives the positive part of the curve for

$r > R$, and (2) the attractive nuclear force, which causes the curve to be negative for $r < R$. As shown in Example 44.7, a typical disintegration energy Q is approximately 5 MeV, which is the approximate kinetic energy of the alpha particle, represented by the lower dashed line in Figure 44.13.

According to classical physics, the alpha particle is trapped in a potential well. How, then, does it ever escape from the nucleus? The answer to this question was first provided by George Gamow (1904–1968) in 1928 and independently by R. W. Gurney (1898–1953) and E. U. Condon (1902–1974) in 1929, using quantum mechanics. In the view of quantum mechanics, there is always some probability that a particle can tunnel through a barrier (Section 41.5). That is exactly how we can describe alpha decay: the alpha particle tunnels through the barrier in Figure 44.13, escaping the nucleus. Furthermore, this model agrees with the observation that higher-energy alpha particles come from nuclei with shorter half-lives. For higher-energy alpha particles in Figure 44.13, the barrier is narrower and the probability is higher that tunneling occurs. The higher probability translates to a shorter half-life.

As an example, consider the decays of ^{238}U and ^{226}Ra in Equations 44.11 and 44.12, along with the corresponding half-lives and alpha-particle energies:

$$^{238}\text{U}: \quad T_{1/2} = 4.47 \times 10^9 \text{ yr} \quad K_{\alpha} = 4.20 \text{ MeV}$$

$$^{226}\text{Ra}: \quad T_{1/2} = 1.60 \times 10^3 \text{ yr} \quad K_{\alpha} = 4.78 \text{ MeV}$$

Notice that a relatively small difference in alpha-particle energy is associated with a tremendous difference of six orders of magnitude in the half-life. The origin of this effect can be understood as follows. Figure 44.13 shows that the curve below an alpha-particle energy of 5 MeV has a slope with a relatively small magnitude. Therefore, a small difference in energy on the vertical axis has a relatively large effect on the width of the potential barrier. Second, recall Equation 41.22, which describes the exponential dependence of the probability of transmission on the barrier width. These two factors combine to give the very sensitive relationship between half-life and alpha-particle energy that the data above suggest.

A life-saving application of alpha decay is the household smoke detector, shown in Figure 44.14. The detector consists of an ionization chamber, a sensitive current detector, and an alarm. A weak radioactive source (usually ^{241}Am) ionizes the air in the chamber of the detector, creating charged particles. A voltage is maintained between the plates inside the chamber, setting up a small but detectable current in the external circuit due to the ions acting as charge carriers between the plates. As long as the current is maintained, the alarm is deactivated. If smoke drifts into the chamber, however, the ions become attached to the smoke particles. These heavier particles do not drift as readily as do the lighter ions, which causes a decrease in the detector current. The external circuit senses this decrease in current and sets off the alarm.

Beta Decay

When a radioactive nucleus undergoes beta decay, the daughter nucleus contains the same number of nucleons as the parent nucleus but the atomic number is changed by 1, which means that the number of protons changes:



where, as mentioned in Section 44.4, e^{-} designates an electron and e^{+} designates a positron, with *beta particle* being the general term referring to either. *Beta decay is not described completely by these expressions.* We shall give reasons for this statement shortly.

As with alpha decay, the nucleon number and total charge are both conserved in beta decays. Because A does not change but Z does, we conclude that in beta decay,

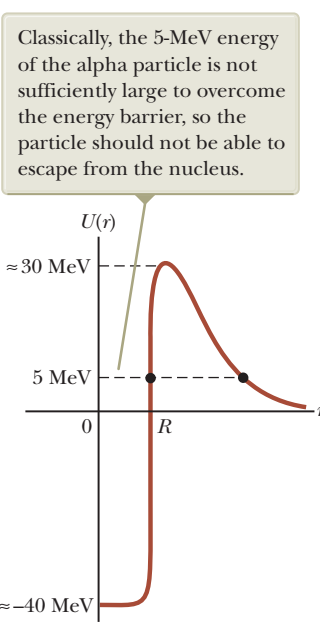
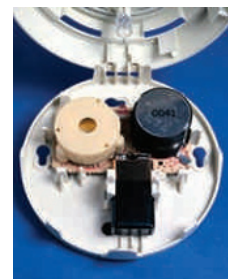


Figure 44.13 Potential energy versus separation distance for a system consisting of an alpha particle and a daughter nucleus. The alpha particle escapes by tunneling through the barrier.



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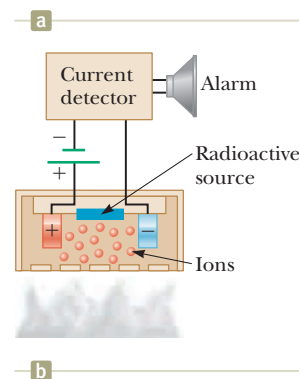
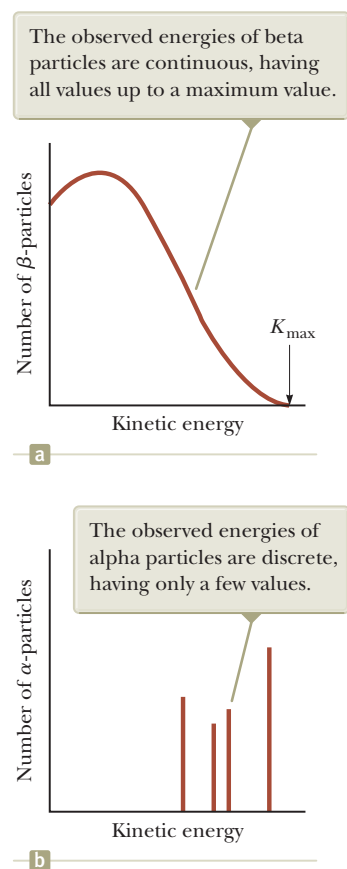


Figure 44.14 (a) A smoke detector uses alpha decay to determine whether smoke is in the air. The alpha source is in the black cylinder at the right. (b) Smoke entering the chamber reduces the detected current, causing the alarm to sound.

**ACTIVE FIGURE 44.15**

(a) Distribution of beta-particle energies in a typical beta decay.
 (b) Distribution of alpha-particle energies in a typical alpha decay.

Properties of the neutrino ▶

- It has zero electric charge.
- Its mass is either zero (in which case it travels at the speed of light) or very small; much recent persuasive experimental evidence suggests that the neutrino mass is not zero. Current experiments place the upper bound of the mass of the neutrino at approximately $7 \text{ eV}/c^2$.
- It has a spin of $\frac{1}{2}$, which allows the law of conservation of angular momentum to be satisfied in beta decay.
- It interacts very weakly with matter and is therefore very difficult to detect.

We can now write the beta-decay processes (Eqs. 44.15 and 44.16) in their correct and complete form:

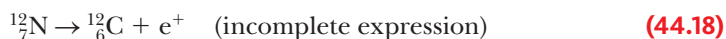
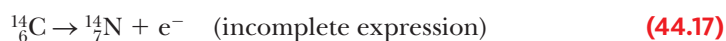
Beta decay processes ▶

as well as those for carbon-14 and nitrogen-12 (Eqs. 44.17 and 44.18):



where the symbol $\bar{\nu}$ represents the **antineutrino**, the antiparticle to the neutrino. We shall discuss antiparticles further in Chapter 46. For now, it suffices to say that a neutrino is emitted in positron decay and an antineutrino is emitted in electron decay. As with alpha decay, the decays listed above are analyzed by applying conservation laws, but relativistic expressions must be used for beta particles because their kinetic energy is large (typically 1 MeV) compared with their rest energy of 0.511 MeV. Active Figure 44.16 shows a pictorial representation of the decays described by Equations 44.21 and 44.22.

either a neutron changes to a proton (Eq. 44.15) or a proton changes to a neutron (Eq. 44.16). Note that the electron or positron emitted in these decays is not present beforehand in the nucleus; it is created in the process of the decay from the rest energy of the decaying nucleus. Two typical beta-decay processes are



Let's consider the energy of the system undergoing beta decay before and after the decay. As with alpha decay, energy of the isolated system must be conserved. Experimentally, it is found that beta particles from a single type of nucleus are emitted over a continuous range of energies (Active Fig. 44.15a), as opposed to alpha decay, in which the alpha particles are emitted with discrete energies (Active Fig. 44.15b). The kinetic energy of the system after the decay is equal to the decrease in rest energy of the system, that is, the Q value. Because all decaying nuclei in the sample have the same initial mass, however, *the Q value must be the same for each decay*. So, why do the emitted particles have the range of kinetic energies shown in Active Figure 44.15a? The law of conservation of energy seems to be violated! It becomes worse: further analysis of the decay processes described by Equations 44.15 and 44.16 shows that the laws of conservation of angular momentum (spin) and linear momentum are also violated!

After a great deal of experimental and theoretical study, Pauli in 1930 proposed that a third particle must be present in the decay products to carry away the “missing” energy and momentum. Fermi later named this particle the **neutrino** (little neutral one) because it had to be electrically neutral and have little or no mass. Although it eluded detection for many years, the neutrino (symbol ν , Greek nu) was finally detected experimentally in 1956 by Frederick Reines (1918–1998), who received the Nobel Prize in Physics for this work in 1995. The neutrino has the following properties:

In Equation 44.19, the number of protons has increased by one and the number of neutrons has decreased by one. We can write the fundamental process of e^- decay in terms of a neutron changing into a proton as follows:

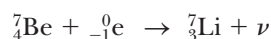


The electron and the antineutrino are ejected from the nucleus, with the net result that there is one more proton and one fewer neutron, consistent with the changes in Z and $A - Z$. A similar process occurs in e^+ decay, with a proton changing into a neutron, a positron, and a neutrino. This latter process can only occur within the nucleus, with the result that the nuclear mass decreases. It cannot occur for an isolated proton because its mass is less than that of the neutron.

A process that competes with e^+ decay is **electron capture**, which occurs when a parent nucleus captures one of its own orbital electrons and emits a neutrino. The final product after decay is a nucleus whose charge is $Z - 1$:



In most cases, it is a K-shell electron that is captured and the process is therefore referred to as **K capture**. One example is the capture of an electron by ${}^7_4\text{Be}$:



Because the neutrino is very difficult to detect, electron capture is usually observed by the x-rays given off as higher-shell electrons cascade downward to fill the vacancy created in the K shell.

Finally, we specify Q values for the beta-decay processes. The Q values for e^- decay and electron capture are given by $Q = (M_X - M_Y)c^2$, where M_X and M_Y are the masses of neutral atoms. In e^- decay, the parent nucleus experiences an increase in atomic number and, for the atom to become neutral, an electron must be absorbed by the atom. If the neutral parent atom and an electron (which will eventually combine with the daughter to form a neutral atom) is the initial system and the final system is the neutral daughter atom and the beta-ejected electron, the system contains a free electron both before and after the decay. Therefore, in subtracting the initial and final masses of the system, this electron mass cancels.

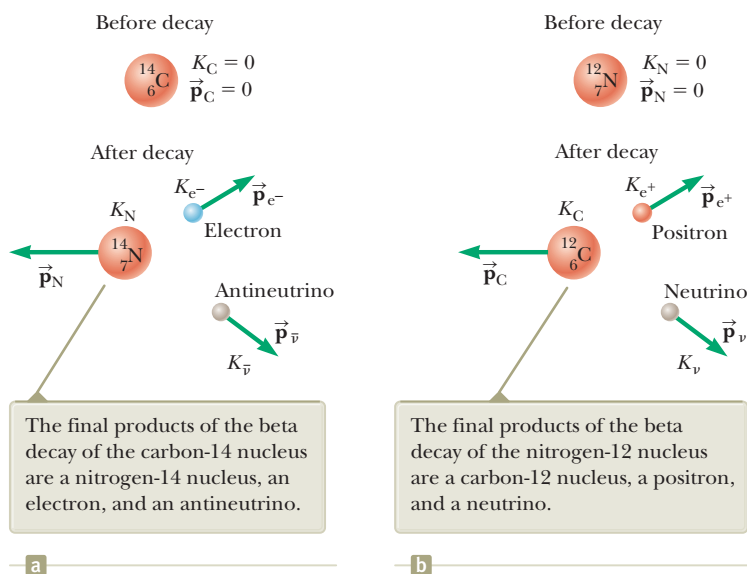
The Q values for e^+ decay are given by $Q = (M_X - M_Y - 2m_e)c^2$. The extra term $-2m_e c^2$ in this expression is necessary because the atomic number of the parent decreases by one when the daughter is formed. After it is formed by the decay, the daughter atom sheds one electron to form a neutral atom. Therefore, the final products are the daughter atom, the shed electron, and the ejected positron.

Pitfall Prevention 44.7

Mass Number of the Electron

An alternative notation for an electron in Equation 44.24 is the symbol ${}_{-1}^0 e$, which does not imply that the electron has zero rest energy. The mass of the electron is so much smaller than that of the lightest nucleon, however, that we approximate it as zero in the context of nuclear decays and reactions.

◀ Electron capture



ACTIVE FIGURE 44.16

- (a) The beta decay of carbon-14.
(b) The beta decay of nitrogen-12.

These relationships are useful in determining whether or not a process is energetically possible. For example, the Q value for proposed e^+ decay for a particular parent nucleus may turn out to be negative. In that case, this decay does not occur. The Q value for electron capture for this parent nucleus, however, may be a positive number, so electron capture can occur even though e^+ decay is not possible. Such is the case for the decay of ${}^7_4\text{Be}$ shown above.

Quick Quiz 44.4 Which of the following is the correct daughter nucleus associated with the beta decay of ${}^{184}_{72}\text{Hf}$? (a) ${}^{183}_{72}\text{Hf}$ (b) ${}^{183}_{73}\text{Ta}$ (c) ${}^{184}_{73}\text{Ta}$

Carbon Dating

The beta decay of ${}^{14}\text{C}$ (Eq. 44.21) is commonly used to date organic samples. Cosmic rays in the upper atmosphere cause nuclear reactions (Section 44.7) that create ${}^{14}\text{C}$. The ratio of ${}^{14}\text{C}$ to ${}^{12}\text{C}$ in the carbon dioxide molecules of our atmosphere has a constant value of approximately $r_0 = 1.3 \times 10^{-12}$. The carbon atoms in all living organisms have this same ${}^{14}\text{C}/{}^{12}\text{C}$ ratio r_0 because the organisms continuously exchange carbon dioxide with their surroundings. When an organism dies, however, it no longer absorbs ${}^{14}\text{C}$ from the atmosphere, and so the ${}^{14}\text{C}/{}^{12}\text{C}$ ratio decreases as the ${}^{14}\text{C}$ decays with a half-life of 5 730 yr. It is therefore possible to measure the age of a material by measuring its ${}^{14}\text{C}$ activity. Using this technique, scientists have been able to identify samples of wood, charcoal, bone, and shell as having lived from 1 000 to 25 000 years ago. This knowledge has helped us reconstruct the history of living organisms—including humans—during this time span.

A particularly interesting example is the dating of the Dead Sea Scrolls. This group of manuscripts was discovered by a shepherd in 1947. Translation showed them to be religious documents, including most of the books of the Old Testament. Because of their historical and religious significance, scholars wanted to know their age. Carbon dating applied to the material in which they were wrapped established their age at approximately 1 950 yr.

Conceptual Example 44.8

The Age of Iceman

In 1991, German tourists discovered the well-preserved remains of a man, now called “Ötzi the Iceman,” trapped in a glacier in the Italian Alps. (See the photograph at the opening of this chapter.) Radioactive dating with ${}^{14}\text{C}$ revealed that this person was alive approximately 5 300 years ago. Why did scientists date a sample of Ötzi using ${}^{14}\text{C}$ rather than ${}^{11}\text{C}$, which is a beta emitter having a half-life of 20.4 min?

SOLUTION

Because ${}^{14}\text{C}$ has a half-life of 5 730 yr, the fraction of ${}^{14}\text{C}$ nuclei remaining after thousands of years is high enough to allow accurate measurements of changes in the sample’s activity. Because ${}^{11}\text{C}$ has a very short half-life, it is not useful; its activity decreases to a vanishingly small value over the age of the sample, making it impossible to detect.

An isotope used to date a sample must be present in a known amount in the sample when it is formed. As a general rule, the isotope chosen to date a sample should also have a half-life that is on the same order of magnitude as the age of the sample. If the half-life is much less than the

age of the sample, there won’t be enough activity left to measure because almost all the original radioactive nuclei will have decayed. If the half-life is much greater than the age of the sample, the amount of decay that has taken place since the sample died will be too small to measure. For example, if you have a specimen estimated to have died 50 years ago, neither ${}^{14}\text{C}$ (5 730 yr) nor ${}^{11}\text{C}$ (20 min) is suitable. If you know your sample contains hydrogen, however, you can measure the activity of ${}^3\text{H}$ (tritium), a beta emitter that has a half-life of 12.3 yr.

Example 44.9 Radioactive Dating

A piece of charcoal containing 25.0 g of carbon is found in some ruins of an ancient city. The sample shows a ^{14}C activity R of 250 decays/min. How long has the tree from which this charcoal came been dead?

SOLUTION

Conceptualize Because the charcoal was found in ancient ruins, we expect the current activity to be smaller than the initial activity. If we can determine the initial activity, we can find out how long the wood has been dead.

Categorize The text of the question helps us categorize this example as a carbon dating problem.

Analyze Solve Equation 44.7 for t :

$$(1) \quad t = -\frac{1}{\lambda} \ln \left(\frac{R}{R_0} \right)$$

Evaluate the ratio R/R_0 using Equation 44.7, the initial value of the $^{14}\text{C}/^{12}\text{C}$ ratio r_0 , the number of moles n of carbon, and Avogadro's number N_A :

$$\frac{R}{R_0} = \frac{R}{\lambda N_0(^{14}\text{C})} = \frac{R}{\lambda r_0 N_0(^{12}\text{C})} = \frac{R}{\lambda r_0 n N_A}$$

Replace the number of moles in terms of the molar mass M of carbon and the mass m of the sample and substitute for the decay constant λ :

$$\frac{R}{R_0} = \frac{R}{(\ln 2 / T_{1/2}) r_0 (m/M) N_A} = \frac{R M T_{1/2}}{r_0 m N_A \ln 2}$$

Substitute numerical values:

$$\frac{R}{R_0} = \frac{(250 \text{ min}^{-1})(12.0 \text{ g/mol})(5730 \text{ yr})}{(1.3 \times 10^{-12})(25.0 \text{ g})(6.022 \times 10^{23} \text{ mol}^{-1}) \ln 2} \left(\frac{3.156 \times 10^7 \text{ s}}{1 \text{ yr}} \right) \left(\frac{1 \text{ min}}{60 \text{ s}} \right)$$

$$= 0.667$$

Substitute this ratio into Equation (1) and substitute for the decay constant λ :

$$t = -\frac{1}{\lambda} \ln \left(\frac{R}{R_0} \right) = -\frac{T_{1/2}}{\ln 2} \ln \left(\frac{R}{R_0} \right)$$

$$= -\frac{5730 \text{ yr}}{\ln 2} \ln (0.667) = 3.4 \times 10^3 \text{ yr}$$

Finalize Note that the time interval found here is on the same order of magnitude as the half-life, so ^{14}C is a valid isotope to use for this sample, as discussed in Conceptual Example 44.8.

Gamma Decay

Very often, a nucleus that undergoes radioactive decay is left in an excited energy state. The nucleus can then undergo a second decay to a lower-energy state, perhaps to the ground state, by emitting a high-energy photon:



where X^* indicates a nucleus in an excited state. The typical half-life of an excited nuclear state is 10^{-10} s. Photons emitted in such a de-excitation process are called gamma rays. Such photons have very high energy (1 MeV to 1 GeV) relative to the energy of visible light (approximately 1 eV). Recall from Section 42.3 that the energy of a photon emitted or absorbed by an atom equals the difference in energy between the two electronic states involved in the transition. Similarly, a gamma-ray photon has an energy hf that equals the energy difference ΔE between two nuclear energy levels. When a nucleus decays by emitting a gamma ray, the only change in

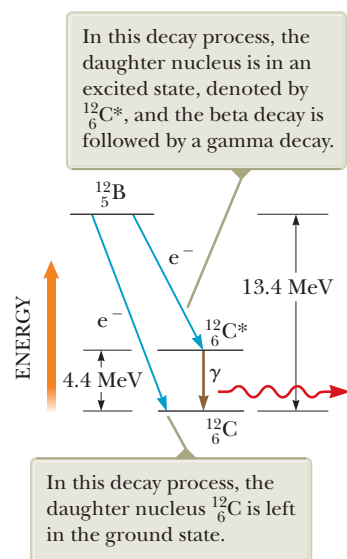


Figure 44.17 An energy-level diagram showing the initial nuclear state of a ^{12}B nucleus and two possible lower-energy states of the ^{12}C nucleus.

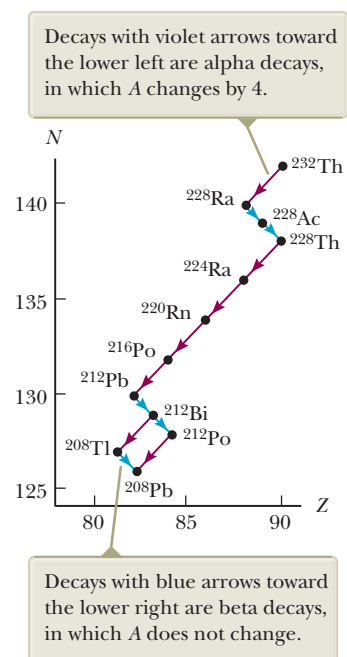


Figure 44.18 Successive decays for the ^{232}Th series.

TABLE 44.3 Various Decay Pathways

Alpha decay	${}^A_Z\text{X} \rightarrow {}^{A-4}_{Z-2}\text{Y} + {}^4_2\text{He}$
Beta decay (e^-)	${}^A_Z\text{X} \rightarrow {}^A_{Z+1}\text{Y} + e^- + \bar{\nu}$
Beta decay (e^+)	${}^A_Z\text{X} \rightarrow {}^A_{Z-1}\text{Y} + e^+ + \nu$
Electron capture	${}^A_Z\text{X} + e^- \rightarrow {}^A_{Z-1}\text{Y} + \nu$
Gamma decay	${}^A_Z\text{X}^* \rightarrow {}^A_Z\text{X} + \gamma$

the nucleus is that it ends up in a lower-energy state. There are no changes in Z , N , or A .

A nucleus may reach an excited state as the result of a violent collision with another particle. More common, however, is for a nucleus to be in an excited state after it has undergone alpha or beta decay. The following sequence of events represents a typical situation in which gamma decay occurs:



Figure 44.17 shows the decay scheme for ^{12}B , which undergoes beta decay to either of two levels of ^{12}C . It can either (1) decay directly to the ground state of ^{12}C by emitting a 13.4-MeV electron or (2) undergo beta decay to an excited state of $^{12}\text{C}^*$ followed by gamma decay to the ground state. The latter process results in the emission of a 9.0-MeV electron and a 4.4-MeV photon.

The various pathways by which a radioactive nucleus can undergo decay are summarized in Table 44.3.

44.6 Natural Radioactivity

Radioactive nuclei are generally classified into two groups: (1) unstable nuclei found in nature, which give rise to **natural radioactivity**, and (2) unstable nuclei produced in the laboratory through nuclear reactions, which exhibit **artificial radioactivity**.

As Table 44.4 shows, there are three series of naturally occurring radioactive nuclei. Each series starts with a specific long-lived radioactive isotope whose half-life exceeds that of any of its unstable descendants. The three natural series begin with the isotopes ^{238}U , ^{235}U , and ^{232}Th , and the corresponding stable end products are three isotopes of lead: ^{206}Pb , ^{207}Pb , and ^{208}Pb . The fourth series in Table 44.4 begins with ^{237}Np and has as its stable end product ^{209}Bi . The element ^{237}Np is a *transuranic* element (one having an atomic number greater than that of uranium) not found in nature. This element has a half-life of “only” 2.14×10^6 years.

Figure 44.18 shows the successive decays for the ^{232}Th series. First, ^{232}Th undergoes alpha decay to ^{228}Ra . Next, ^{228}Ra undergoes two successive beta decays to ^{228}Th . The series continues and finally branches when it reaches ^{212}Bi . At this point, there are two decay possibilities. The sequence shown in Figure 44.18 is characterized by a mass-number decrease of either 4 (for alpha decays) or 0 (for beta or gamma decays). The two uranium series are more complex than the ^{232}Th series. In addition, several naturally occurring radioactive isotopes, such as ^{14}C and ^{40}K , are not part of any decay series.

Because of these radioactive series, our environment is constantly replenished with radioactive elements that would otherwise have disappeared long ago. For example, because our solar system is approximately 5×10^9 years old, the supply of

TABLE 44.4 The Four Radioactive Series

Series	Starting Isotope	Half-life (years)	Stable End Product
Uranium } Natural	$^{238}_{92}\text{U}$	4.47×10^9	$^{206}_{82}\text{Pb}$
	$^{235}_{92}\text{U}$	7.04×10^8	$^{207}_{82}\text{Pb}$
	$^{232}_{90}\text{Th}$	1.41×10^{10}	$^{208}_{82}\text{Pb}$
Neptunium	$^{237}_{93}\text{Np}$	2.14×10^6	$^{209}_{83}\text{Bi}$

^{226}Ra (whose half-life is only 1 600 years) would have been depleted by radioactive decay long ago if it were not for the radioactive series starting with ^{238}U .

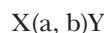
44.7 Nuclear Reactions

We have studied radioactivity, which is a spontaneous process in which the structure of a nucleus changes. It is also possible to stimulate changes in the structure of nuclei by bombarding them with energetic particles. Such collisions, which change the identity of the target nuclei, are called **nuclear reactions**. Rutherford was the first to observe them, in 1919, using naturally occurring radioactive sources for the bombarding particles. Since then, thousands of nuclear reactions have been observed following the development of charged-particle accelerators in the 1930s. With today's advanced technology in particle accelerators and particle detectors, it is possible to achieve particle energies of at least $1\,000\text{ GeV} = 1\text{ TeV}$. These high-energy particles are used to create new particles whose properties are helping to solve the mysteries of the nucleus.

Consider a reaction in which a target nucleus X is bombarded by a particle a, resulting in a daughter nucleus Y and an outgoing particle b:



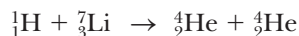
Sometimes this reaction is written in the more compact form



In Section 44.5, the Q value, or disintegration energy, of a radioactive decay was defined as the rest energy transformed to kinetic energy as a result of the decay process. Likewise, we define the **reaction energy** Q associated with a nuclear reaction as *the difference between the initial and final rest energy resulting from the reaction*:

$$Q = (M_a + M_X - M_Y - M_b)c^2 \quad (44.29) \quad \leftarrow \text{Reaction energy } Q$$

As an example, consider the reaction $^7\text{Li}(p, \alpha)^4\text{He}$. The notation p indicates a proton, which is a hydrogen nucleus. Therefore, we can write this reaction in the expanded form



The Q value for this reaction is 17.3 MeV. A reaction such as this one, for which Q is positive, is called **exothermic**. A reaction for which Q is negative is called **endothermic**. To satisfy conservation of momentum, an endothermic reaction does not occur unless the bombarding particle has a kinetic energy greater than Q . (See Problem 58.) The minimum energy necessary for such a reaction to occur is called the **threshold energy**.

If particles a and b in a nuclear reaction are identical so that X and Y are also necessarily identical, the reaction is called a **scattering event**. If the kinetic energy of the system (a and X) before the event is the same as that of the system (b and Y) after the event, it is classified as *elastic scattering*. If the kinetic energy of the system after the event is less than that before the event, the reaction is described as *inelastic scattering*. In this case, the target nucleus has been raised to an excited state by the event, which accounts for the difference in energy. The final system now consists of b and an excited nucleus Y^* , and eventually it will become b, Y, and γ , where γ is the gamma-ray photon that is emitted when the system returns to the ground state. This elastic and inelastic terminology is identical to that used in describing collisions between macroscopic objects as discussed in Section 9.4.

In addition to energy and momentum, the total charge and total number of nucleons must be conserved in any nuclear reaction. For example, consider the

reaction ${}^1_1\text{H} + {}^{19}_9\text{F} \rightarrow {}^{16}_8\text{O} + {}^4_2\text{He}$, which has a Q value of 8.11 MeV. We can show this reaction more completely as



The total number of nucleons before the reaction ($1 + 19 = 20$) is equal to the total number after the reaction ($16 + 4 = 20$). Furthermore, the total charge is the same before ($1 + 9$) and after ($8 + 2$) the reaction.

44.8 Nuclear Magnetic Resonance and Magnetic Resonance Imaging

In this section, we describe an important application of nuclear physics in medicine called magnetic resonance imaging. To understand this application, we first discuss the spin angular momentum of the nucleus. This discussion has parallels with the discussion of spin for atomic electrons.

In Chapter 42, we discussed that the electron has an intrinsic angular momentum, called spin. Nuclei also have spin because their component particles—neutrons and protons—each have spin $\frac{1}{2}$ as well as orbital angular momentum within the nucleus. All types of angular momentum obey the quantum rules that were outlined for orbital and spin angular momentum in Chapter 42. In particular, two quantum numbers associated with the angular momentum determine the allowed values of the magnitude of the angular momentum vector and its direction in space. The magnitude of the nuclear angular momentum is $\sqrt{I(I+1)}\hbar$, where I is called the **nuclear spin quantum number** and may be an integer or a half-integer, depending on how the individual proton and neutron spins combine. The quantum number I is the analog to ℓ for the electron in an atom as discussed in Section 42.6. Furthermore, there is a quantum number m_I that is the analog to m_ℓ , in that the allowed projections of the nuclear spin angular momentum vector on the z axis are $m_I\hbar$. The values of m_I range from $-I$ to $+I$ in steps of 1. (In fact, for *any* type of spin with a quantum number S , there is a quantum number m_S that ranges in value from $-S$ to $+S$ in steps of 1.) Therefore, the maximum value of the z component of the spin angular momentum vector is $I\hbar$. Figure 44.19 is a vector model (see Section 42.6) illustrating the possible orientations of the nuclear spin vector and its projections along the z axis for the case in which $I = \frac{3}{2}$.

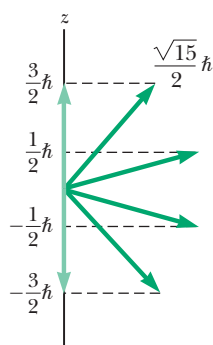


Figure 44.19 A vector model showing possible orientations of the nuclear spin angular momentum vector and its projections along the z axis for the case $I = \frac{3}{2}$.

Nuclear spin has an associated nuclear magnetic moment, similar to that of the electron. The spin magnetic moment of a nucleus is measured in terms of the **nuclear magneton** μ_n , a unit of moment defined as

Nuclear magneton ►

$$\mu_n \equiv \frac{e\hbar}{2m_p} = 5.05 \times 10^{-27} \text{ J/T} \quad (44.31)$$

where m_p is the mass of the proton. This definition is analogous to that of the Bohr magneton μ_B , which corresponds to the spin magnetic moment of a free electron (see Section 42.6). Note that μ_n is smaller than μ_B ($= 9.274 \times 10^{-24} \text{ J/T}$) by a factor of 1 836 because of the large difference between the proton mass and the electron mass.

The magnetic moment of a free proton is $2.792 8\mu_n$. Unfortunately, there is no general theory of nuclear magnetism that explains this value. The neutron also has a magnetic moment, which has a value of $-1.913 5\mu_n$. The negative sign indicates that this moment is opposite the spin angular momentum of the neutron. The existence of a magnetic moment for the neutron is surprising in view of the neutron being uncharged. That suggests that the neutron is not a fundamental particle but rather has an underlying structure consisting of charged constituents. We shall explore this structure in Chapter 46.

The potential energy associated with a magnetic dipole moment $\vec{\mu}$ in an external magnetic field \vec{B} is given by $-\vec{\mu} \cdot \vec{B}$ (Eq. 29.18). When the magnetic moment $\vec{\mu}$ is lined up with the field as closely as quantum physics allows, the potential energy of the dipole–field system has its minimum value E_{\min} . When $\vec{\mu}$ is antiparallel to the field as possible, the potential energy has its maximum value E_{\max} . In general, there are other energy states between these values corresponding to the quantized directions of the magnetic moment with respect to the field. For a nucleus with spin $\frac{1}{2}$, there are only two allowed states, with energies E_{\min} and E_{\max} . These two energy states are shown in Figure 44.20.

It is possible to observe transitions between these two spin states using a technique called **NMR**, for **nuclear magnetic resonance**. A constant magnetic field (\vec{B} in Fig. 44.20) is introduced to define a z axis and split the energies of the spin states. A second, weaker, oscillating magnetic field is then applied perpendicular to \vec{B} , creating a cloud of radio-frequency photons around the sample. When the frequency of the oscillating field is adjusted so that the photon energy matches the energy difference between the spin states, there is a net absorption of photons by the nuclei that can be detected electronically.

Figure 44.21 is a simplified diagram of the apparatus used in nuclear magnetic resonance. The energy absorbed by the nuclei is supplied by the tunable oscillator producing the oscillating magnetic field. Nuclear magnetic resonance and a related technique called *electron spin resonance* are extremely important methods for studying nuclear and atomic systems and the ways in which these systems interact with their surroundings.

A widely used medical diagnostic technique called **MRI**, for **magnetic resonance imaging**, is based on nuclear magnetic resonance. Because nearly two-thirds of the atoms in the human body are hydrogen (which gives a strong NMR signal), MRI works exceptionally well for viewing internal tissues. The patient is placed inside a large solenoid that supplies a magnetic field that is constant in time but whose magnitude varies spatially across the body. Because of the variation in the field, hydrogen atoms in different parts of the body have different energy splittings between spin states, so the resonance signal can be used to provide information about the positions of the protons. A computer is used to analyze the position information to provide data for constructing a final image. Contrast in the final image among different types of tissues is created by computer analysis of the time intervals for the nuclei to return to the lower-energy spin state between pulses of radio-frequency photons. Contrast can be enhanced with the use of contrast agents such as gadolinium compounds or iron oxide nanoparticles taken orally or injected intravenously. An MRI scan showing incredible detail in internal body structure is shown in Figure 44.22.

The main advantage of MRI over other imaging techniques is that it causes minimal cellular damage. The photons associated with the radio-frequency signals used in MRI have energies of only about 10^{-7} eV. Because molecular bond strengths are much larger (approximately 1 eV), the radio-frequency radiation causes little cellular damage. In comparison, x-rays have energies ranging from 10^4 to 10^6 eV and can cause considerable cellular damage. Therefore, despite some individuals' fears of the word *nuclear* associated with MRI, the radio-frequency radiation involved is overwhelmingly safer than the x-rays that these individuals might accept more readily. A disadvantage of MRI is that the equipment required to conduct the procedure is very expensive, so MRI images are costly.

The magnetic field produced by the solenoid is sufficient to lift a car, and the radio signal is about the same magnitude as that from a small commercial broadcasting station. Although MRI is inherently safe in normal use, the strong magnetic field of the solenoid requires diligent care to ensure that no ferromagnetic materials are located in the room near the MRI apparatus. Several accidents have occurred, such as a 2000 incident in which a gun pulled from a police officer's hand discharged upon striking the machine.

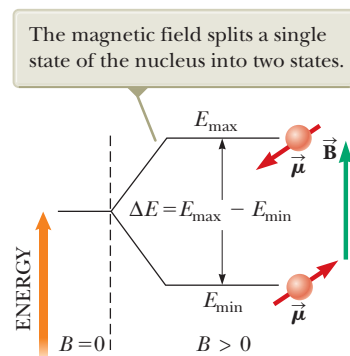


Figure 44.20 A nucleus with spin $\frac{1}{2}$ is placed in a magnetic field.

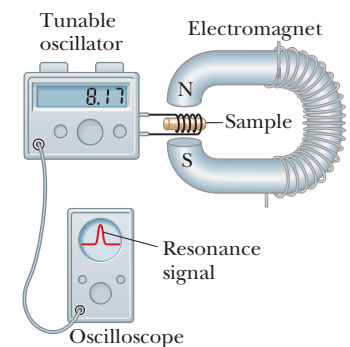


Figure 44.21 Experimental arrangement for nuclear magnetic resonance. The radio-frequency magnetic field created by the coil surrounding the sample and provided by the variable-frequency oscillator is perpendicular to the constant magnetic field created by the electromagnet. When the nuclei in the sample meet the resonance condition, the nuclei absorb energy from the radio-frequency field of the coil; this absorption changes the characteristics of the circuit in which the coil is included. Most modern NMR spectrometers use superconducting magnets at fixed field strengths and operate at frequencies of approximately 200 MHz.



Figure 44.22 A color-enhanced MRI scan of a human brain.

Summary

Definitions

A nucleus is represented by the symbol A_ZX , where A is the **mass number** (the total number of nucleons) and Z is the **atomic number** (the total number of protons). The total number of neutrons in a nucleus is the **neutron number** N , where $A = N + Z$. Nuclei having the same Z value but different A and N values are **isotopes** of each other.

The magnetic moment of a nucleus is measured in terms of the **nuclear magneton** μ_n , where

$$\mu_n \equiv \frac{e\hbar}{2m_p} = 5.05 \times 10^{-27} \text{ J/T} \quad (44.31)$$

Concepts and Principles

Assuming nuclei are spherical, their radius is given by

$$r = aA^{1/3} \quad (44.1)$$

where $a = 1.2$ fm.

Nuclei are stable because of the **nuclear force** between nucleons. This short-range force dominates the Coulomb repulsive force at distances of less than about 2 fm and is independent of charge. Light stable nuclei have equal numbers of protons and neutrons. Heavy stable nuclei have more neutrons than protons. The most stable nuclei have Z and N values that are both even.

The difference between the sum of the masses of a group of separate nucleons and the mass of the compound nucleus containing these nucleons, when multiplied by c^2 , gives the **binding energy** E_b of the nucleus. The binding energy of a nucleus can be calculated in MeV using the expression

$$E_b = [ZM(\text{H}) + Nm_n - M({}^A_ZX)] \times 931.494 \text{ MeV/u} \quad (44.2)$$

where $M(\text{H})$ is the atomic mass of the neutral hydrogen atom, $M({}^A_ZX)$ represents the atomic mass of an atom of the isotope A_ZX , and m_n is the mass of the neutron.

The **liquid-drop model** of nuclear structure treats the nucleons as molecules in a drop of liquid. The four main contributions influencing binding energy are the volume effect, the surface effect, the Coulomb repulsion effect, and the symmetry effect. Summing such contributions results in the **semiempirical binding-energy formula**:

$$E_b = C_1A - C_2A^{2/3} - C_3\frac{Z(Z-1)}{A^{1/3}} - C_4\frac{(N-Z)^2}{A} \quad (44.3)$$

The **shell model**, or **independent-particle model**, assumes each nucleon exists in a shell and can only have discrete energy values. The stability of certain nuclei can be explained with this model.

A radioactive substance decays by **alpha decay**, **beta decay**, or **gamma decay**. An alpha particle is the ${}^4\text{He}$ nucleus, a beta particle is either an electron (e^-) or a positron (e^+), and a gamma particle is a high-energy photon.

If a radioactive material contains N_0 radioactive nuclei at $t = 0$, the number N of nuclei remaining after a time t has elapsed is

$$N = N_0e^{-\lambda t} \quad (44.6)$$

where λ is the **decay constant**, a number equal to the probability per second that a nucleus will decay. The **decay rate**, or **activity**, of a radioactive substance is

$$R = \left| \frac{dN}{dt} \right| = R_0e^{-\lambda t} \quad (44.7)$$

where $R_0 = \lambda N_0$ is the activity at $t = 0$. The **half-life** $T_{1/2}$ is the time interval required for half of a given number of radioactive nuclei to decay, where

$$T_{1/2} = \frac{0.693}{\lambda} \quad (44.8)$$

In alpha decay, a helium nucleus is ejected from the parent nucleus with a discrete set of kinetic energies. A nucleus undergoing beta decay emits either an electron (e^-) and an antineutrino ($\bar{\nu}$) or a positron (e^+) and a neutrino (ν). The electron or positron is ejected with a continuous range of energies. In **electron capture**, the nucleus of an atom absorbs one of its own electrons and emits a neutrino. In gamma decay, a nucleus in an excited state decays to its ground state and emits a gamma ray.

Nuclear reactions can occur when a target nucleus X is bombarded by a particle a, resulting in a daughter nucleus Y and an outgoing particle b:



The mass energy conversion in such a reaction, called the **reaction energy** Q , is

$$Q = (M_a + M_X - M_Y - M_b)c^2 \quad (44.29)$$

Objective Questions

denotes answer available in Student Solutions Manual/Study Guide

- In the decay ${}_{90}^{234}\text{Th} \rightarrow {}_Z^A\text{Ra} + {}_2^4\text{He}$, identify the mass number and the atomic number of the Ra nucleus: (a) $A = 230$, $Z = 92$ (b) $A = 238$, $Z = 88$ (c) $A = 230$, $Z = 88$ (d) $A = 234$, $Z = 88$ (e) $A = 238$, $Z = 86$
- When ${}_{60}^{144}\text{Nd}$ decays to ${}_{58}^{140}\text{Ce}$, identify the particle that is released. (a) a proton (b) an alpha particle (c) an electron (d) a neutron (e) a neutrino
- When ${}_{15}^{32}\text{P}$ decays to ${}_{16}^{32}\text{S}$, which of the following particles is emitted? (a) a proton (b) an alpha particle (c) an electron (d) a gamma ray (e) an antineutrino
- (i) To predict the behavior of a nucleus in a fission reaction, which model would be more appropriate, (a) the liquid-drop model or (b) the shell model? (ii) Which model would be more successful in predicting the magnetic moment of a given nucleus? Choose from the same answers as in part (i). (iii) Which could better explain the gamma-ray spectrum of an excited nucleus? Choose from the same answers as in part (i).
- Two samples of the same radioactive nuclide are prepared. Sample G has twice the initial activity of sample H. (i) How does the half-life of G compare with the half-life of H? (a) It is two times larger. (b) It is the same. (c) It is half as large. (ii) After each has passed through five half-lives, how do their activities compare? (a) G has more than twice the activity of H. (b) G has twice the activity of H. (c) G and H have the same activity. (d) G has lower activity than H.
- If a radioactive nuclide ${}_Z^AX$ decays by emitting a gamma ray, what happens? (a) The resulting nuclide has a different Z value. (b) The resulting nuclide has the same A and Z values. (c) The resulting nuclide has a different A value. (d) Both A and Z decrease by one. (e) None of those statements is correct.
- Does a nucleus designated as ${}_{18}^{40}\text{X}$ contain (a) 20 neutrons and 20 protons, (b) 22 protons and 18 neutrons, (c) 18 protons and 22 neutrons, (d) 18 protons and 40 neutrons, or (e) 40 protons and 18 neutrons?
- When the ${}_{36}^{95}\text{Kr}$ nucleus undergoes beta decay by emitting an electron and an antineutrino, does the daughter nucleus (Rb) contain (a) 58 neutrons and 37 protons, (b) 58 protons and 37 neutrons, (c) 54 neutrons and 41 protons, or (d) 55 neutrons and 40 protons?
- What is the Q value for the reaction ${}^9\text{Be} + \alpha \rightarrow {}^{12}\text{C} + n$? (a) 8.4 MeV (b) 7.3 MeV (c) 6.2 MeV (d) 5.7 MeV (e) 4.2 MeV
- The half-life of radium-224 is about 3.6 days. What approximate fraction of a sample remains undecayed after two weeks? (a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{8}$ (d) $\frac{1}{16}$ (e) $\frac{1}{32}$
- A free neutron has a half-life of 614 s. It undergoes beta decay by emitting an electron. Can a free proton undergo a similar decay? (a) yes, the same decay (b) yes, but by emitting a positron (c) yes, but with a very different half-life (d) no
- Which of the following quantities represents the reaction energy of a nuclear reaction? (a) (final mass – initial mass)/ c^2 (b) (initial mass – final mass)/ c^2 (c) (final mass – initial mass) c^2 (d) (initial mass – final mass) c^2 (e) none of those quantities
- In nuclear magnetic resonance, suppose we increase the value of the constant magnetic field. As a result, the frequency of the photons that are absorbed in a particular transition changes. How is the frequency of the photons absorbed related to the magnetic field? (a) The frequency is proportional to the square of the magnetic field. (b) The frequency is directly proportional to the magnetic field. (c) The frequency is independent of the magnetic field. (d) The frequency is inversely proportional to the magnetic field. (e) The frequency is proportional to the reciprocal of the square of the magnetic field.

Conceptual Questions

□ denotes answer available in Student Solutions Manual/Study Guide

- In Rutherford's experiment, assume an alpha particle is headed directly toward the nucleus of an atom. Why doesn't the alpha particle make physical contact with the nucleus?
- Explain why nuclei that are well off the line of stability in Figure 44.4 tend to be unstable.
- A student claims that a heavy form of hydrogen decays by alpha emission. How do you respond?
- In beta decay, the energy of the electron or positron emitted from the nucleus lies somewhere in a relatively large range of possibilities. In alpha decay, however, the alpha-particle energy can only have discrete values. Explain this difference.
- Can carbon-14 dating be used to measure the age of a rock? Explain.
- In positron decay, a proton in the nucleus becomes a neutron and its positive charge is carried away by the positron. A neutron, though, has a larger rest energy than a proton. How is that possible?
- Compare and contrast the properties of a photon and a neutrino.
- Why do nearly all the naturally occurring isotopes lie above the $N = Z$ line in Figure 44.4?
- Why are very heavy nuclei unstable?
- "If no more people were to be born, the law of population growth would strongly resemble the radioactive decay law." Discuss this statement.
- Consider two heavy nuclei X and Y having similar mass numbers. If X has the higher binding energy, which nucleus tends to be more unstable? Explain your answer.
- What fraction of a radioactive sample has decayed after two half-lives have elapsed?
- Figure CQ44.13 shows a watch from the early 20th century. The numbers and the hands of the watch are painted with a paint that contains a small amount of natural radium $^{226}_{88}\text{Ra}$ mixed with a phosphorescent material. The decay of the radium causes the phosphorescent material to glow continuously. The radioactive nuclide $^{226}_{88}\text{Ra}$ has a half-life of approximately 1.60×10^3 years. Being that the solar system is approximately 5 billion years old, why was this isotope still available in the 20th century for use on this watch?
- Can a nucleus emit alpha particles that have different energies? Explain.
- If a nucleus such as ^{226}Ra initially at rest undergoes alpha decay, which has more kinetic energy after the decay, the alpha particle or the daughter nucleus? Explain your answer.
- Suppose it could be shown that the cosmic-ray intensity at the Earth's surface was much greater 10 000 years ago. How would this difference affect what we accept as valid carbon-dated values of the age of ancient samples of once-living matter? Explain your answer.
- (a) How many values of I_z are possible for $I = \frac{5}{2}$? (b) For $I = 3$?

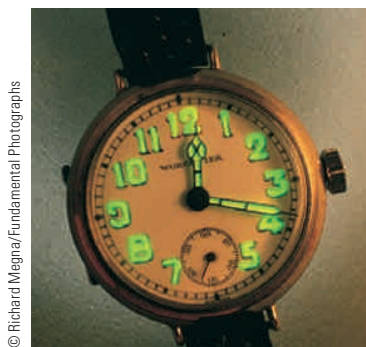


Figure CQ44.13

Problems

WebAssign The problems found in this chapter may be assigned online in Enhanced WebAssign

1. denotes straightforward problem; 2. denotes intermediate problem; 3. denotes challenging problem

1. full solution available in the Student Solutions Manual/Study Guide

1. denotes problems most often assigned in Enhanced WebAssign; these provide students with targeted feedback and either a Master It tutorial or a Watch It solution video.

Q/C denotes asking for quantitative and conceptual reasoning

S denotes symbolic reasoning problem

M denotes Master It tutorial available in Enhanced WebAssign

GP denotes guided problem

shaded denotes "paired problems" that develop reasoning with symbols and numerical values

Section 44.1 Some Properties of Nuclei

1. Consider the ${}^{65}_{29}\text{Cu}$ nucleus. Find approximate values for its (a) radius, (b) volume, and (c) density.
2. **Q/C** (a) Determine the mass number of a nucleus whose radius is approximately equal to two-thirds the radius of ${}^{230}_{88}\text{Ra}$. (b) Identify the element. (c) Are any other answers possible? Explain.
3. **M** (a) Use energy methods to calculate the distance of closest approach for a head-on collision between an alpha particle having an initial energy of 0.500 MeV and a gold nucleus (${}^{197}\text{Au}$) at rest. Assume the gold nucleus remains at rest during the collision. (b) What minimum initial speed must the alpha particle have to approach as close as 300 fm to the gold nucleus?
4. (a) What is the order of magnitude of the number of protons in your body? (b) Of the number of neutrons? (c) Of the number of electrons?
5. Find the nuclear radii of (a) ${}^2_1\text{H}$, (b) ${}^{60}_{27}\text{Co}$, (c) ${}^{197}_{79}\text{Au}$, and (d) ${}^{239}_{94}\text{Pu}$.
6. Using $2.30 \times 10^{17} \text{ kg/m}^3$ as the density of nuclear matter, find the radius of a sphere of such matter that would have a mass equal to that of a baseball, 0.145 kg.
7. A star ending its life with a mass of four to eight times the Sun's mass is expected to collapse and then undergo a supernova event. In the remnant that is not carried away by the supernova explosion, protons and electrons combine to form a neutron star with approximately twice the mass of the Sun. Such a star can be thought of as a gigantic atomic nucleus. Assume $r = aA^{1/3}$ (Eq. 44.1). If a star of mass $3.98 \times 10^{30} \text{ kg}$ is composed entirely of neutrons ($m_n = 1.67 \times 10^{-27} \text{ kg}$), what would its radius be?
8. **Q/C** Figure P44.8 shows the potential energy for two protons as a function of separation distance. In the text, it was claimed that, to be visible on such a graph, the peak in the curve is exaggerated by a factor of ten. (a) Find the electric potential energy of a pair of protons separated by 4.00 fm. (b) Verify that the peak in Figure P44.8 is exaggerated by a factor of ten.

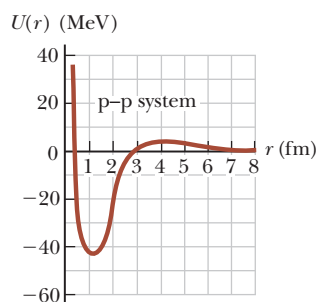


Figure P44.8

9. **Review.** Singly ionized carbon is accelerated through 1 000 V and passed into a mass spectrometer to determine the isotopes present (see Chapter 29). The magnitude of the magnetic field in the spectrometer is 0.200 T. The orbit radius for a ${}^{12}\text{C}$ isotope as it passes through the field is $r = 7.89 \text{ cm}$. Find the radius of the orbit of a ${}^{13}\text{C}$ isotope.
10. **S Review.** Singly ionized carbon is accelerated through a potential difference ΔV and passed into a mass spectrometer to determine the isotopes present (see Chapter 29). The magnitude of the magnetic field in the spectrometer is B . The orbit radius for an isotope of mass m_1 as it passes through the field is r_1 . Find the radius of the orbit of an isotope of mass m_2 .
11. Assume a hydrogen atom is a sphere with diameter 0.100 nm and a hydrogen molecule consists of two such spheres in contact. (a) What fraction of the space in a tank of hydrogen gas at 0°C and 1.00 atm is occupied by the hydrogen molecules themselves? (b) What fraction of the space within one hydrogen atom is occupied by its nucleus, of radius 1.20 fm?
12. **Q/C** In a Rutherford scattering experiment, alpha particles having kinetic energy of 7.70 MeV are fired toward a gold nucleus that remains at rest during the collision. The alpha particles come as close as 29.5 fm to the gold nucleus before turning around. (a) Calculate the de Broglie wavelength for the 7.70-MeV alpha particle and compare it with the distance of closest approach, 29.5 fm. (b) Based on this comparison, why is it proper to treat the alpha particle as a particle and not as a wave in the Rutherford scattering experiment?
13. **Review.** Two golf balls each have a 4.30-cm diameter and are 1.00 m apart. What would be the gravitational force exerted by each ball on the other if the balls were made of nuclear matter?

Section 44.2 Nuclear Binding Energy

14. **Q/C** The peak of the graph of nuclear binding energy per nucleon occurs near ${}^{56}\text{Fe}$, which is why iron is prominent in the spectrum of the Sun and stars. Show that ${}^{56}\text{Fe}$ has a higher binding energy per nucleon than its neighbors ${}^{55}\text{Mn}$ and ${}^{59}\text{Co}$.
15. Calculate the binding energy per nucleon for (a) ${}^2\text{H}$, (b) ${}^4\text{He}$, (c) ${}^{56}\text{Fe}$, and (d) ${}^{238}\text{U}$.
16. **Q/C** (a) Calculate the difference in binding energy per nucleon for the nuclei ${}^{23}_{11}\text{Na}$ and ${}^{23}_{12}\text{Mg}$. (b) How do you account for the difference?
17. A pair of nuclei for which $Z_1 = N_2$ and $Z_2 = N_1$ are called *mirror isobars* (the atomic and neutron numbers are interchanged). Binding-energy measurements on these nuclei can be used to obtain evidence of the charge independence

of nuclear forces (that is, proton–proton, proton–neutron, and neutron–neutron nuclear forces are equal). Calculate the difference in binding energy for the two mirror isobars $^{15}_8\text{O}$ and $^{15}_7\text{N}$. The electric repulsion among eight protons rather than seven accounts for the difference.

18. **Q/C** The energy required to construct a uniformly charged sphere of total charge Q and radius R is $U = 3k_e Q^2/5R$, where k_e is the Coulomb constant (see Problem 73). Assume a ^{40}Ca nucleus contains 20 protons uniformly distributed in a spherical volume. (a) How much energy is required to counter their electrical repulsion according to the above equation? (b) Calculate the binding energy of ^{40}Ca . (c) Explain what you can conclude from comparing the result of part (b) with that of part (a).
19. Nuclei having the same mass numbers are called *isobars*. The isotope $^{139}_{57}\text{La}$ is stable. A radioactive isobar, $^{139}_{59}\text{Pr}$, is located below the line of stable nuclei as shown in Figure P44.19 and decays by e^+ emission. Another radioactive isobar of $^{139}_{57}\text{La}$, $^{139}_{55}\text{Cs}$, decays by e^- emission and is located above the line of stable nuclei in Figure P44.19. (a) Which of these three isobars has the highest neutron-to-proton ratio? (b) Which has the greatest binding energy per nucleon? (c) Which do you expect to be heavier, $^{139}_{59}\text{Pr}$ or $^{139}_{55}\text{Cs}$?

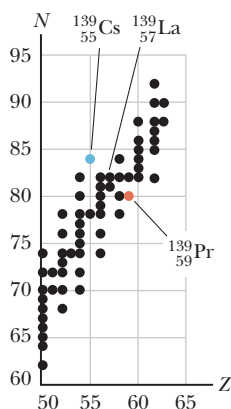


Figure P44.19

20. Calculate the minimum energy required to remove a neutron from the $^{43}_{20}\text{Ca}$ nucleus.

Section 44.3 Nuclear Models

21. (a) Use the semiempirical binding-energy formula (Eq. 44.3) to compute the binding energy for $^{56}_{26}\text{Fe}$. (b) What percentage is contributed to the binding energy by each of the four terms?
22. Using the graph in Figure 44.5, estimate how much energy is released when a nucleus of mass number 200 fissions into two nuclei each of mass number 100.
23. **Q/C S** (a) In the liquid-drop model of nuclear structure, why does the surface-effect term $-C_2A^{2/3}$ have a negative sign? (b) **What If?** The binding energy of the nucleus increases as the volume-to-surface area ratio increases. Calculate this ratio for both spherical and cubical shapes and explain which is more plausible for nuclei.

Section 44.4 Radioactivity

24. **S** From the equation expressing the law of radioactive decay, derive the following useful expressions for the decay constant and the half-life, in terms of the time interval Δt during which the decay rate decreases from R_0 to R :

$$\lambda = \frac{1}{\Delta t} \ln \left(\frac{R_0}{R} \right) \quad T_{1/2} = \frac{(\ln 2) \Delta t}{\ln (R_0/R)}$$

25. A sample of radioactive material contains 1.00×10^{15} atoms and has an activity of 6.00×10^{11} Bq. What is its half-life?
26. **M** A freshly prepared sample of a certain radioactive isotope has an activity of 10.0 mCi. After 4.00 h, its activity is 8.00 mCi. Find (a) the decay constant and (b) the half-life. (c) How many atoms of the isotope were contained in the freshly prepared sample? (d) What is the sample's activity 30.0 h after it is prepared?
27. The half-life of ^{131}I is 8.04 days. (a) Calculate the decay constant for this nuclide. (b) Find the number of ^{131}I nuclei necessary to produce a sample with an activity of 6.40 mCi. (c) A sample of ^{131}I with this initial activity decays for 40.2 d. What is the activity at the end of that period?
28. **Q/C** Tritium has a half-life of 12.33 years. What fraction of the nuclei in a tritium sample will remain (a) after 5.00 yr? (b) After 10.0 yr? (c) After 123.3 yr? (d) According to Equation 44.6, an infinite amount of time is required for the entire sample to decay. Discuss whether that is realistic.

29. **M** The radioactive isotope ^{198}Au has a half-life of 64.8 h. A sample containing this isotope has an initial activity ($t = 0$) of $40.0 \mu\text{Ci}$. Calculate the number of nuclei that decay in the time interval between $t_1 = 10.0$ h and $t_2 = 12.0$ h.

30. **S** A radioactive nucleus has half-life $T_{1/2}$. A sample containing these nuclei has initial activity R_0 at $t = 0$. Calculate the number of nuclei that decay during the interval between the later times t_1 and t_2 .

31. Consider a radioactive sample. Determine the ratio of the number of nuclei decaying during the first half of its half-life to the number of nuclei decaying during the second half of its half-life.

32. **Q/C** (a) The daughter nucleus formed in radioactive decay is often radioactive. Let N_{10} represent the number of parent nuclei at time $t = 0$, $N_1(t)$ the number of parent nuclei at time t , and λ_1 the decay constant of the parent. Suppose the number of daughter nuclei at time $t = 0$ is zero. Let $N_2(t)$ be the number of daughter nuclei at time t and let λ_2 be the decay constant of the daughter. Show that $N_2(t)$ satisfies the differential equation

$$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2$$

(b) Verify by substitution that this differential equation has the solution

$$N_2(t) = \frac{N_{10}\lambda_1}{\lambda_1 - \lambda_2} (e^{-\lambda_2 t} - e^{-\lambda_1 t})$$

This equation is the law of successive radioactive decays. (c) ^{218}Po decays into ^{214}Pb with a half-life of 3.10 min, and ^{214}Pb decays into ^{214}Bi with a half-life of 26.8 min. On the same axes, plot graphs of $N_1(t)$ for ^{218}Po and $N_2(t)$ for ^{214}Pb . Let $N_{10} = 1\,000$ nuclei and choose values of t from 0 to 36 min in 2-min intervals. (d) The curve for ^{214}Pb obtained in part (c) at first rises to a maximum and then starts to decay. At what instant t_m is the number of ^{214}Pb nuclei a maximum? (e) By applying the condition for a maximum $dN_2/dt = 0$, derive a symbolic equation for t_m in terms of λ_1 and λ_2 . (f) Explain whether the value obtained in part (c) agrees with this equation.

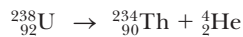
Section 44.5 The Decay Processes

33. The ^{14}C isotope undergoes beta decay according to the process given by Equation 44.21. Find the Q value for this process.
34. Identify the unknown nuclide or particle (X).
 (a) $\text{X} \rightarrow {}^{65}_{28}\text{Ni} + \gamma$ (b) ${}^{215}_{84}\text{Po} \rightarrow \text{X} + \alpha$ (c) $\text{X} \rightarrow {}^{55}_{26}\text{Fe} + e^+ + \nu$
35. Determine which decays can occur spontaneously.
 (a) ${}^{40}_{20}\text{Ca} \rightarrow e^+ + {}^{40}_{19}\text{K}$ (b) ${}^{98}_{44}\text{Ru} \rightarrow {}^4_2\text{He} + {}^{94}_{42}\text{Mo}$
 (c) ${}^{144}_{60}\text{Nd} \rightarrow {}^4_2\text{He} + {}^{140}_{58}\text{Ce}$
36. A ^3H nucleus beta decays into ^3He by creating an electron and an antineutrino according to the reaction



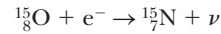
Determine the total energy released in this decay.

37. Find the energy released in the alpha decay



38. **GP** A living specimen in equilibrium with the atmosphere contains one atom of ^{14}C (half-life = 5 730 yr) for every 7.70×10^{11} stable carbon atoms. An archeological sample of wood (cellulose, $\text{C}_{12}\text{H}_{22}\text{O}_{11}$) contains 21.0 mg of carbon. When the sample is placed inside a shielded beta counter with 88.0% counting efficiency, 837 counts are accumulated in one week. We wish to find the age of the sample. (a) Find the number of carbon atoms in the sample. (b) Find the number of carbon-14 atoms in the sample. (c) Find the decay constant for carbon-14 in inverse seconds. (d) Find the initial number of decays per week just after the specimen died. (e) Find the corrected number of decays per week from the current sample. (f) From the answers to parts (d) and (e), find the time interval in years since the specimen died.

39. The nucleus $^{15}_8\text{O}$ decays by electron capture. The nuclear reaction is written



- (a) Write the process going on for a single particle within the nucleus. (b) Disregarding the daughter's recoil, determine the energy of the neutrino.
40. **Q/C** A sample consists of 1.00×10^6 radioactive nuclei with a half-life of 10.0 h. No other nuclei are present at time $t = 0$. The stable daughter nuclei accumulate in the sample as time goes on. (a) Derive an equation giving the number of daughter nuclei N_d as a function of time. (b) Sketch or describe a graph of the number of daughter nuclei as a function of time. (c) What are the maximum and minimum numbers of daughter nuclei, and when do they occur? (d) What are the maximum and minimum rates of change in the number of daughter nuclei, and when do they occur?

Section 44.6 Natural Radioactivity

41. Enter the correct nuclide symbol in each open tan rectangle in Figure P44.41, which shows the sequences of decays in the natural radioactive series starting with the long-lived isotope uranium-235 and ending with the stable nucleus lead-207.

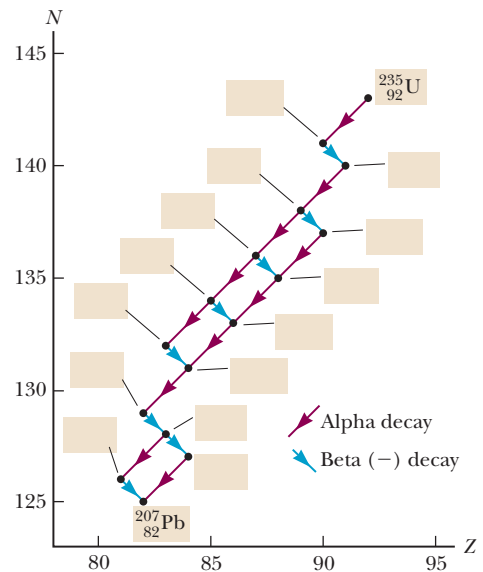


Figure P44.41

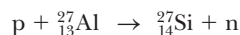
42. **Q/C** The most common isotope of radon is ^{222}Rn , which has half-life 3.82 days. (a) What fraction of the nuclei that were on the Earth one week ago are now undecayed? (b) Of those that existed one year ago? (c) In view of these results, explain why radon remains a problem, contributing significantly to our background radiation exposure.

43. Uranium is naturally present in rock and soil. At one step in its series of radioactive decays, ^{238}U produces the chemically inert gas radon-222, with a half-life of 3.82 days. The radon seeps out of the ground to mix into the atmosphere, typically making open air radioactive with activity 0.3 pCi/L. In homes, ^{222}Rn can be a serious pollutant, accumulating to reach much higher activities in enclosed spaces, sometimes reaching 4.00 pCi/L. If the radon radioactivity exceeds 4.00 pCi/L, the U.S. Environmental Protection Agency suggests taking action to reduce it such as by reducing infiltration of air from the ground. (a) Convert the activity 4.00 pCi/L to units of becquerels per cubic meter. (b) How many ^{222}Rn atoms are in 1 m^3 of air displaying this activity? (c) What fraction of the mass of the air does the radon constitute?

44. A rock sample contains traces of ^{238}U , ^{235}U , ^{232}Th , ^{208}Pb , ^{207}Pb , and ^{206}Pb . Analysis shows that the ratio of the amount of ^{238}U to ^{206}Pb is 1.164. (a) Assuming the rock originally contained no lead, determine the age of the rock. (b) What should be the ratios of ^{235}U to ^{207}Pb and of ^{232}Th to ^{208}Pb so that they would yield the same age for the rock? Ignore the minute amounts of the intermediate decay products in the decay chains. *Note:* This form of multiple dating gives reliable geological dates.

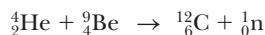
Section 44.7 Nuclear Reactions

45. A beam of 6.61-MeV protons is incident on a target of $^{27}_{13}\text{Al}$. Those that collide produce the reaction

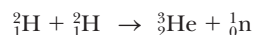


Ignoring any recoil of the product nucleus, determine the kinetic energy of the emerging neutrons.

46. (a) One method of producing neutrons for experimental use is bombardment of light nuclei with alpha particles. In the method used by James Chadwick in 1932, alpha particles emitted by polonium are incident on beryllium nuclei:



What is the Q value of this reaction? (b) Neutrons are also often produced by small-particle accelerators. In one design, deuterons accelerated in a Van de Graaff generator bombard other deuterium nuclei and cause the reaction

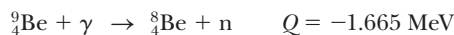
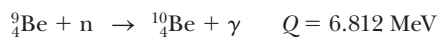


Calculate the Q value of the reaction. (c) Is the reaction in part (b) exothermic or endothermic?

47. Identify the unknown nuclides and particles X and X' in the nuclear reactions (a) $X + {}^4_2\text{He} \rightarrow {}^{24}_{12}\text{Mg} + {}^1_0n$, (b) ${}^{235}_{92}\text{U} + {}^1_0n \rightarrow {}^{90}_{38}\text{Sr} + X + 2({}^1_0n)$, and (c) $2({}^1_1\text{H}) \rightarrow {}^2_1\text{H} + X + X'$.

48. Natural gold has only one isotope, $^{197}_{79}\text{Au}$. If natural gold is irradiated by a flux of slow neutrons, electrons are emitted. (a) Write the reaction equation. (b) Calculate the maximum energy of the emitted electrons.

49. The following reactions are observed:



Calculate the masses of ${}^8\text{Be}$ and ${}^{10}\text{Be}$ in unified mass units to four decimal places from these data.

Section 44.8 Nuclear Magnetic Resonance and Magnetic Resonance Imaging

50. Construct a diagram like that of Figure 44.19 for the cases when I equals (a) $\frac{5}{2}$ and (b) 4.

51. The radio frequency at which a nucleus having a magnetic moment of magnitude μ displays resonance absorption between spin states is called the Larmor frequency and is given by

$$f = \frac{\Delta E}{h} = \frac{2\mu B}{h}$$

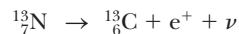
Calculate the Larmor frequency for (a) free neutrons in a magnetic field of 1.00 T, (b) free protons in a magnetic field of 1.00 T, and (c) free protons in the Earth's magnetic field at a location where the magnitude of the field is $50.0\ \mu\text{T}$.

Additional Problems

52. *Why is the following situation impossible?* A ${}^{10}\text{B}$ nucleus is struck by an incoming alpha particle. As a result, a proton and a ${}^{12}\text{C}$ nucleus leave the site after the reaction.

53. (a) Find the radius of the ${}^{12}_6\text{C}$ nucleus. (b) Find the force of repulsion between a proton at the surface of a ${}^{12}_6\text{C}$ nucleus and the remaining five protons. (c) How much work (in MeV) has to be done to overcome this electric repulsion in transporting the last proton from a large distance up to the surface of the nucleus? (d) Repeat parts (a), (b), and (c) for ${}^{238}_{92}\text{U}$.

54. **Q/C** (a) Why is the beta decay $p \rightarrow n + e^+ + \nu$ forbidden for a free proton? (b) **What If?** Why is the same reaction possible if the proton is bound in a nucleus? For example, the following reaction occurs:

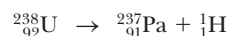


(c) How much energy is released in the reaction given in part (b)?

55. **Review.** Consider the Bohr model of the hydrogen atom, with the electron in the ground state. The magnetic field at the nucleus produced by the orbiting electron has a value of 12.5 T. (See Problem 4 in Chapter 30.) The proton can have its magnetic moment aligned in either of two directions perpendicular to the plane of the electron's orbit.

The interaction of the proton's magnetic moment with the electron's magnetic field causes a difference in energy between the states with the two different orientations of the proton's magnetic moment. Find that energy difference in electron volts.

56. Show that the ^{238}U isotope cannot spontaneously emit a proton by analyzing the hypothetical process



Note: The ^{237}Pa isotope has a mass of 237.051 144 u.

57. **S** As part of his discovery of the neutron in 1932, James Chadwick determined the mass of the newly identified particle by firing a beam of fast neutrons, all having the same speed, at two different targets and measuring the maximum recoil speeds of the target nuclei. The maximum speeds arise when an elastic head-on collision occurs between a neutron and a stationary target nucleus. (a) Represent the masses and final speeds of the two target nuclei as m_1 , v_1 , m_2 , and v_2 and assume Newtonian mechanics applies. Show that the neutron mass can be calculated from the equation

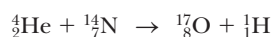
$$m_n = \frac{m_1 v_1 - m_2 v_2}{v_2 - v_1}$$

(b) Chadwick directed a beam of neutrons (produced from a nuclear reaction) on paraffin, which contains hydrogen. The maximum speed of the protons ejected was found to be 3.30×10^7 m/s. Because the velocity of the neutrons could not be determined directly, a second experiment was performed using neutrons from the same source and nitrogen nuclei as the target. The maximum recoil speed of the nitrogen nuclei was found to be 4.70×10^6 m/s. The masses of a proton and a nitrogen nucleus were taken as 1.00 u and 14.0 u, respectively. What was Chadwick's value for the neutron mass?

58. **S** When the nuclear reaction represented by Equation 44.28 is endothermic, the reaction energy Q is negative. For the reaction to proceed, the incoming particle must have a minimum energy called the threshold energy, E_{th} . Some fraction of the energy of the incident particle is transferred to the compound nucleus to conserve momentum. Therefore, E_{th} must be greater than Q . (a) Show that

$$E_{\text{th}} = -Q \left(1 + \frac{M_a}{M_X} \right)$$

(b) Calculate the threshold energy of the incident alpha particle in the reaction



59. In an experiment on the transport of nutrients in a plant's root structure, two radioactive nuclides X and Y are used. Initially, 2.50 times more nuclei of type X are present than

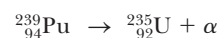
of type Y. At a time 3.00 d later, there are 4.20 times more nuclei of type X than of type Y. Isotope Y has a half-life of 1.60 d. What is the half-life of isotope X?

60. **S** In an experiment on the transport of nutrients in a plant's root structure, two radioactive nuclides X and Y are used. Initially, the ratio of the number of nuclei of type X present to that of type Y is r_1 . After a time interval Δt , the ratio of the number of nuclei of type X present to that of type Y is r_2 . Isotope Y has a half-life of T_Y . What is the half-life of isotope X?

61. **Review.** (a) Is the mass of a hydrogen atom in its ground state larger or smaller than the sum of the masses of a proton and an electron? (b) What is the mass difference? (c) How large is the difference as a percentage of the total mass? (d) Is it large enough to affect the value of the atomic mass listed to six decimal places in Table 44.2?

62. *Why is the following situation impossible?* In an effort to study positronium, a scientist places ^{57}Co and ^{14}C in proximity. The ^{57}Co nuclei decay by e^+ emission, and the ^{14}C nuclei decay by e^- emission. Some of the positrons and electrons from these decays combine to form sufficient amounts of positronium for the scientist to gather data.

63. A by-product of some fission reactors is the isotope $^{239}_{94}\text{Pu}$, an alpha emitter having a half-life of 24 120 yr:



Consider a sample of 1.00 kg of pure $^{239}_{94}\text{Pu}$ at $t = 0$. Calculate (a) the number of $^{239}_{94}\text{Pu}$ nuclei present at $t = 0$ and (b) the initial activity in the sample. (c) **What If?** For what time interval does the sample have to be stored if a "safe" activity level is 0.100 Bq?

64. After the sudden release of radioactivity from the Chernobyl nuclear reactor accident in 1986, the radioactivity of milk in Poland rose to 2 000 Bq/L due to iodine-131 present in the grass eaten by dairy cattle. Radioactive iodine, with half-life 8.04 days, is particularly hazardous because the thyroid gland concentrates iodine. The Chernobyl accident caused a measurable increase in thyroid cancers among children in Poland and many other Eastern European countries. (a) For comparison, find the activity of milk due to potassium. Assume 1.00 liter of milk contains 2.00 g of potassium, of which 0.011 7% is the isotope ^{40}K with half-life 1.28×10^9 yr. (b) After what elapsed time would the activity due to iodine fall below that due to potassium?

65. A theory of nuclear astrophysics proposes that all the elements heavier than iron are formed in supernova explosions ending the lives of massive stars. Assume equal amounts of ^{235}U and ^{238}U were created at the time of the explosion and the present $^{235}\text{U}/^{238}\text{U}$ ratio on the Earth is 0.007 25. The half-lives of ^{235}U and ^{238}U are 0.704×10^9 yr

and 4.47×10^9 yr, respectively. How long ago did the star(s) explode that released the elements that formed the Earth?

66. The activity of a radioactive sample was measured over 12 h, with the net count rates shown in the accompanying table. (a) Plot the logarithm of the counting rate as a function of time. (b) Determine the decay constant and half-life of the radioactive nuclei in the sample. (c) What counting rate would you expect for the sample at $t = 0$? (d) Assuming the efficiency of the counting instrument is 10.0%, calculate the number of radioactive atoms in the sample at $t = 0$.

Time (h)	Counting Rate (counts/min)
1.00	3 100
2.00	2 450
4.00	1 480
6.00	910
8.00	545
10.0	330
12.0	200

67. When, after a reaction or disturbance of any kind, a nucleus is left in an excited state, it can return to its normal (ground) state by emission of a gamma-ray photon (or several photons). This process is illustrated by Equation 44.25. The emitting nucleus must recoil to conserve both energy and momentum. (a) Show that the recoil energy of the nucleus is

$$E_r = \frac{(\Delta E)^2}{2Mc^2}$$

where ΔE is the difference in energy between the excited and ground states of a nucleus of mass M . (b) Calculate the recoil energy of the ^{57}Fe nucleus when it decays by gamma emission from the 14.4-keV excited state. For this calculation, take the mass to be 57 u. *Suggestion:* Assume $hf \ll Mc^2$.

68. **Q.C** In a piece of rock from the Moon, the ^{87}Rb content is assayed to be 1.82×10^{10} atoms per gram of material and the ^{87}Sr content is found to be 1.07×10^9 atoms per gram. The relevant decay relating these nuclides is $^{87}\text{Rb} \rightarrow ^{87}\text{Sr} + e^- + \bar{\nu}$. The half-life of the decay is 4.75×10^{10} yr. (a) Calculate the age of the rock. (b) **What If?** Could the material in the rock actually be much older? What assumption is implicit in using the radioactive dating method?
69. **M** Free neutrons have a characteristic half-life of 10.4 min. What fraction of a group of free neutrons with kinetic energy 0.040 0 eV decays before traveling a distance of 10.0 km?
70. On July 4, 1054, a brilliant light appeared in the constellation Taurus the Bull. The supernova, which could be seen in daylight for some days, was recorded by Arab and Chinese astronomers. As it faded, it remained visible for years, dimming for a time with the 77.1-day half-life of the radioactive cobalt-56 that had been created in the explosion.

(a) The remains of the star now form the Crab nebula (see the photograph opening Chapter 34). In it, the cobalt-56 has now decreased to what fraction of its original activity? (b) Suppose that an American, of the people called the Anasazi, made a charcoal drawing of the supernova. The carbon-14 in the charcoal has now decayed to what fraction of its original activity?

71. When a nucleus decays, it can leave the daughter nucleus in an excited state. The $^{93}_{43}\text{Tc}$ nucleus (molar mass 92.910 2 g/mol) in the ground state decays by electron capture and e^+ emission to energy levels of the daughter (molar mass 92.906 8 g/mol in the ground state) at 2.44 MeV, 2.03 MeV, 1.48 MeV, and 1.35 MeV. (a) Identify the daughter nuclide. (b) To which of the listed levels of the daughter are electron capture and e^+ decay of $^{93}_{43}\text{Tc}$ allowed?
72. The radioactive isotope ^{137}Ba has a relatively short half-life and can be easily extracted from a solution containing its parent ^{137}Cs . This barium isotope is commonly used in an undergraduate laboratory exercise for demonstrating the radioactive decay law. Undergraduate students using modest experimental equipment took the data presented in Figure P44.72. Determine the half-life for the decay of ^{137}Ba using their data.

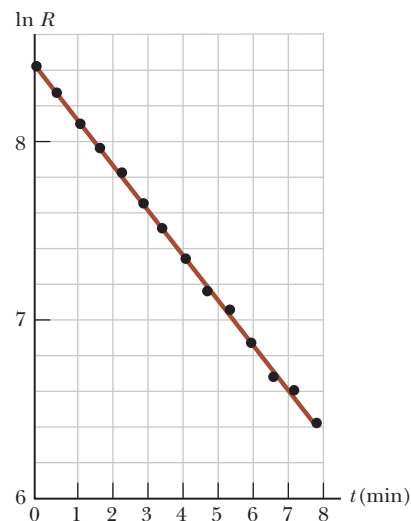


Figure P44.72

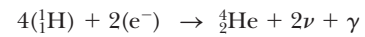
Challenge Problems

73. **S Review.** Consider a model of the nucleus in which the positive charge (Ze) is uniformly distributed throughout a sphere of radius R . By integrating the energy density $\frac{1}{2}\epsilon_0 E^2$ over all space, show that the electric potential energy may be written

$$U = \frac{3Z^2 e^2}{20\pi\epsilon_0 R} = \frac{3k_e Z^2 e^2}{5R}$$

Problem 72 in Chapter 25 derived the same result by a different method.

74. **Q/C** After determining that the Sun has existed for hundreds of millions of years, but before the discovery of nuclear physics, scientists could not explain why the Sun has continued to burn for such a long time interval. For example, if it were a coal fire, it would have burned up in about 3 000 yr. Assume the Sun, whose mass is equal to 1.99×10^{30} kg, originally consisted entirely of hydrogen and its total power output is 3.85×10^{26} W. (a) Assuming the energy-generating mechanism of the Sun is the fusion of hydrogen into helium via the net reaction



calculate the energy (in joules) given off by this reaction. (b) Take the mass of one hydrogen atom to be equal to 1.67×10^{-27} kg. Determine how many hydrogen atoms constitute the Sun. (c) If the total power output remains constant, after what time interval will all the hydrogen be converted into helium, making the Sun die? (d) How does your answer to part (c) compare with current estimates of the expected life of the Sun, which are 4 billion to 7 billion years?