CHAPTER

38

Diffraction Patterns and Polarization

The Hubble Space Telescope does its viewing above the atmosphere and does not suffer from the atmospheric blurring, caused by air turbulence, that plagues ground-based telescopes. Despite this advantage, it does have limitations due to diffraction effects. In this chapter, we show how the wave nature of light limits the ability of any optical system to distinguish between closely spaced objects. *(NASA Hubble Space Telescope Collection)*

When plane light waves pass through a small aperture in an opaque barrier, the aper-ture acts as if it were a point source of light, with waves entering the shadow region behind the barrier. This phenomenon, known as diffraction, was first mentioned in Section 35.3, and can be described only with a wave model for light. In this chapter, we investigate the features of the *diffraction pattern* that occurs when the light from the aperture is allowed to fall upon a screen.

In Chapter 34, we learned that electromagnetic waves are transverse. That is, the electric and magnetic field vectors associated with electromagnetic waves are perpendicular to the direction of wave propagation. In this chapter, we show that under certain conditions these transverse waves with electric field vectors in all possible transverse directions can be *polarized* in various ways. In other words, only certain directions of the electric field vectors are present in the polarized wave.

38.1 Introduction to Diffraction Patterns

In Sections 35.3 and 37.1, we discussed that light of wavelength comparable to or larger than the width of a slit spreads out in all forward directions upon passing through the slit. This phenomenon is called *diffraction*. When light passes through a narrow slit, it spreads beyond the narrow path defined by the slit into regions that would be in shadow if light traveled in straight lines. Other waves, such as sound waves and water waves, also have this property of spreading when passing through apertures or by sharp edges.

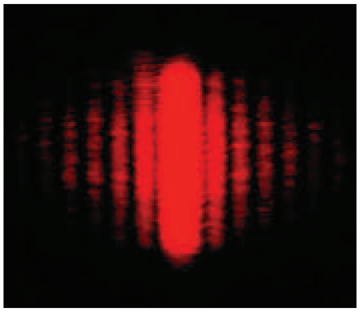

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Figure 38.1 The diffraction pattern that appears on a screen when light passes through a narrow vertical slit. The pattern consists of a broad central fringe and a series of less intense and narrower side fringes.

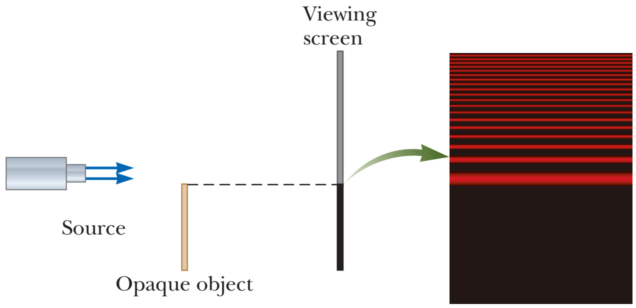
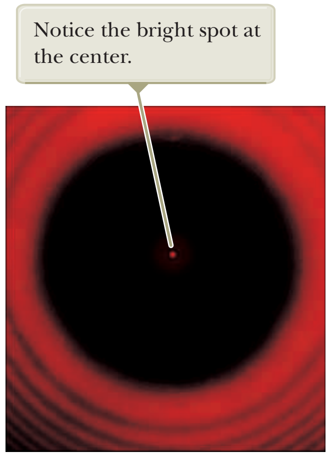


Figure 38.2 Light from a small source passes by the edge of an opaque object and continues on to a screen. A diffraction pattern consisting of bright and dark fringes appears on the screen in the region above the edge of the object.

You might expect that the light passing through a small opening would simply result in a broad region of light on a screen due to the spreading of the light as it passes through the opening. We find something more interesting, however. A **diffraction pattern** consisting of light and dark areas is observed, somewhat similar to the interference patterns discussed earlier. For example, when a narrow slit is placed between a distant light source (or a laser beam) and a screen, the light produces a diffraction pattern like that shown in Figure 38.1. The pattern consists of a broad, intense central band (called the **central maximum**) flanked by a series of narrower, less intense additional bands (called **side maxima** or **secondary maxima**) and a series of intervening dark bands (or **minima**). Figure 38.2 shows a diffraction pattern associated with light passing by the edge of an object. Again we see bright and dark fringes, which is reminiscent of an interference pattern.

Figure 38.3 shows a diffraction pattern associated with the shadow of a penny. A bright spot occurs at the center, and circular fringes extend outward from the shadow's edge. We can explain the central bright spot by using the wave theory of light, which predicts constructive interference at this point. From the viewpoint of ray optics (in which light is viewed as rays traveling in straight lines), we expect the center of the shadow to be dark because that part of the viewing screen is completely shielded by the penny.



P. M. Rinard, *Am. J. Phys.* 44: 70 1976

Figure 38.3 Diffraction pattern created by the illumination of a penny, with the penny positioned midway between the screen and light source.

Shortly before the central bright spot was first observed, one of the supporters of ray optics, Simeon Poisson, argued that if Augustin Fresnel's wave theory of light were valid, a central bright spot should be observed in the shadow of a circular object illuminated by a point source of light. To Poisson's astonishment, the spot was observed by Dominique Arago shortly thereafter. Therefore, Poisson's prediction reinforced the wave theory rather than disproving it.

38.2 Diffraction Patterns from Narrow Slits

Let's consider a common situation, that of light passing through a narrow opening modeled as a slit and projected onto a screen. To simplify our analysis, we assume the observing screen is far from the slit and the rays reaching the screen are approximately parallel. (This situation can also be achieved experimentally by using a converging lens to focus the parallel rays on a nearby screen.) In this model, the pattern on the screen is called a **Fraunhofer diffraction pattern.**[1]

Figure 38.4a (page 1162) shows light entering a single slit from the left and diffracting as it propagates toward a screen. Figure 38.4b shows the fringe structure of

[1]If the screen is brought close to the slit (and no lens is used), the pattern is a *Fresnel* diffraction pattern. The Fresnel pattern is more difficult to analyze, so we shall restrict our discussion to Fraunhofer diffraction.

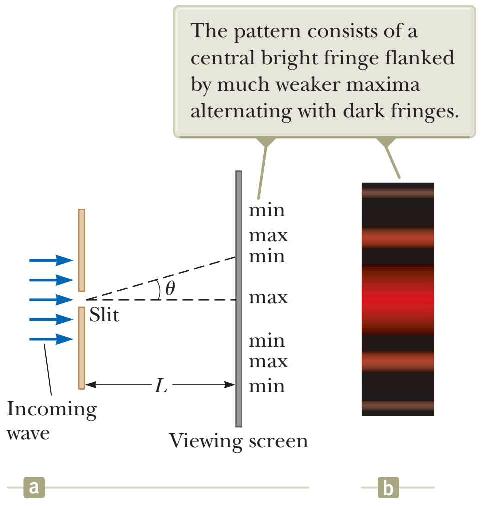


Figure 38.4 (a) Geometry for analyzing the Fraunhofer diffraction pattern of a single slit. (Drawing not to scale.) (b) Simulation of a single-slit Fraunhofer diffraction pattern.

Pitfall Prevention 38.1

Diffraction Versus Diffraction Pattern *Diffraction* refers to the general behavior of waves spreading out as they pass through a slit. We used diffraction in explaining the existence of an interference pattern in Chapter 37. A *diffraction pattern* is actually a misnomer, but is deeply entrenched in the language of physics. The diffraction pattern seen on a screen when a single slit is illuminated is actually another interference pattern. The interference is between parts of the incident light illuminating different regions of the slit.

a Fraunhofer diffraction pattern. A bright fringe is observed along the axis at $\theta = 0$, with alternating dark and bright fringes on each side of the central bright fringe.

Until now, we have assumed slits are point sources of light. In this section, we abandon that assumption and see how the finite width of slits is the basis for understanding Fraunhofer diffraction. We can explain some important features of this phenomenon by examining waves coming from various portions of the slit as shown in Figure 38.5. According to Huygens's principle, each portion of the slit acts as a source of light waves. Hence, light from one portion of the slit can interfere with light from another portion, and the resultant light intensity on a viewing screen depends on the direction θ. Based on this analysis, we recognize that a diffraction pattern is actually an interference pattern in which the different sources of light are different portions of the single slit! Therefore, the diffraction patterns we discuss in this chapter are applications of the waves in interference analysis model.

To analyze the diffraction pattern, let's divide the slit into two halves as shown in Figure 38.5. Keeping in mind that all the waves are in phase as they leave the slit, consider rays 1 and 3. As these two rays travel toward a viewing screen far to the right of the figure, ray 1 travels farther than ray 3 by an amount equal to the path difference $(a/2)\sin\theta$, where a is the width of the slit. Similarly, the path difference between rays 2 and 4 is also $(a/2)\sin\theta$, as is that between rays 3 and 5. If this path difference is exactly half a wavelength (corresponding to a phase difference of 180°), the pairs of waves cancel each other and destructive interference results. This cancellation occurs for any two rays that originate at points separated by half the slit width because the phase difference between two such points is 180°. Therefore, waves from the upper half of the slit interfere destructively with waves from the lower half when

$$\frac{a}{2}\sin\theta = \frac{\lambda}{2}$$

or, if we consider waves at angle θ both above the dashed line in Figure 38.5 and below,

$$\sin\theta = \pm\frac{\lambda}{a}$$

Dividing the slit into four equal parts and using similar reasoning, we find that the viewing screen is also dark when

$$\sin\theta = \pm 2\frac{\lambda}{a}$$

Likewise, dividing the slit into six equal parts shows that darkness occurs on the screen when

$$\sin\theta = \pm 3\frac{\lambda}{a}$$

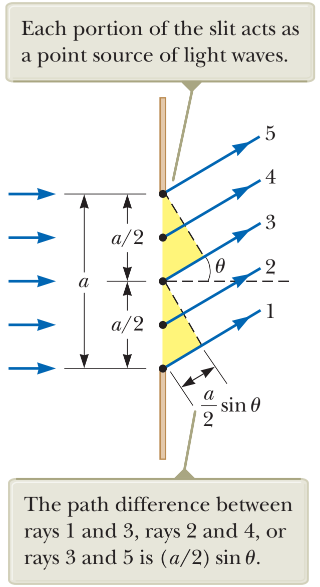


Figure 38.5 Paths of light rays that encounter a narrow slit of width a and diffract toward a screen in the direction described by angle θ (not to scale).

Therefore, the general condition for destructive interference is

$$\sin\theta_{\text{dark}} = m\frac{\lambda}{a} \qquad m = \pm1, \pm2, \pm3, \ldots \tag{38.1}$$

Condition for destructive interference for a single slit

This equation gives the values of θ_{dark} for which the diffraction pattern has zero light intensity, that is, when a dark fringe is formed. It tells us nothing, however, about the variation in light intensity along the screen. The general features of the intensity distribution are shown in Figure 38.4. A broad, central bright fringe is observed; this fringe is flanked by much weaker bright fringes alternating with dark fringes. The various dark fringes occur at the values of θ_{dark} that satisfy Equation 38.1. Each bright-fringe peak lies approximately halfway between its bordering dark-fringe minima. Notice that the central bright maximum is twice as wide as the secondary maxima. There is no central dark fringe, represented by the absence of $m = 0$ in Equation 38.1.

Pitfall Prevention 38.2

Similar Equation Warning! Equation 38.1 has exactly the same form as Equation 37.2, with d, the slit separation, used in Equation 37.2 and a, the slit width, used in Equation 38.1. Equation 37.2, however, describes the *bright* regions in a two-slit interference pattern, whereas Equation 38.1 describes the *dark* regions in a single-slit diffraction pattern.

Quick Quiz 38.1 Suppose the slit width in Figure 38.4 is made half as wide. Does the central bright fringe **(a)** become wider, **(b)** remain the same, or **(c)** become narrower?

Example 38.1 Where Are the Dark Fringes? AM

Light of wavelength 580 nm is incident on a slit having a width of 0.300 mm. The viewing screen is 2.00 m from the slit. Find the width of the central bright fringe.

SOLUTION

Conceptualize Based on the problem statement, we imagine a single-slit diffraction pattern similar to that in Figure 38.4.

Categorize We categorize this example as a straightforward application of our discussion of single-slit diffraction patterns, which comes from the *waves in interference* analysis model.

Analyze Evaluate Equation 38.1 for the two dark fringes that flank the central bright fringe, which correspond to $m = \pm1$:

$$\sin\theta_{\text{dark}} = \pm\frac{\lambda}{a}$$

Let y represent the vertical position along the viewing screen in Figure 38.4a, measured from the point on the screen directly behind the slit. Then, $\tan\theta_{\text{dark}} = y_1/L$, where the subscript 1 refers to the first dark fringe. Because θ_{dark} is very small, we can use the approximation $\sin\theta_{\text{dark}} \approx \tan\theta_{\text{dark}}$; therefore, $y_1 = L\sin\theta_{\text{dark}}$.

The width of the central bright fringe is twice the absolute value of y_1:

$$2|y_1| = 2|L\sin\theta_{\text{dark}}| = 2\left|\pm L\frac{\lambda}{a}\right| = 2L\frac{\lambda}{a} = 2(2.00\text{ m})\frac{580\times10^{-9}\text{ m}}{0.300\times10^{-3}\text{ m}}$$

$$= 7.73\times10^{-3}\text{ m} = \boxed{7.73\text{ mm}}$$

Finalize Notice that this value is much greater than the width of the slit. Let's explore below what happens if we change the slit width.

WHAT IF? What if the slit width is increased by an order of magnitude to 3.00 mm? What happens to the diffraction pattern?

Answer Based on Equation 38.1, we expect that the angles at which the dark bands appear will decrease as a increases. Therefore, the diffraction pattern narrows.

Repeat the calculation with the larger slit width:

$$2|y_1| = 2L\frac{\lambda}{a} = 2(2.00\text{ m})\frac{580\times10^{-9}\text{ m}}{3.00\times10^{-3}\text{ m}} = 7.73\times10^{-4}\text{ m} = \boxed{0.773\text{ mm}}$$

Notice that this result is *smaller* than the width of the slit. In general, for large values of a, the various maxima and minima are so closely spaced that only a large, central bright area resembling the geometric image of the slit is observed. This concept is very important in the performance of optical instruments such as telescopes.

Intensity of Single-Slit Diffraction Patterns

Analysis of the intensity variation in a diffraction pattern from a single slit of width a shows that the intensity is given by

Intensity of a single-slit Fraunhofer diffraction pattern

$$I = I_{\max}\left[\frac{\sin(\pi a \sin\theta/\lambda)}{\pi a \sin\theta/\lambda}\right]^2 \tag{38.2}$$

where $I_{\max}$ is the intensity at $\theta = 0$ (the central maximum) and λ is the wavelength of light used to illuminate the slit. This expression shows that *minima* occur when

$$\frac{\pi a \sin\theta_{\text{dark}}}{\lambda} = m\pi$$

or

Condition for intensity minima for a single slit

$$\sin\theta_{\text{dark}} = m\frac{\lambda}{a} \qquad m = \pm1, \pm2, \pm3, \ldots$$

in agreement with Equation 38.1.

Figure 38.6a represents a plot of the intensity in the single-slit pattern as given by Equation 38.2, and Figure 38.6b is a simulation of a single-slit Fraunhofer diffraction pattern. Notice that most of the light intensity is concentrated in the central bright fringe.

Intensity of Two-Slit Diffraction Patterns

When more than one slit is present, we must consider not only diffraction patterns due to the individual slits but also the interference patterns due to the waves coming from different slits. Notice the curved dashed lines in Figure 37.7 in Chapter 37, which indicate a decrease in intensity of the interference maxima as θ increases. This decrease is due to a diffraction pattern. The interference patterns in that figure are located entirely within the central bright fringe of the diffraction pattern, so the only hint of the diffraction pattern we see is the falloff in intensity toward the outside of the pattern. To determine the effects of both two-slit interference and a single-slit diffraction pattern from each slit from a wider viewpoint than that in Figure 37.7, we combine Equations 37.14 and 38.2:

$$I = I_{\max}\cos^2\left(\frac{\pi d\sin\theta}{\lambda}\right)\left[\frac{\sin(\pi a\sin\theta/\lambda)}{\pi a\sin\theta/\lambda}\right]^2 \tag{38.3}$$

Although this expression looks complicated, it merely represents the single-slit diffraction pattern (the factor in square brackets) acting as an "envelope" for a two-slit interference pattern (the cosine-squared factor) as shown in Figure 38.7. The broken

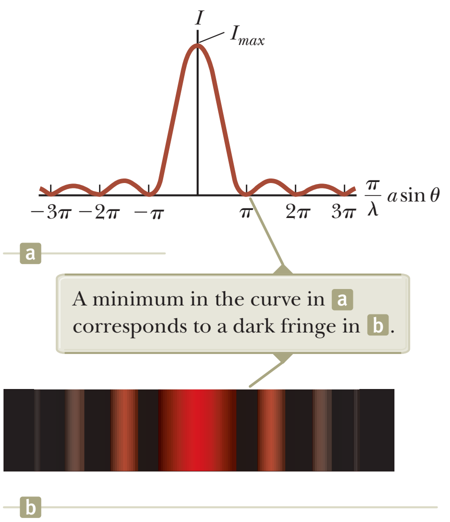


Figure 38.6 (a) A plot of light intensity I versus $(\pi/\lambda)a\sin\theta$ for the single-slit Fraunhofer diffraction pattern. (b) Simulation of a single-slit Fraunhofer diffraction pattern.

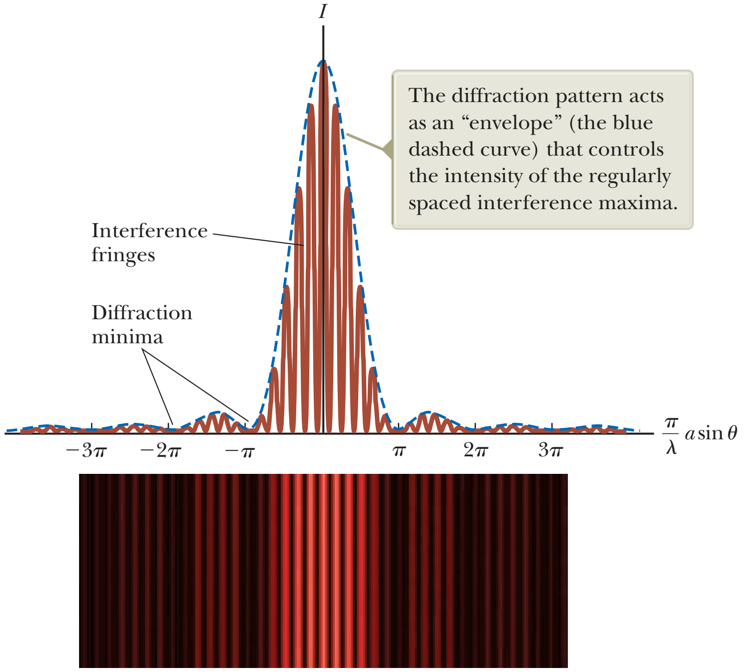


Figure 38.7 The combined effects of two-slit and single-slit interference. This pattern is produced when 650-nm light waves pass through two 3.0-μm slits that are 18 μm apart.

blue curve in Figure 38.7 represents the factor in square brackets in Equation 38.3. The cosine-squared factor by itself would give a series of peaks all with the same height as the highest peak of the red-brown curve in Figure 38.7. Because of the effect of the square-bracket factor, however, these peaks vary in height as shown.

Equation 37.2 indicates the conditions for interference maxima as $d \sin\theta = m\lambda$, where d is the distance between the two slits. Equation 38.1 specifies that the first diffraction minimum occurs when $a \sin\theta = \lambda$, where a is the slit width. Dividing Equation 37.2 by Equation 38.1 (with $m = 1$) allows us to determine which interference maximum coincides with the first diffraction minimum:

$$\frac{d \sin\theta}{a \sin\theta} = \frac{m\lambda}{\lambda}$$

$$\frac{d}{a} = m \tag{38.4}$$

In Figure 38.7, $d/a = 18\ \mu\text{m}/3.0\ \mu\text{m} = 6$. Therefore, the sixth interference maximum (if we count the central maximum as $m = 0$) is aligned with the first diffraction minimum and is dark.

Quick Quiz 38.2 Consider the central peak in the diffraction envelope in Figure 38.7 and look closely at the horizontal scale. Suppose the wavelength of the light is changed to 450 nm. What happens to this central peak? **(a)** The width of the peak decreases, and the number of interference fringes it encloses decreases. **(b)** The width of the peak decreases, and the number of interference fringes it encloses increases. **(c)** The width of the peak decreases, and the number of interference fringes it encloses remains the same. **(d)** The width of the peak increases, and the number of interference fringes it encloses decreases. **(e)** The width of the peak increases, and the number of interference fringes it encloses increases. **(f)** The width of the peak increases, and the number of interference fringes it encloses remains the same. **(g)** The width of the peak remains the same, and the number of interference fringes it encloses decreases. **(h)** The width of the peak remains the same, and the number of interference fringes it encloses increases. **(i)** The width of the peak remains the same, and the number of interference fringes it encloses remains the same.

38.3 Resolution of Single-Slit and Circular Apertures

The ability of optical systems to distinguish between closely spaced objects is limited because of the wave nature of light. To understand this limitation, consider Figure 38.8, which shows two light sources far from a narrow slit of width a. The sources can be two noncoherent point sources S_1 and S_2; for example, they could be two distant stars. If no interference occurred between light passing through different parts of the slit, two distinct bright spots (or images) would be observed on the viewing screen. Because of such interference, however, each source is imaged as a bright central region flanked by weaker bright and dark fringes, a diffraction pattern. What is observed on the screen is the sum of two diffraction patterns: one from S_1 and the other from S_2.

If the two sources are far enough apart to keep their central maxima from overlapping as in Figure 38.8a, their images can be distinguished and are said to be *resolved*. If the sources are close together as in Figure 38.8b, however, the two central maxima overlap and the images are not resolved. To determine whether two images are resolved, the following condition is often used:

> When the central maximum of one image falls on the first minimum of another image, the images are said to be just resolved. This limiting condition of resolution is known as **Rayleigh's criterion.**

From Rayleigh's criterion, we can determine the minimum angular separation θ_{min} subtended by the sources at the slit in Figure 38.8 for which the images are just resolved. Equation 38.1 indicates that the first minimum in a single-slit diffraction pattern occurs at the angle for which

$$\sin\theta = \frac{\lambda}{a}$$

where a is the width of the slit. According to Rayleigh's criterion, this expression gives the smallest angular separation for which the two images are resolved. Because $\lambda << a$ in most situations, $\sin\theta$ is small and we can use the approximation $\sin\theta \approx \theta$. Therefore, the limiting angle of resolution for a slit of width a is

$$\theta_{min} = \frac{\lambda}{a} \tag{38.5}$$

where θ_{min} is expressed in radians. Hence, the angle subtended by the two sources at the slit must be greater than λ/a if the images are to be resolved.

Many optical systems use circular apertures rather than slits. The diffraction pattern of a circular aperture as shown in the photographs of Figure 38.9 consists of

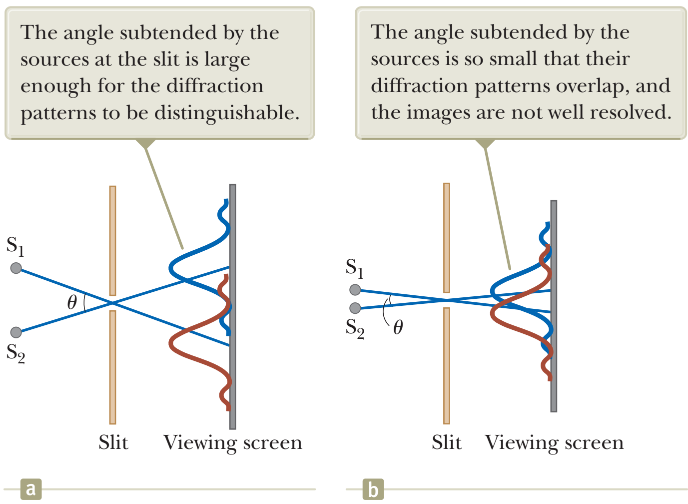


Figure 38.8 Two point sources far from a narrow slit each produce a diffraction pattern. (a) The sources are separated by a large angle. (b) The sources are separated by a small angle. (Notice that the angles are greatly exaggerated. The drawing is not to scale.)

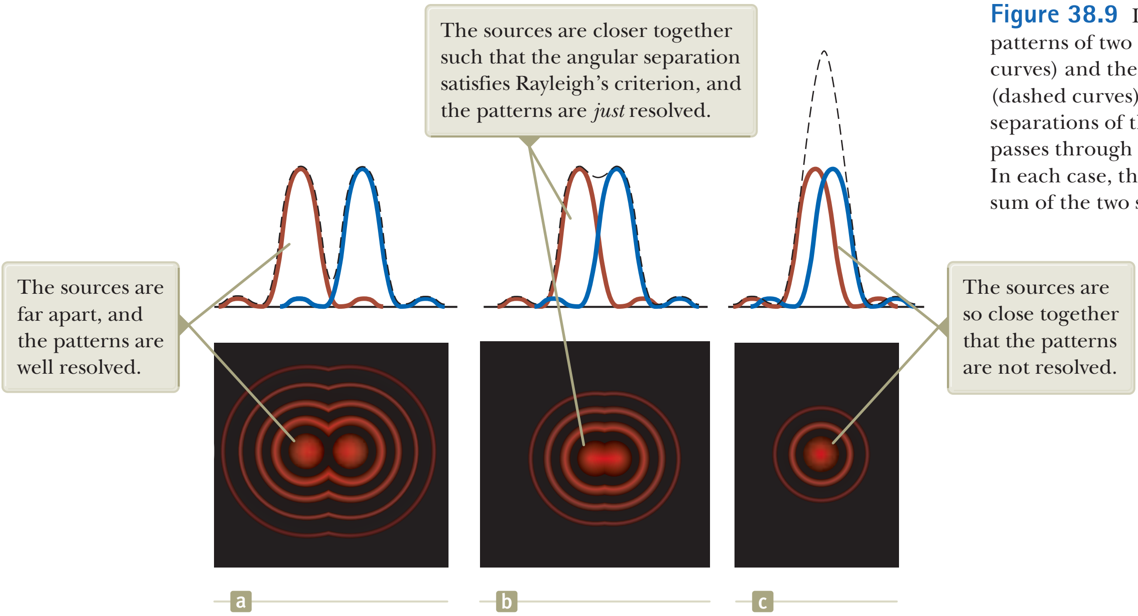


Figure 38.9 Individual diffraction patterns of two point sources (solid curves) and the resultant patterns (dashed curves) for various angular separations of the sources as the light passes through a circular aperture. In each case, the dashed curve is the sum of the two solid curves.

a central circular bright disk surrounded by progressively fainter bright and dark rings. Figure 38.9 shows diffraction patterns for three situations in which light from two point sources passes through a circular aperture. When the sources are far apart, their images are well resolved (Fig. 38.9a). When the angular separation of the sources satisfies Rayleigh's criterion, the images are just resolved (Fig. 38.9b). Finally, when the sources are close together, the images are said to be unresolved (Fig. 38.9c) and the pattern looks like that of a single source.

Analysis shows that the limiting angle of resolution of the circular aperture is

$$\theta_{\min} = 1.22\frac{\lambda}{D} \tag{38.6}$$

◀ **Limiting angle of resolution for a circular aperture**

where D is the diameter of the aperture. This expression is similar to Equation 38.5 except for the factor 1.22, which arises from a mathematical analysis of diffraction from the circular aperture.

Quick Quiz 38.3 Cat's eyes have pupils that can be modeled as vertical slits. At night, would cats be more successful in resolving **(a)** headlights on a distant car or **(b)** vertically separated lights on the mast of a distant boat?

Quick Quiz 38.4 Suppose you are observing a binary star with a telescope and are having difficulty resolving the two stars. You decide to use a colored filter to maximize the resolution. (A filter of a given color transmits only that color of light.) What color filter should you choose? **(a)** blue **(b)** green **(c)** yellow **(d)** red

Example 38.2 Resolution of the Eye

Light of wavelength 500 nm, near the center of the visible spectrum, enters a human eye. Although pupil diameter varies from person to person, let's estimate a daytime diameter of 2 mm.

(A) Estimate the limiting angle of resolution for this eye, assuming its resolution is limited only by diffraction.

SOLUTION

Conceptualize Identify the pupil of the eye as the aperture through which the light travels. Light passing through this small aperture causes diffraction patterns to occur on the retina.

Categorize We determine the result using equations developed in this section, so we categorize this example as a substitution problem.

continued

38.2 continued

Use Equation 38.6, taking λ = 500 nm and D = 2 mm:

$$\theta_{\min} = 1.22\frac{\lambda}{D} = 1.22\left(\frac{5.00 \times 10^{-7}\text{ m}}{2 \times 10^{-3}\text{ m}}\right)$$

$$= \boxed{3 \times 10^{-4}\text{ rad}} \approx \boxed{1\text{ min of arc}}$$

(B) Determine the minimum separation distance d between two point sources that the eye can distinguish if the point sources are a distance L = 25 cm from the observer (Fig. 38.10).

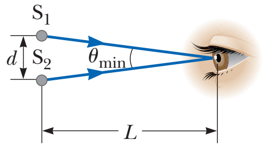


Figure 38.10 (Example 38.2) Two point sources separated by a distance d as observed by the eye.

SOLUTION

Noting that $\theta_{\min}$ is small, find d:

$$\sin\theta_{\min} \approx \theta_{\min} \approx \frac{d}{L} \rightarrow d = L\theta_{\min}$$

Substitute numerical values:

$$d = (25\text{ cm})(3 \times 10^{-4}\text{ rad}) = \boxed{8 \times 10^{-3}\text{ cm}}$$

This result is approximately equal to the thickness of a human hair.

Example 38.3 Resolution of a Telescope

Each of the two telescopes at the Keck Observatory on the dormant Mauna Kea volcano in Hawaii has an effective diameter of 10 m. What is its limiting angle of resolution for 600-nm light?

SOLUTION

Conceptualize Identify the aperture through which the light travels as the opening of the telescope. Light passing through this aperture causes diffraction patterns to occur in the final image.

Categorize We determine the result using equations developed in this section, so we categorize this example as a substitution problem.

Use Equation 38.6, taking $\lambda = 6.00 \times 10^{-7}$ m and D = 10 m:

$$\theta_{\min} = 1.22\frac{\lambda}{D} = 1.22\left(\frac{6.00 \times 10^{-7}\text{ m}}{10\text{ m}}\right)$$

$$= \boxed{7.3 \times 10^{-8}\text{ rad}} \approx \boxed{0.015\text{ s of arc}}$$

Any two stars that subtend an angle greater than or equal to this value are resolved (if atmospheric conditions are ideal).

WHAT IF? What if we consider radio telescopes? They are much larger in diameter than optical telescopes, but do they have better angular resolutions than optical telescopes? For example, the radio telescope at Arecibo, Puerto Rico, has a diameter of 305 m and is designed to detect radio waves of 0.75-m wavelength. How does its resolution compare with that of one of the Keck telescopes?

Answer The increase in diameter might suggest that radio telescopes would have better resolution than a Keck telescope, but Equation 38.6 shows that $\theta_{\min}$ depends on *both* diameter and wavelength. Calculating the minimum angle of resolution for the radio telescope, we find

$$\theta_{\min} = 1.22\frac{\lambda}{D} = 1.22\left(\frac{0.75\text{ m}}{305\text{ m}}\right)$$

$$= 3.0 \times 10^{-3}\text{ rad} \approx 10\text{ min of arc}$$

This limiting angle of resolution is measured in *minutes* of arc rather than the *seconds* of arc for the optical telescope. Therefore, the change in wavelength more than compensates for the increase in diameter. The limiting angle of resolution for the Arecibo radio telescope is more than 40 000 times larger (that is, *worse*) than the Keck minimum.

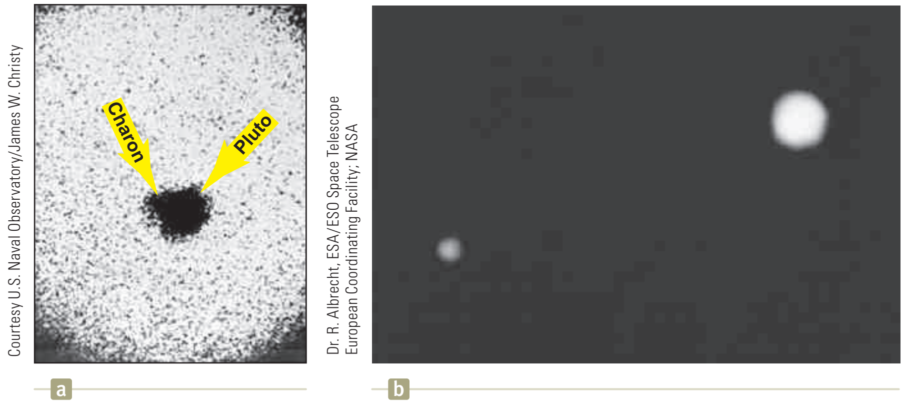


Figure 38.11 (a) The photograph on which Charon, the moon of Pluto, was discovered in 1978. From an Earth-based telescope, atmospheric blurring results in Charon appearing only as a subtle bump on the edge of Pluto. (b) A Hubble Space Telescope photo of Pluto and Charon, clearly resolving the two objects.

A telescope such as the one discussed in Example 38.3 can never reach its diffraction limit because the limiting angle of resolution is always set by atmospheric blurring at optical wavelengths. This seeing limit is usually about 1 s of arc and is never smaller than about 0.1 s of arc. The atmospheric blurring is caused by variations in index of refraction with temperature variations in the air. This blurring is one reason for the superiority of photographs from orbiting telescopes, which view celestial objects from a position above the atmosphere.

As an example of the effects of atmospheric blurring, consider telescopic images of Pluto and its moon, Charon. Figure 38.11a, an image taken in 1978, represents the discovery of Charon. In this photograph, taken from an Earth-based telescope, atmospheric turbulence causes the image of Charon to appear only as a bump on the edge of Pluto. In comparison, Figure 38.11b shows a photograph taken from the Hubble Space Telescope. Without the problems of atmospheric turbulence, Pluto and its moon are clearly resolved.

38.4 The Diffraction Grating

The **diffraction grating,** a useful device for analyzing light sources, consists of a large number of equally spaced parallel slits. A *transmission grating* can be made by cutting parallel grooves on a glass plate with a precision ruling machine. The spaces between the grooves are transparent to the light and hence act as separate slits. A *reflection grating* can be made by cutting parallel grooves on the surface of a reflective material. The reflection of light from the spaces between the grooves is specular, and the reflection from the grooves cut into the material is diffuse. Therefore, the spaces between the grooves act as parallel sources of reflected light like the slits in a transmission grating. Current technology can produce gratings that have very small slit spacings. For example, a typical grating ruled with 5 000 grooves/cm has a slit spacing $d = (1/5\,000)$ cm $= 2.00 \times 10^{-4}$ cm.

Pitfall Prevention 38.3

A Diffraction Grating Is an Interference Grating As with *diffraction pattern, diffraction grating* is a misnomer, but is deeply entrenched in the language of physics. The diffraction grating depends on diffraction in the same way as the double slit, spreading the light so that light from different slits can interfere. It would be more correct to call it an *interference grating,* but *diffraction grating* is the name in use.

A section of a diffraction grating is illustrated in Figure 38.12 (page 1170). A plane wave is incident from the left, normal to the plane of the grating. The pattern observed on the screen far to the right of the grating is the result of the combined effects of interference and diffraction. Each slit produces diffraction, and the diffracted beams interfere with one another to produce the final pattern.

The waves from all slits are in phase as they leave the slits. For an arbitrary direction θ measured from the horizontal, however, the waves must travel different path lengths before reaching the screen. Notice in Figure 38.12 that the path difference δ between rays from any two adjacent slits is equal to $d \sin \theta$. If this path difference equals one wavelength or some integral multiple of a wavelength, waves from all slits are in phase at the screen and a bright fringe is observed. Therefore, the condition for *maxima* in the interference pattern at the angle θ_{bright} is

$$d \sin \theta_{\text{bright}} = m\lambda \quad m = 0, \pm 1, \pm 2, \pm 3, \ldots \tag{38.7}$$

◀ **Condition for interference maxima for a grating**

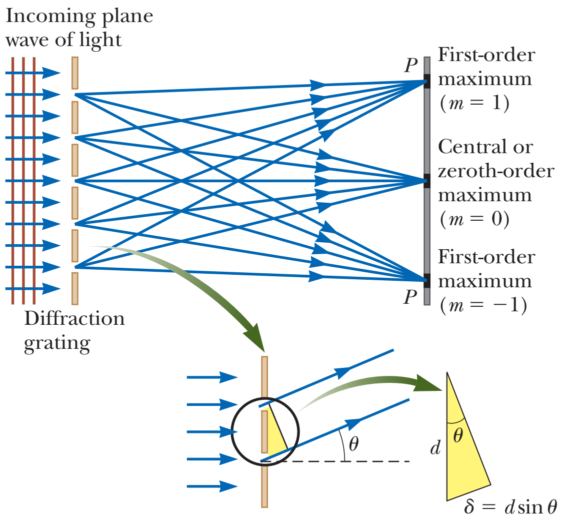


Figure 38.12 Side view of a diffraction grating. The slit separation is d, and the path difference between adjacent slits is $d \sin \theta$.

We can use this expression to calculate the wavelength if we know the grating spacing d and the angle θ_{bright}. If the incident radiation contains several wavelengths, the mth-order maximum for each wavelength occurs at a specific angle. All wavelengths are seen at $\theta = 0$, corresponding to $m = 0$, the zeroth-order maximum. The first-order maximum ($m = 1$) is observed at an angle that satisfies the relationship $\sin \theta_{\text{bright}} = \lambda/d$, the second-order maximum ($m = 2$) is observed at a larger angle θ_{bright}, and so on. For the small values of d typical in a diffraction grating, the angles θ_{bright} are large, as we see in Example 38.5.

The intensity distribution for a diffraction grating obtained with the use of a monochromatic source is shown in Figure 38.13. Notice the sharpness of the principal maxima and the broadness of the dark areas compared with the broad bright fringes characteristic of the two-slit interference pattern (see Fig. 37.6). You should also review Figure 37.7, which shows that the width of the intensity maxima decreases as the number of slits increases. Because the principal maxima are so sharp, they are much brighter than two-slit interference maxima.

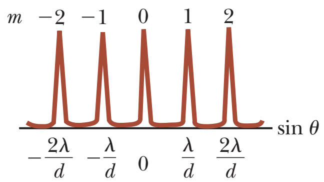


Figure 38.13 Intensity versus $\sin \theta$ for a diffraction grating. The zeroth-, first-, and second-order maxima are shown.

Quick Quiz 38.5 Ultraviolet light of wavelength 350 nm is incident on a diffraction grating with slit spacing d and forms an interference pattern on a screen a distance L away. The angular positions θ_{bright} of the interference maxima are large. The locations of the bright fringes are marked on the screen. Now red light of wavelength 700 nm is used with a diffraction grating to form another diffraction pattern on the screen. Will the bright fringes of this pattern be located at the marks on the screen if **(a)** the screen is moved to a distance $2L$ from the grating, **(b)** the screen is moved to a distance $L/2$ from the grating, **(c)** the grating is replaced with one of slit spacing $2d$, **(d)** the grating is replaced with one of slit spacing $d/2$, or **(e)** nothing is changed?

Conceptual Example 38.4 A Compact Disc Is a Diffraction Grating

Light reflected from the surface of a compact disc is multicolored as shown in Figure 38.14. The colors and their intensities depend on the orientation of the CD relative to the eye and relative to the light source. Explain how that works.

SOLUTION

The surface of a CD has a spiral grooved track (with adjacent grooves having a separation on

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Figure 38.14 (Conceptual Example 38.4) A compact disc observed under white light. The colors observed in the reflected light and their intensities depend on the orientation of the CD relative to the eye and relative to the light source.

38.4 continued

the order of 1 μm). Therefore, the surface acts as a reflection grating. The light reflecting from the regions between these closely spaced grooves interferes constructively only in certain directions that depend on the wavelength and the direction of the incident light. Any section of the CD serves as a diffraction grating for white light, sending different colors in different directions. The different colors you see upon viewing one section change when the light source, the CD, or you change position. This change in position causes the angle of incidence or the angle of the diffracted light to be altered.

Example 38.5 The Orders of a Diffraction Grating

Monochromatic light from a helium–neon laser ($\lambda = 632.8$ nm) is incident normally on a diffraction grating containing 6 000 grooves per centimeter. Find the angles at which the first- and second-order maxima are observed.

SOLUTION

Conceptualize Study Figure 38.12 and imagine that the light coming from the left originates from the helium–neon laser. Let's evaluate the possible values of the angle θ for constructive interference.

Categorize We determine results using equations developed in this section, so we categorize this example as a substitution problem.

Calculate the slit separation as the inverse of the number of grooves per centimeter:

$$d = \frac{1}{6\,000}\text{ cm} = 1.667 \times 10^{-4}\text{ cm} = 1\,667\text{ nm}$$

Solve Equation 38.7 for $\sin\theta$ and substitute numerical values for the first-order maximum ($m = 1$) to find θ_1:

$$\sin\theta_1 = \frac{(1)\lambda}{d} = \frac{632.8\text{ nm}}{1\,667\text{ nm}} = 0.379\,7$$

$$\theta_1 = \boxed{22.31°}$$

Repeat for the second-order maximum ($m = 2$):

$$\sin\theta_2 = \frac{(2)\lambda}{d} = \frac{2(632.8\text{ nm})}{1\,667\text{ nm}} = 0.759\,4$$

$$\theta_2 = \boxed{49.41°}$$

WHAT IF? What if you looked for the third-order maximum? Would you find it?

Answer For $m = 3$, we find $\sin\theta_3 = 1.139$. Because $\sin\theta$ cannot exceed unity, this result does not represent a realistic solution. Hence, only zeroth-, first-, and second-order maxima can be observed for this situation.

Applications of Diffraction Gratings

A schematic drawing of a simple apparatus used to measure angles in a diffraction pattern is shown in Figure 38.15 (page 1172). This apparatus is a *diffraction grating spectrometer.* The light to be analyzed passes through a slit, and a collimated beam of light is incident on the grating. The diffracted light leaves the grating at angles that satisfy Equation 38.7, and a telescope is used to view the image of the slit. The wavelength can be determined by measuring the precise angles at which the images of the slit appear for the various orders.

The spectrometer is a useful tool in *atomic spectroscopy,* in which the light from an atom is analyzed to find the wavelength components. These wavelength components can be used to identify the atom. We shall investigate atomic spectra in Chapter 42 of the extended version of this text.

Another application of diffraction gratings is the *grating light valve* (GLV), which competes in some video display applications with the digital micromirror devices (DMDs) discussed in Section 35.4. A GLV is a silicon microchip fitted with an array

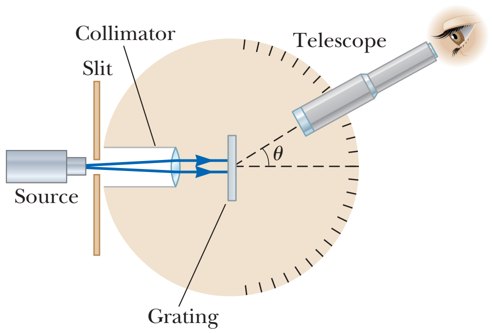


Figure 38.15 Diagram of a diffraction grating spectrometer. The collimated beam incident on the grating is spread into its various wavelength components with constructive interference for a particular wavelength occurring at the angles θ_{bright} that satisfy the equation $d \sin \theta_{\text{bright}} = m\lambda$, where $m = 0, \pm 1, \pm 2, \ldots$.

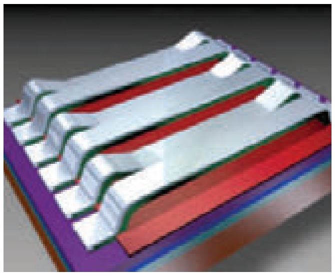


Figure 38.16 A small portion of a grating light valve. The alternating reflective ribbons at different levels act as a diffraction grating, offering very high-speed control of the direction of light toward a digital display device.

of parallel silicon nitride ribbons coated with a thin layer of aluminum (Fig. 38.16). Each ribbon is approximately 20 μm long and 5 μm wide and is separated from the silicon substrate by an air gap on the order of 100 nm. With no voltage applied, all ribbons are at the same level. In this situation, the array of ribbons acts as a flat surface, specularly reflecting incident light.

When a voltage is applied between a ribbon and the electrode on the silicon substrate, an electric force pulls the ribbon downward, closer to the substrate. Alternate ribbons can be pulled down, while those in between remain in an elevated configuration. As a result, the array of ribbons acts as a diffraction grating such that the constructive interference for a particular wavelength of light can be directed toward a screen or other optical display system. If one uses three such devices—one each for red, blue, and green light—full-color display is possible.

In addition to its use in video display, the GLV has found applications in laser optical navigation sensor technology, computer-to-plate commercial printing, and other types of imaging.

Another interesting application of diffraction gratings is **holography,** the production of three-dimensional images of objects. The physics of holography was developed by Dennis Gabor (1900–1979) in 1948 and resulted in the Nobel Prize in Physics for Gabor in 1971. The requirement of coherent light for holography delayed the realization of holographic images from Gabor's work until the development of lasers in the 1960s. Figure 38.17 shows a single hologram viewed from two different positions and the three-dimensional character of its image. Notice in particular the difference in the view through the magnifying glass in Figures 38.17a and 38.17b.

Figure 38.18 shows how a hologram is made. Light from the laser is split into two parts by a half-silvered mirror at B. One part of the beam reflects off the object to be photographed and strikes an ordinary photographic film. The other half of the beam is diverged by lens L_2, reflects from mirrors M_1 and M_2, and finally

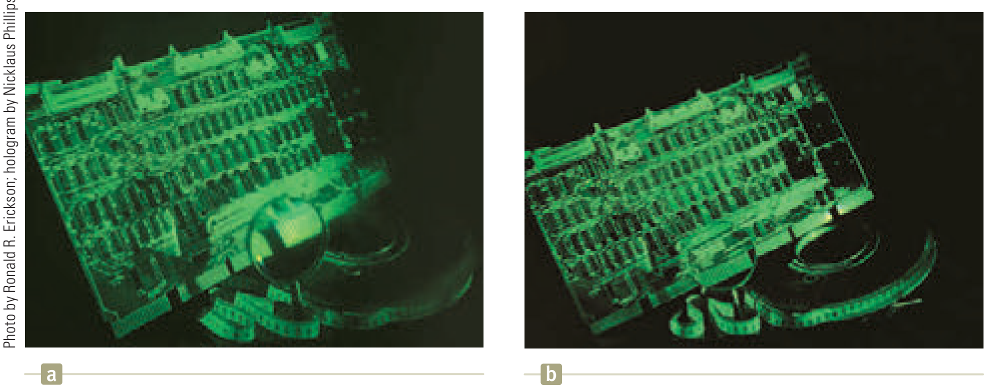


Figure 38.17 In this hologram, a circuit board is shown from two different views. Notice the difference in the appearance of the measuring tape and the view through the magnifying lens in (a) and (b).

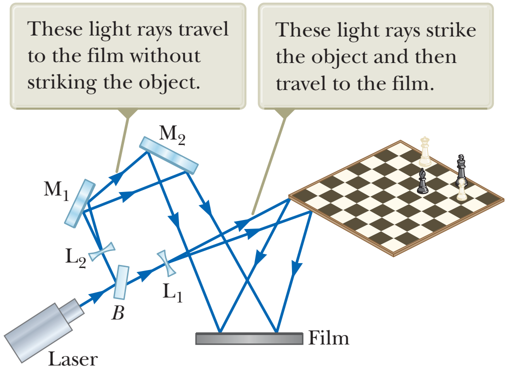


Figure 38.18 Experimental arrangement for producing a hologram.

strikes the film. The two beams overlap to form an extremely complicated interference pattern on the film. Such an interference pattern can be produced only if the phase relationship of the two waves is constant throughout the exposure of the film. This condition is met by illuminating the scene with light coming through a pinhole or with coherent laser radiation. The hologram records not only the intensity of the light scattered from the object (as in a conventional photograph), but also the phase difference between the reference beam and the beam scattered from the object. Because of this phase difference, an interference pattern is formed that produces an image in which all three-dimensional information available from the perspective of any point on the hologram is preserved.

In a normal photographic image, a lens is used to focus the image so that each point on the object corresponds to a single point on the photograph. Notice that there is no lens used in Figure 38.18 to focus the light onto the film. Therefore, light from each point on the object reaches *all* points on the film. As a result, each region of the photographic film on which the hologram is recorded contains information about all illuminated points on the object, which leads to a remarkable result: if a small section of the hologram is cut from the film, the complete image can be formed from the small piece! (The quality of the image is reduced, but the entire image is present.)

A hologram is best viewed by allowing coherent light to pass through the developed film as one looks back along the direction from which the beam comes. The interference pattern on the film acts as a diffraction grating. Figure 38.19 shows two rays of light striking and passing through the film. For each ray, the $m = 0$ and $m = \pm 1$ rays in the diffraction pattern are shown emerging from the right side of the film. The $m = +1$ rays converge to form a real image of the scene, which is not the image that is normally viewed. By extending the light rays corresponding to $m = -1$ behind the film, we see that there is a virtual image located there, with light coming from it in exactly the same way that light came from the actual object

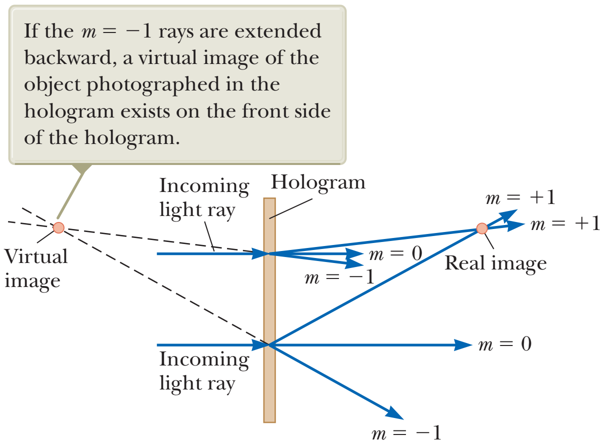


Figure 38.19 Two light rays strike a hologram at normal incidence. For each ray, outgoing rays corresponding to $m = 0$ and $m = \pm 1$ are shown.

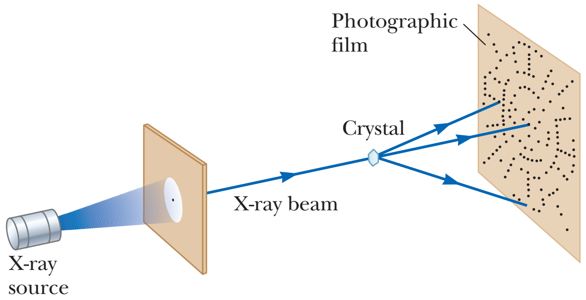


Figure 38.20 Schematic diagram of the technique used to observe the diffraction of x-rays by a crystal. The array of spots formed on the film is called a Laue pattern.

when the film was exposed. This image is what one sees when looking through the holographic film.

Holograms are finding a number of applications. You may have a hologram on your credit card. This special type of hologram is called a *rainbow hologram* and is designed to be viewed in reflected white light.

38.5 Diffraction of X-Rays by Crystals

In principle, the wavelength of any electromagnetic wave can be determined if a grating of the proper spacing (on the order of λ) is available. X-rays, discovered by Wilhelm Roentgen (1845–1923) in 1895, are electromagnetic waves of very short wavelength (on the order of 0.1 nm). It would be impossible to construct a grating having such a small spacing by the cutting process described at the beginning of Section 38.4. The atomic spacing in a solid is known to be about 0.1 nm, however. In 1913, Max von Laue (1879–1960) suggested that the regular array of atoms in a crystal could act as a three-dimensional diffraction grating for x-rays. Subsequent experiments confirmed this prediction. The diffraction patterns from crystals are complex because of the three-dimensional nature of the crystal structure. Nevertheless, x-ray diffraction has proved to be an invaluable technique for elucidating these structures and for understanding the structure of matter.

Figure 38.20 shows one experimental arrangement for observing x-ray diffraction from a crystal. A collimated beam of monochromatic x-rays is incident on a crystal. The diffracted beams are very intense in certain directions, corresponding to constructive interference from waves reflected from layers of atoms in the crystal. The diffracted beams, which can be detected by a photographic film, form an array of spots known as a *Laue pattern* as in Figure 38.21a. One can deduce the crystalline structure by analyzing the positions and intensities of the various spots in the pattern. Figure 38.21b shows a Laue pattern from a crystalline enzyme, using a wide range of wavelengths so that a swirling pattern results.

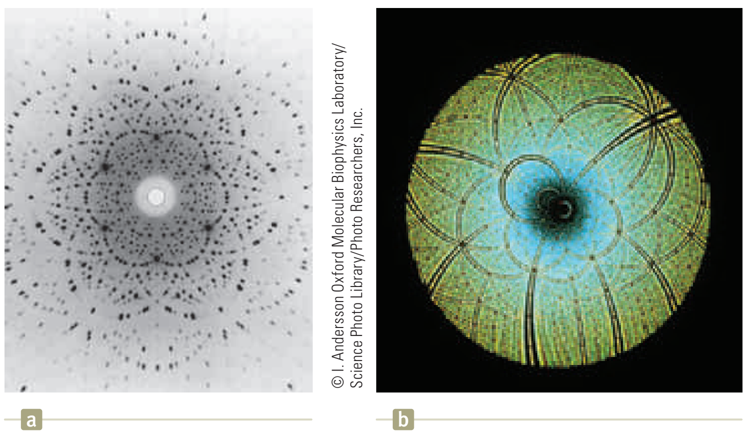


Figure 38.21 (a) A Laue pattern of a single crystal of the mineral beryl (beryllium aluminum silicate). Each dot represents a point of constructive interference. (b) A Laue pattern of the enzyme Rubisco, produced with a wideband x-ray spectrum. This enzyme is present in plants and takes part in the process of photosynthesis. The Laue pattern is used to determine the crystal structure of Rubisco.

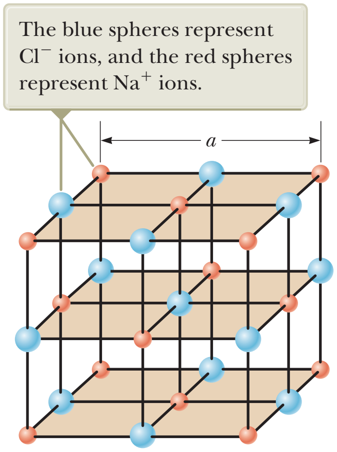


Figure 38.22 Crystalline structure of sodium chloride (NaCl). The length of the cube edge is $a = 0.562\ 737$ nm.

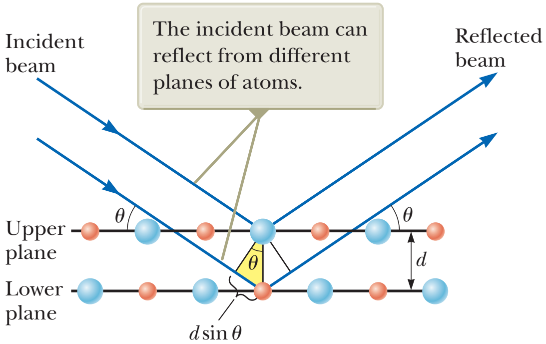


Figure 38.23 A two-dimensional description of the reflection of an x-ray beam from two parallel crystalline planes separated by a distance d. The beam reflected from the lower plane travels farther than the beam reflected from the upper plane by a distance $2d \sin \theta$.

The arrangement of atoms in a crystal of sodium chloride (NaCl) is shown in Figure 38.22. Each unit cell (the geometric solid that repeats throughout the crystal) is a cube having an edge length a. A careful examination of the NaCl structure shows that the ions lie in discrete planes (the shaded areas in Fig. 38.22). Now suppose an incident x-ray beam makes an angle θ with one of the planes as in Figure 38.23. The beam can be reflected from both the upper plane and the lower one, but the beam reflected from the lower plane travels farther than the beam reflected from the upper plane. The effective path difference is $2d \sin \theta$. The two beams reinforce each other (constructive interference) when this path difference equals some integer multiple of λ. The same is true for reflection from the entire family of parallel planes. Hence, the condition for *constructive* interference (maxima in the reflected beam) is

Pitfall Prevention 38.4

Different Angles Notice in Figure 38.23 that the angle θ is measured from the reflecting surface rather than from the normal as in the case of the law of reflection in Chapter 35. With slits and diffraction gratings, we also measured the angle θ from the normal to the array of slits. Because of historical tradition, the angle is measured differently in Bragg diffraction, so interpret Equation 38.8 with care.

$$2d \sin \theta = m\lambda \quad m = 1, 2, 3, \ldots \tag{38.8}$$

◀ **Bragg's law**

This condition is known as **Bragg's law,** after W. L. Bragg (1890–1971), who first derived the relationship. If the wavelength and diffraction angle are measured, Equation 38.8 can be used to calculate the spacing between atomic planes.

38.6 Polarization of Light Waves

In Chapter 34, we described the transverse nature of light and all other electromagnetic waves. Polarization, discussed in this section, is firm evidence of this transverse nature.

An ordinary beam of light consists of a large number of waves emitted by the atoms of the light source. Each atom produces a wave having some particular orientation of the electric field vector $\vec{\mathbf{E}}$, corresponding to the direction of atomic vibration. The *direction of polarization* of each individual wave is defined to be the direction in which the electric field is vibrating. In Figure 38.24, this direction happens to lie along the y axis. All individual electromagnetic waves traveling in the x direction have an $\vec{\mathbf{E}}$ vector parallel to the yz plane, but this vector could be at any possible angle with respect to the y axis. Because all directions of vibration from a wave source are possible, the resultant electromagnetic wave is a superposition of waves vibrating in many different directions. The result is an **unpolarized** light beam, represented in Figure 38.25a (page 1176). The direction of wave propagation in this figure is perpendicular to the page. The arrows show a few possible

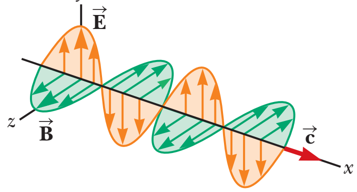


Figure 38.24 Schematic diagram of an electromagnetic wave propagating at velocity $\vec{\mathbf{c}}$ in the x direction. The electric field vibrates in the xy plane, and the magnetic field vibrates in the xz plane.

directions of the electric field vectors for the individual waves making up the resultant beam. At any given point and at some instant of time, all these individual electric field vectors add to give one resultant electric field vector.

As noted in Section 34.3, a wave is said to be **linearly polarized** if the resultant electric field $\vec{\mathbf{E}}$ vibrates in the same direction *at all times* at a particular point as shown in Figure 38.25b. (Sometimes, such a wave is described as *plane-polarized,* or simply *polarized.*) The plane formed by $\vec{\mathbf{E}}$ and the direction of propagation is called the *plane of polarization* of the wave. If the wave in Figure 38.24 represents the resultant of all individual waves, the plane of polarization is the *xy* plane.

A linearly polarized beam can be obtained from an unpolarized beam by removing all waves from the beam except those whose electric field vectors oscillate in a single plane. We now discuss four processes for producing polarized light from unpolarized light.

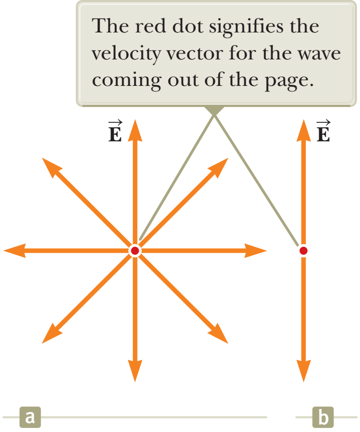


Figure 38.25 (a) A representation of an unpolarized light beam viewed along the direction of propagation. The transverse electric field can vibrate in any direction in the plane of the page with equal probability. (b) A linearly polarized light beam with the electric field vibrating in the vertical direction.

Polarization by Selective Absorption

The most common technique for producing polarized light is to use a material that transmits waves whose electric fields vibrate in a plane parallel to a certain direction and that absorbs waves whose electric fields vibrate in all other directions.

In 1938, E. H. Land (1909–1991) discovered a material, which he called *Polaroid,* that polarizes light through selective absorption. This material is fabricated in thin sheets of long-chain hydrocarbons. The sheets are stretched during manufacture so that the long-chain molecules align. After a sheet is dipped into a solution containing iodine, the molecules become good electrical conductors. Conduction takes place primarily along the hydrocarbon chains because electrons can move easily only along the chains. If light whose electric field vector is parallel to the chains is incident on the material, the electric field accelerates electrons along the chains and energy is absorbed from the radiation. Therefore, the light does not pass through the material. Light whose electric field vector is perpendicular to the chains passes through the material because electrons cannot move from one molecule to the next. As a result, when unpolarized light is incident on the material, the exiting light is polarized perpendicular to the molecular chains.

It is common to refer to the direction perpendicular to the molecular chains as the *transmission axis.* In an ideal polarizer, all light with $\vec{\mathbf{E}}$ parallel to the transmission axis is transmitted and all light with $\vec{\mathbf{E}}$ perpendicular to the transmission axis is absorbed.

Figure 38.26 represents an unpolarized light beam incident on a first polarizing sheet, called the *polarizer.* Because the transmission axis is oriented vertically in the figure, the light transmitted through this sheet is polarized vertically. A second polarizing sheet, called the *analyzer,* intercepts the beam. In Figure 38.26, the analyzer transmission axis is set at an angle θ to the polarizer axis. We call the electric field vector of the first transmitted beam $\vec{\mathbf{E}}_0$. The component of $\vec{\mathbf{E}}_0$ perpendicular to the analyzer axis is completely absorbed. The component of $\vec{\mathbf{E}}_0$ parallel to the

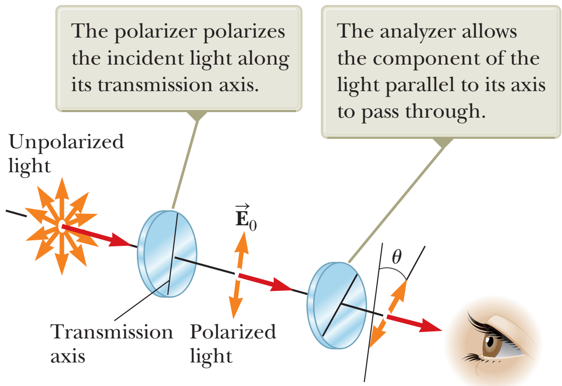


Figure 38.26 Two polarizing sheets whose transmission axes make an angle θ with each other. Only a fraction of the polarized light incident on the analyzer is transmitted through it.

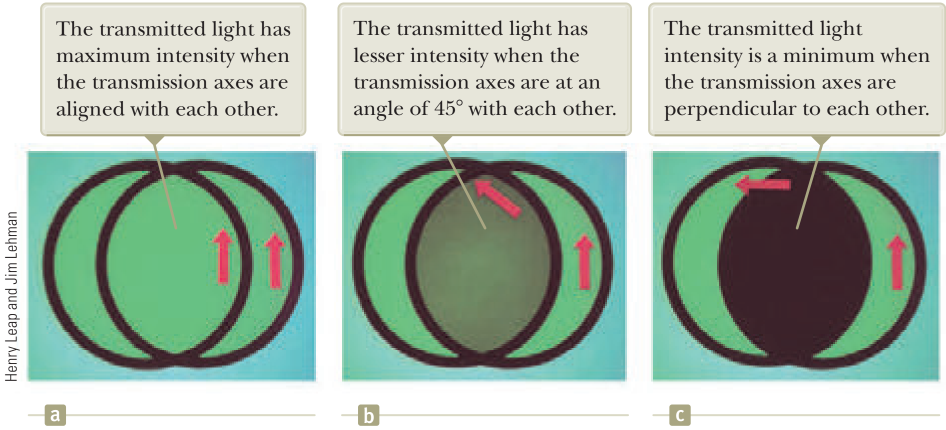


Figure 38.27 The intensity of light transmitted through two polarizers depends on the relative orientation of their transmission axes. The red arrows indicate the transmission axes of the polarizers.

analyzer axis, which is transmitted through the analyzer, is $E_0 \cos\theta$. Because the intensity of the transmitted beam varies as the square of its magnitude, we conclude that the intensity I of the (polarized) beam transmitted through the analyzer varies as

$$I = I_{\max} \cos^2\theta \qquad (38.9)$$

◀ Malus's law

where $I_{\max}$ is the intensity of the polarized beam incident on the analyzer. This expression, known as **Malus's law,**[2] applies to any two polarizing materials whose transmission axes are at an angle θ to each other. This expression shows that the intensity of the transmitted beam is maximum when the transmission axes are parallel ($\theta = 0$ or $180°$) and is zero (complete absorption by the analyzer) when the transmission axes are perpendicular to each other. This variation in transmitted intensity through a pair of polarizing sheets is illustrated in Figure 38.27. Because the average value of $\cos^2\theta$ is $\frac{1}{2}$, the intensity of initially unpolarized light is reduced by a factor of one-half as the light passes through a single ideal polarizer.

Polarization by Reflection

When an unpolarized light beam is reflected from a surface, the polarization of the reflected light depends on the angle of incidence. If the angle of incidence is 0°, the reflected beam is unpolarized. For other angles of incidence, the reflected light is polarized to some extent, and for one particular angle of incidence, the reflected light is completely polarized. Let's now investigate reflection at that special angle.

Suppose an unpolarized light beam is incident on a surface as in Figure 38.28a (page 1178). Each individual electric field vector can be resolved into two components: one parallel to the surface (and perpendicular to the page in Fig. 38.28, represented by the dots) and the other (represented by the orange arrows) perpendicular both to the first component and to the direction of propagation. Therefore, the polarization of the entire beam can be described by two electric field components in these directions. It is found that the parallel component represented by the dots reflects more strongly than the other component represented by the arrows, resulting in a partially polarized reflected beam. Furthermore, the refracted beam is also partially polarized.

Now suppose the angle of incidence θ_1 is varied until the angle between the reflected and refracted beams is 90° as in Figure 38.28b. At this particular angle of incidence, the reflected beam is completely polarized (with its electric field vector parallel to the surface) and the refracted beam is still only partially polarized. The angle of incidence at which this polarization occurs is called the **polarizing angle** θ_p.

[2]Named after its discoverer, E. L. Malus (1775–1812). Malus discovered that reflected light was polarized by viewing it through a calcite ($CaCO_3$) crystal.

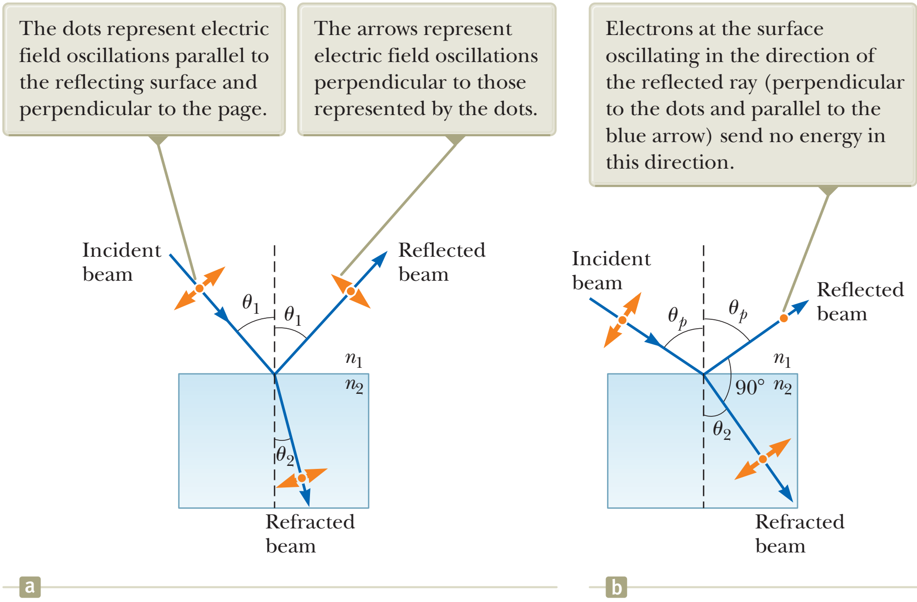


Figure 38.28 (a) When unpolarized light is incident on a reflecting surface, the reflected and refracted beams are partially polarized. (b) The reflected beam is completely polarized when the angle of incidence equals the polarizing angle θ_p, which satisfies the equation $n_2/n_1 = \tan\theta_p$. At this incident angle, the reflected and refracted rays are perpendicular to each other.

We can obtain an expression relating the polarizing angle to the index of refraction of the reflecting substance by using Figure 38.28b. From this figure, we see that $\theta_p + 90° + \theta_2 = 180°$; therefore, $\theta_2 = 90° - \theta_p$. Using Snell's law of refraction (Eq. 35.8) gives

$$\frac{n_2}{n_1} = \frac{\sin\theta_1}{\sin\theta_2} = \frac{\sin\theta_p}{\sin\theta_2}$$

Because $\sin\theta_2 = \sin(90° - \theta_p) = \cos\theta_p$, we can write this expression as $n_2/n_1 = \sin\theta_p/\cos\theta_p$, which means that

Brewster's law ▶

$$\tan\theta_p = \frac{n_2}{n_1} \tag{38.10}$$

This expression is called **Brewster's law,** and the polarizing angle θ_p is sometimes called **Brewster's angle,** after its discoverer, David Brewster (1781–1868). Because n varies with wavelength for a given substance, Brewster's angle is also a function of wavelength.

We can understand polarization by reflection by imagining that the electric field in the incident light sets electrons at the surface of the material in Figure 38.28b into oscillation. The component directions of oscillation are (1) parallel to the arrows shown on the refracted beam of light and therefore parallel to the reflected beam and (2) perpendicular to the page. The oscillating electrons act as dipole antennas radiating light with a polarization parallel to the direction of oscillation. Consult Figure 34.12, which shows the pattern of radiation from a dipole antenna. Notice that there is no radiation at an angle of $\theta = 0$, that is, along the oscillation direction of the antenna. Therefore, for the oscillations in direction 1, there is no radiation in the direction along the reflected ray. For oscillations in direction 2, the electrons radiate light with a polarization perpendicular to the page. Therefore, the light reflected from the surface at this angle is completely polarized parallel to the surface.

Polarization by reflection is a common phenomenon. Sunlight reflected from water, glass, and snow is partially polarized. If the surface is horizontal, the electric field vector of the reflected light has a strong horizontal component. Sunglasses made of polarizing material reduce the glare of reflected light. The transmission axes of such lenses are oriented vertically so that they absorb the strong horizontal component of the reflected light. If you rotate sunglasses through 90°, they are not as effective at blocking the glare from shiny horizontal surfaces.

Polarization by Double Refraction

Solids can be classified on the basis of internal structure. Those in which the atoms are arranged in a specific order are called *crystalline;* the NaCl structure of Figure 38.22 is one example of a crystalline solid. Those solids in which the atoms are distributed randomly are called *amorphous.* When light travels through an amorphous material such as glass, it travels with a speed that is the same in all directions. That is, glass has a single index of refraction. In certain crystalline materials such as calcite and quartz, however, the speed of light is not the same in all directions. In these materials, the speed of light depends on the direction of propagation *and* on the plane of polarization of the light. Such materials are characterized by two indices of refraction. Hence, they are often referred to as **double-refracting** or **birefringent** materials.

When unpolarized light enters a birefringent material, it may split into an **ordinary (O) ray** and an **extraordinary (E) ray.** These two rays have mutually perpendicular polarizations and travel at different speeds through the material. The two speeds correspond to two indices of refraction, n_O for the ordinary ray and n_E for the extraordinary ray.

There is one direction, called the **optic axis,** along which the ordinary and extraordinary rays have the same speed. If light enters a birefringent material at an angle to the optic axis, however, the different indices of refraction will cause the two polarized rays to split and travel in different directions as shown in Figure 38.29.

The index of refraction n_O for the ordinary ray is the same in all directions. If one could place a point source of light inside the crystal as in Figure 38.30, the ordinary waves would spread out from the source as spheres. The index of refraction n_E varies with the direction of propagation. A point source sends out an extraordinary wave having wave fronts that are elliptical in cross section. The difference in speed for the two rays is a maximum in the direction perpendicular to the optic axis. For example, in calcite, $n_O = 1.658$ at a wavelength of 589.3 nm and n_E varies from 1.658 along the optic axis to 1.486 perpendicular to the optic axis. Values for n_O and the extreme value of n_E for various double-refracting crystals are given in Table 38.1.

If you place a calcite crystal on a sheet of paper and then look through the crystal at any writing on the paper, you would see two images as shown in Figure 38.31. As can be seen from Figure 38.29, these two images correspond to one formed by the ordinary ray and one formed by the extraordinary ray. If the two images are viewed through a sheet of rotating polarizing glass, they alternately appear and disappear because the ordinary and extraordinary rays are plane-polarized along mutually perpendicular directions.

Some materials such as glass and plastic become birefringent when stressed. Suppose an unstressed piece of plastic is placed between a polarizer and an analyzer so that light passes from polarizer to plastic to analyzer. When the plastic is unstressed and the analyzer axis is perpendicular to the polarizer axis, none of the polarized light passes through the analyzer. In other words, the unstressed plastic has no effect on the light passing through it. If the plastic is stressed, however, regions of greatest stress become birefringent and the polarization of the light passing through the plastic changes. Hence, a series of bright and dark bands is observed in the transmitted light, with the bright bands corresponding to regions of greatest stress.

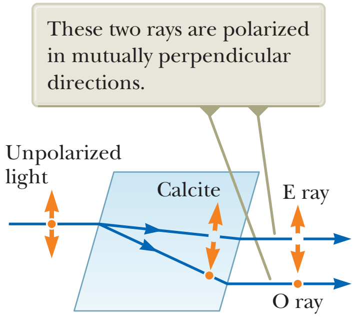


Figure 38.29 Unpolarized light incident at an angle to the optic axis in a calcite crystal splits into an ordinary (O) ray and an extraordinary (E) ray (not to scale).

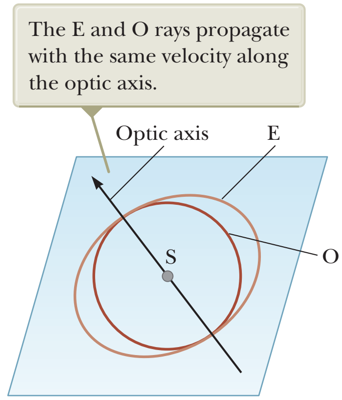
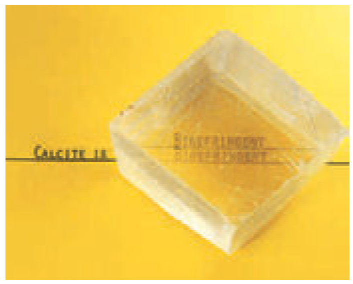


Figure 38.30 A point source S inside a double-refracting crystal produces a spherical wave front corresponding to the ordinary (O) ray and an elliptical wave front corresponding to the extraordinary (E) ray.



Henry Leap and Jim Lehman

Figure 38.31 A calcite crystal produces a double image because it is a birefringent (double-refracting) material.

Table 38.1 Indices of Refraction for Some Double-Refracting Crystals at a Wavelength of 589.3 nm

Crystal	n_O	n_E	n_O/n_E
Calcite ($CaCO_3$)	1.658	1.486	1.116
Quartz (SiO_2)	1.544	1.553	0.994
Sodium nitrate ($NaNO_3$)	1.587	1.336	1.188
Sodium sulfite ($NaSO_3$)	1.565	1.515	1.033
Zinc chloride ($ZnCl_2$)	1.687	1.713	0.985
Zinc sulfide (ZnS)	2.356	2.378	0.991

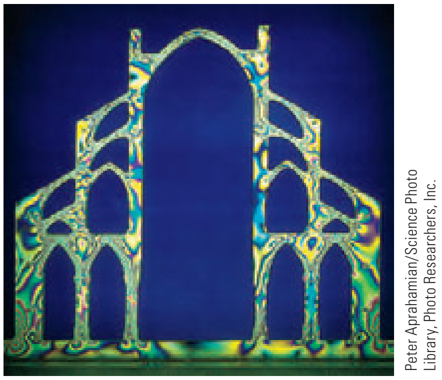


Figure 38.32 A plastic model of an arch structure under load conditions. The pattern is produced when the plastic model is viewed between a polarizer and analyzer oriented perpendicular to each other. Such patterns are useful in the optimal design of architectural components.

Engineers often use this technique, called *optical stress analysis,* in designing structures ranging from bridges to small tools. They build a plastic model and analyze it under different load conditions to determine regions of potential weakness and failure under stress. An example of a plastic model under stress is shown in Figure 38.32.

Polarization by Scattering

When light is incident on any material, the electrons in the material can absorb and reradiate part of the light. Such absorption and reradiation of light by electrons in the gas molecules that make up air is what causes sunlight reaching an observer on the Earth to be partially polarized. You can observe this effect—called **scattering**—by looking directly up at the sky through a pair of sunglasses whose lenses are made of polarizing material. Less light passes through at certain orientations of the lenses than at others.

Figure 38.33 illustrates how sunlight becomes polarized when it is scattered. The phenomenon is similar to that creating completely polarized light upon reflection from a surface at Brewster's angle. An unpolarized beam of sunlight traveling in the horizontal direction (parallel to the ground) strikes a molecule of one of the gases that make up air, setting the electrons of the molecule into vibration. These vibrating charges act like the vibrating charges in an antenna. The horizontal component of the electric field vector in the incident wave results in a horizontal component of the vibration of the charges, and the vertical component of the vector results in a vertical component of vibration. If the observer in Figure 38.33 is looking straight up (perpendicular to the original direction of propagation of the light), the vertical oscillations of the charges send no radiation toward the observer. Therefore, the observer sees light that is completely polarized in the horizontal direction as indicated by the orange arrows. If the observer looks in other directions, the light is partially polarized in the horizontal direction.

Variations in the color of scattered light in the atmosphere can be understood as follows. When light of various wavelengths λ is incident on gas molecules of diameter d, where $d << \lambda$, the relative intensity of the scattered light varies as $1/\lambda^4$. The condition $d << \lambda$ is satisfied for scattering from oxygen (O_2) and nitrogen (N_2) molecules in the atmosphere, whose diameters are about 0.2 nm. Hence, short wavelengths (violet light) are scattered more efficiently than long wavelengths (red light). Therefore, when sunlight is scattered by gas molecules in the air, the short-wavelength radiation (violet) is scattered more intensely than the long-wavelength radiation (red).

When you look up into the sky in a direction that is not toward the Sun, you see the scattered light, which is predominantly violet. Your eyes, however, are not very sensitive to violet light. Light of the next color in the spectrum, blue, is scattered with less intensity than violet, but your eyes are far more sensitive to blue light than to violet light. Hence, you see a blue sky. If you look toward the west at sunset (or toward the east at sunrise), you are looking in a direction toward the Sun and are seeing light that has passed through a large distance of air. Most of the blue light has been scattered by the air between you and the Sun. The light that survives this

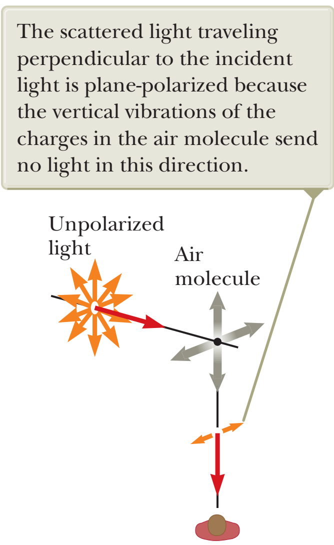


Figure 38.33 The scattering of unpolarized sunlight by air molecules.

trip through the air to you has had much of its blue component scattered and is therefore heavily weighted toward the red end of the spectrum; as a result, you see the red and orange colors of sunset (or sunrise).

Optical Activity

Many important applications of polarized light involve materials that display **optical activity.** A material is said to be optically active if it rotates the plane of polarization of any light transmitted through the material. The angle through which the light is rotated by a specific material depends on the length of the path through the material and on concentration if the material is in solution. One optically active material is a solution of the common sugar dextrose. A standard method for determining the concentration of sugar solutions is to measure the rotation produced by a fixed length of the solution.

Molecular asymmetry determines whether a material is optically active. For example, some proteins are optically active because of their spiral shape.

The liquid crystal displays found in most calculators have their optical activity changed by the application of electric potential across different parts of the display. Try using a pair of polarizing sunglasses to investigate the polarization used in the display of your calculator.

Quick Quiz 38.6 A polarizer for microwaves can be made as a grid of parallel metal wires approximately 1 cm apart. Is the electric field vector for microwaves transmitted through this polarizer **(a)** parallel or **(b)** perpendicular to the metal wires?

Quick Quiz 38.7 You are walking down a long hallway that has many light fixtures in the ceiling and a very shiny, newly waxed floor. When looking at the floor, you see reflections of every light fixture. Now you put on sunglasses that are polarized. Some of the reflections of the light fixtures can no longer be seen. (Try it!) Are the reflections that disappear those **(a)** nearest to you, **(b)** farthest from you, or **(c)** at an intermediate distance from you?

Summary

Concepts and Principles

Diffraction is the deviation of light from a straight-line path when the light passes through an aperture or around an obstacle. Diffraction is due to the wave nature of light.

The **Fraunhofer diffraction pattern** produced by a single slit of width a on a distant screen consists of a central bright fringe and alternating bright and dark fringes of much lower intensities. The angles θ_{dark} at which the diffraction pattern has zero intensity, corresponding to destructive interference, are given by

$$\sin\theta_{\text{dark}} = m\frac{\lambda}{a} \quad m = \pm1, \pm2, \pm3, \ldots \tag{38.1}$$

Rayleigh's criterion, which is a limiting condition of resolution, states that two images formed by an aperture are just distinguishable if the central maximum of the diffraction pattern for one image falls on the first minimum of the diffraction pattern for the other image. The limiting angle of resolution for a slit of width a is $\theta_{\text{min}} = \lambda/a$, and the limiting angle of resolution for a circular aperture of diameter D is given by $\theta_{\text{min}} = 1.22\lambda/D$.

A **diffraction grating** consists of a large number of equally spaced, identical slits. The condition for intensity maxima in the interference pattern of a diffraction grating for normal incidence is

$$d\sin\theta_{\text{bright}} = m\lambda \quad m = 0, \pm1, \pm2, \pm3, \ldots \tag{38.7}$$

where d is the spacing between adjacent slits and m is the order number of the intensity maximum.

continued

When polarized light of intensity I_{max} is emitted by a polarizer and then is incident on an analyzer, the light transmitted through the analyzer has an intensity equal to $I_{max} \cos^2 \theta$, where θ is the angle between the polarizer and analyzer transmission axes.

In general, reflected light is partially polarized. Reflected light, however, is completely polarized when the angle of incidence is such that the angle between the reflected and refracted beams is 90°. This angle of incidence, called the **polarizing angle** θ_p, satisfies **Brewster's law:**

$$\tan \theta_p = \frac{n_2}{n_1} \quad \textbf{(38.10)}$$

where n_1 is the index of refraction of the medium in which the light initially travels and n_2 is the index of refraction of the reflecting medium.

Objective Questions

1. denotes answer available in *Student Solutions Manual/Study Guide*

1. Certain sunglasses use a polarizing material to reduce the intensity of light reflected as glare from water or automobile windshields. What orientation should the polarizing filters have to be most effective? (a) The polarizers should absorb light with its electric field horizontal. (b) The polarizers should absorb light with its electric field vertical. (c) The polarizers should absorb both horizontal and vertical electric fields. (d) The polarizers should not absorb either horizontal or vertical electric fields.

2. What is most likely to happen to a beam of light when it reflects from a shiny metallic surface at an arbitrary angle? Choose the best answer. (a) It is totally absorbed by the surface. (b) It is totally polarized. (c) It is unpolarized. (d) It is partially polarized. (e) More information is required.

3. In Figure 38.4, assume the slit is in a barrier that is opaque to x-rays as well as to visible light. The photograph in Figure 38.4b shows the diffraction pattern produced with visible light. What will happen if the experiment is repeated with x-rays as the incoming wave and with no other changes? (a) The diffraction pattern is similar. (b) There is no noticeable diffraction pattern but rather a projected shadow of high intensity on the screen, having the same width as the slit. (c) The central maximum is much wider, and the minima occur at larger angles than with visible light. (d) No x-rays reach the screen.

4. A Fraunhofer diffraction pattern is produced on a screen located 1.00 m from a single slit. If a light source of wavelength 5.00×10^{-7} m is used and the distance from the center of the central bright fringe to the first dark fringe is 5.00×10^{-3} m, what is the slit width? (a) 0.010 0 mm (b) 0.100 mm (c) 0.200 mm (d) 1.00 mm (e) 0.005 00 mm

5. Consider a wave passing through a single slit. What happens to the width of the central maximum of its diffraction pattern as the slit is made half as wide? (a) It becomes one-fourth as wide. (b) It becomes one-half as wide. (c) Its width does not change. (d) It becomes twice as wide. (e) It becomes four times as wide.

6. Assume Figure 38.1 was photographed with red light of a single wavelength λ_0. The light passed through a single slit of width a and traveled distance L to the screen where the photograph was made. Consider the width of the central bright fringe, measured between the centers of the dark fringes on both sides of it. Rank from largest to smallest the widths of the central fringe in the following situations and note any cases of equality. (a) The experiment is performed as photographed. (b) The experiment is performed with light whose frequency is increased by 50%. (c) The experiment is performed with light whose wavelength is increased by 50%. (d) The experiment is performed with the original light and with a slit of width $2a$. (e) The experiment is performed with the original light and slit and with distance $2L$ to the screen.

7. If plane polarized light is sent through two polarizers, the first at 45° to the original plane of polarization and the second at 90° to the original plane of polarization, what fraction of the original polarized intensity passes through the last polarizer? (a) 0 (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{8}$ (e) $\frac{1}{10}$

8. Why is it advantageous to use a large-diameter objective lens in a telescope? (a) It diffracts the light more effectively than smaller-diameter objective lenses. (b) It increases its magnification. (c) It enables you to see more objects in the field of view. (d) It reflects unwanted wavelengths. (e) It increases its resolution.

9. What combination of optical phenomena causes the bright colored patterns sometimes seen on wet streets covered with a layer of oil? Choose the best answer. (a) diffraction and polarization (b) interference and diffraction (c) polarization and reflection (d) refraction and diffraction (e) reflection and interference

10. When you receive a chest x-ray at a hospital, the x-rays pass through a set of parallel ribs in your chest. Do your ribs act as a diffraction grating for x-rays? (a) Yes. They produce diffracted beams that can be observed separately. (b) Not to a measurable extent. The ribs are too far apart. (c) Essentially not. The ribs are too close together. (d) Essentially not. The ribs are too few in number. (e) Absolutely not. X-rays cannot diffract.

11. When unpolarized light passes through a diffraction grating, does it become polarized? (a) No, it does not. (b) Yes, it does, with the transmission axis parallel to the slits or grooves in the grating. (c) Yes, it does, with the transmission axis perpendicular to the slits or grooves in the grating. (d) It possibly does because an electric field above some threshold is blocked out by the grating if the field is perpendicular to the slits.

12. Off in the distance, you see the headlights of a car, but they are indistinguishable from the single headlight of a motorcycle. Assume the car's headlights are now switched from low beam to high beam so that the light intensity you receive becomes three times greater. What then happens to your ability to resolve the two light sources? (a) It increases by a factor of 9. (b) It increases by a factor of 3. (c) It remains the same. (d) It becomes one-third as good. (e) It becomes one-ninth as good.

Conceptual Questions

$\boxed{1.}$ denotes answer available in *Student Solutions Manual/Study Guide*

1. The atoms in a crystal lie in planes separated by a few tenths of a nanometer. Can they produce a diffraction pattern for visible light as they do for x-rays? Explain your answer with reference to Bragg's law.

2. Holding your hand at arm's length, you can readily block sunlight from reaching your eyes. Why can you not block sound from reaching your ears this way?

3. How could the index of refraction of a flat piece of opaque obsidian glass be determined?

4. (a) Is light from the sky polarized? (b) Why is it that clouds seen through Polaroid glasses stand out in bold contrast to the sky?

5. A laser beam is incident at a shallow angle on a horizontal machinist's ruler that has a finely calibrated scale. The engraved rulings on the scale give rise to a diffraction pattern on a vertical screen. Discuss how you can use this technique to obtain a measure of the wavelength of the laser light.

6. If a coin is glued to a glass sheet and this arrangement is held in front of a laser beam, the projected shadow has diffraction rings around its edge and a bright spot in the center. How are these effects possible?

7. Fingerprints left on a piece of glass such as a windowpane often show colored spectra like that from a diffraction grating. Why?

8. A laser produces a beam a few millimeters wide, with uniform intensity across its width. A hair is stretched vertically across the front of the laser to cross the beam. (a) How is the diffraction pattern it produces on a distant screen related to that of a vertical slit equal in width to the hair? (b) How could you determine the width of the hair from measurements of its diffraction pattern?

9. A radio station serves listeners in a city to the northeast of its broadcast site. It broadcasts from three adjacent towers on a mountain ridge, along a line running east to west, in what's called a *phased array*. Show that by introducing time delays among the signals the individual towers radiate, the station can maximize net intensity in the direction toward the city (and in the opposite direction) and minimize the signal transmitted in other directions.

10. John William Strutt, Lord Rayleigh (1842–1919), invented an improved foghorn. To warn ships of a coastline, a foghorn should radiate sound in a wide horizontal sheet over the ocean's surface. It should not waste energy by broadcasting sound upward or downward. Rayleigh's foghorn trumpet is shown in two possible configurations, horizontal and vertical, in Figure CQ38.10. Which is the correct orientation? Decide whether the long dimension of the rectangular opening should be horizontal or vertical and argue for your decision.

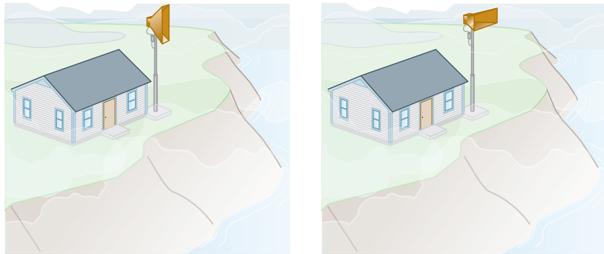
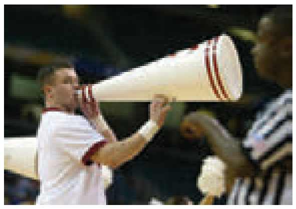


Figure CQ38.10

$\boxed{11.}$ Why can you hear around corners, but not see around corners?

12. Figure CQ38.12 shows a megaphone in use. Construct a theoretical description of how a megaphone works. You may assume the sound of your voice radiates just through the opening of your mouth. Most of the information in speech is carried not in a signal at the fundamental frequency, but in noises and in harmonics, with frequencies of a few thousand hertz. Does your theory allow any prediction that is simple to test?



Doug Pensigner/Getty Images

Figure CQ38.12

Problems

ENHANCED WebAssign The problems found in this chapter may be assigned online in Enhanced WebAssign

1. straightforward; **2.** intermediate; **3.** challenging

$\boxed{1.}$ full solution available in the *Student Solutions Manual/Study Guide*

AMT Analysis Model tutorial available in Enhanced WebAssign

GP Guided Problem

M Master It tutorial available in Enhanced WebAssign

W Watch It video solution available in Enhanced WebAssign

Section 38.2 Diffraction Patterns from Narrow Slits

1. M Light of wavelength 587.5 nm illuminates a slit of width 0.75 mm. (a) At what distance from the slit should a screen be placed if the first minimum in the diffraction pattern is to be 0.85 mm from the central maximum? (b) Calculate the width of the central maximum.

2. W Helium–neon laser light (λ = 632.8 nm) is sent through a 0.300-mm-wide single slit. What is the width of the central maximum on a screen 1.00 m from the slit?

3. Sound with a frequency 650 Hz from a distant source passes through a doorway 1.10 m wide in a sound-absorbing wall. Find (a) the number and (b) the angular directions of the diffraction minima at listening positions along a line parallel to the wall.

4. A horizontal laser beam of wavelength 632.8 nm has a circular cross section 2.00 mm in diameter. A rectangular aperture is to be placed in the center of the beam so that when the light falls perpendicularly on a wall 4.50 m away, the central maximum fills a rectangle 110 mm wide and 6.00 mm high. The dimensions are measured between the minima bracketing the central maximum. Find the required (a) width and (b) height of the aperture. (c) Is the longer dimension of the central bright patch in the diffraction pattern horizontal or vertical? (d) Is the longer dimension of the aperture horizontal or vertical? (e) Explain the relationship between these two rectangles, using a diagram.

5. Coherent microwaves of wavelength 5.00 cm enter a tall, narrow window in a building otherwise essentially opaque to the microwaves. If the window is 36.0 cm wide, what is the distance from the central maximum to the first-order minimum along a wall 6.50 m from the window?

6. Light of wavelength 540 nm passes through a slit of width 0.200 mm. (a) The width of the central maximum on a screen is 8.10 mm. How far is the screen from the slit? (b) Determine the width of the first bright fringe to the side of the central maximum.

7. M A screen is placed 50.0 cm from a single slit, which is illuminated with light of wavelength 690 nm. If the distance between the first and third minima in the diffraction pattern is 3.00 mm, what is the width of the slit?

8. A screen is placed a distance L from a single slit of width a, which is illuminated with light of wavelength λ. Assume $L >> a$. If the distance between the minima for $m = m_1$ and $m = m_2$ in the diffraction pattern is Δy, what is the width of the slit?

9. Assume light of wavelength 650 nm passes through two slits 3.00 μm wide, with their centers 9.00 μm apart. Make a sketch of the combined diffraction and interference pattern in the form of a graph of intensity versus $\phi = (\pi a \sin\theta)/\lambda$. You may use Figure 38.7 as a starting point.

10. **What If?** Suppose light strikes a single slit of width a at an angle β from the perpendicular direction as shown in Figure P38.10. Show that Equation 38.1, the condition for destructive interference, must be modified to read

$$\sin\theta_{\text{dark}} = m\frac{\lambda}{a} - \sin\beta$$
$$m = \pm1, \pm2, \pm3, \ldots$$

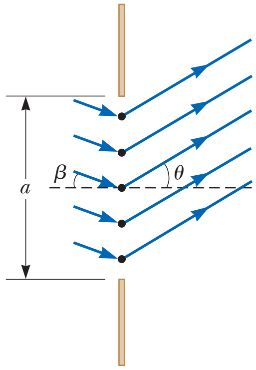


Figure P38.10

11. M A diffraction pattern is formed on a screen 120 cm away from a 0.400-mm-wide slit. Monochromatic 546.1-nm light is used. Calculate the fractional intensity I/I_{max} at a point on the screen 4.10 mm from the center of the principal maximum.

12. GP Coherent light of wavelength 501.5 nm is sent through two parallel slits in an opaque material. Each slit is 0.700 μm wide. Their centers are 2.80 μm apart. The light then falls on a semicylindrical screen, with its axis at the midline between the slits. We would like to describe the appearance of the pattern of light visible on the screen. (a) Find the direction for each two-slit interference maximum on the screen as an angle away from the bisector of the line joining the slits. (b) How many angles are there that represent two-slit interference maxima? (c) Find the direction for each single-slit interference minimum on the screen as an angle away from the bisector of the line joining the slits. (d) How many angles are there that represent single-slit interference minima? (e) How many of the angles in part (d) are identical to those in part (a)? (f) How many bright fringes are visible on the screen? (g) If the intensity of the central fringe is I_{max}, what is the intensity of the last fringe visible on the screen?

13. A beam of monochromatic light is incident on a single slit of width 0.600 mm. A diffraction pattern forms on a wall 1.30 m beyond the slit. The distance between the positions of zero intensity on both sides of the central maximum is 2.00 mm. Calculate the wavelength of the light.

Section 38.3 Resolution of Single-Slit and Circular Apertures

Note: In Problems 14, 19, 22, 23, and 67, you may use the Rayleigh criterion for the limiting angle of resolution of an eye. The standard may be overly optimistic for human vision.

14. The pupil of a cat's eye narrows to a vertical slit of width 0.500 mm in daylight. Assume the average wavelength of the light is 500 nm. What is the angular resolution for horizontally separated mice?

15. The angular resolution of a radio telescope is to be 0.100° when the incident waves have a wavelength of 3.00 mm. What minimum diameter is required for the telescope's receiving dish?

16. A *pinhole camera* has a small circular aperture of diameter D. Light from distant objects passes through the aperture into an otherwise dark box, falling on a screen at the other end of the box. The aperture in a pinhole camera has diameter D = 0.600 mm. Two point sources of light of wavelength 550 nm are at a distance L from the hole. The separation between the sources is 2.80 cm, and they are just resolved by the camera. What is L?

17. The objective lens of a certain refracting telescope has a diameter of 58.0 cm. The telescope is mounted in a satellite that orbits the Earth at an altitude of 270 km to view objects on the Earth's surface. Assuming an average wavelength of 500 nm, find the minimum distance between two objects on the ground if their images are to be resolved by this lens.

18. Yellow light of wavelength 589 nm is used to view an object under a microscope. The objective lens diameter is 9.00 mm. (a) What is the limiting angle of resolution? (b) Suppose it is possible to use visible light of any wavelength. What color should you choose to give the smallest possible angle of resolution, and what is this angle? (c) Suppose water fills the space between the object and the objective. What effect does this change have on the resolving power when 589-nm light is used?

19. What is the approximate size of the smallest object on the Earth that astronauts can resolve by eye when they are orbiting 250 km above the Earth? Assume λ = 500 nm and a pupil diameter of 5.00 mm.

20. M A helium–neon laser emits light that has a wavelength of 632.8 nm. The circular aperture through which the beam emerges has a diameter of 0.500 cm. Estimate the diameter of the beam 10.0 km from the laser.

21. M To increase the resolving power of a microscope, the object and the objective are immersed in oil (n = 1.5). If the limiting angle of resolution without the oil is 0.60 μrad, what is the limiting angle of resolution with the oil? *Hint:* The oil changes the wavelength of the light.

22. Narrow, parallel, glowing gas-filled tubes in a variety of colors form block letters to spell out the name of a nightclub. Adjacent tubes are all 2.80 cm apart. The tubes forming one letter are filled with neon and radiate predominantly red light with a wavelength of 640 nm. For another letter, the tubes emit predominantly blue light at 440 nm. The pupil of a dark-adapted viewer's eye is 5.20 mm in diameter. (a) Which color is easier to resolve? State how you decide. (b) If she is in a certain range of distances away, the viewer can resolve the separate tubes of one color but not the other. The viewer's distance must be in what range for her to resolve the tubes of only one of these two colors?

23. M Impressionist painter Georges Seurat created paintings with an enormous number of dots of pure pigment, each of which was approximately 2.00 mm in diameter. The idea was to have colors such as red and green next to each other to form a scintillating canvas, such as in his masterpiece, *A Sunday Afternoon on the Island of La Grande Jatte* (Fig. P38.23). Assume λ = 500 nm and a pupil diameter of 5.00 mm. Beyond what distance would a viewer be unable to discern individual dots on the canvas?

Figure P38.23

24. W A circular radar antenna on a Coast Guard ship has a diameter of 2.10 m and radiates at a frequency of 15.0 GHz. Two small boats are located 9.00 km away from the ship. How close together could the boats be and still be detected as two objects?

Section 38.4 The Diffraction Grating

Note: In the following problems, assume the light is incident normally on the gratings.

25. A helium–neon laser (λ = 632.8 nm) is used to calibrate a diffraction grating. If the first-order maximum occurs at 20.5°, what is the spacing between adjacent grooves in the grating?

26. M White light is spread out into its spectral components by a diffraction grating. If the grating has 2 000 grooves per centimeter, at what angle does red light of wavelength 640 nm appear in first order?

27. Consider an array of parallel wires with uniform spacing of 1.30 cm between centers. In air at 20.0°C, ultrasound with a frequency of 37.2 kHz from a distant source is incident perpendicular to the array. (a) Find the number of directions on the other side of the array in which there is a maximum of intensity. (b) Find the angle for each of these directions relative to the direction of the incident beam.

28. W Three discrete spectral lines occur at angles of 10.1°, 13.7°, and 14.8° in the first-order spectrum of a grating spectrometer. (a) If the grating has 3 660 slits/cm, what are the wavelengths of the light? (b) At what angles are these lines found in the second-order spectrum?

29. The laser in a compact disc player must precisely follow the spiral track on the CD, along which the distance between one loop of the spiral and the next is only about 1.25 μm. Figure P38.29 (page 1186) shows how a diffraction grating is used to provide information to keep the beam on track. The laser light passes through a diffraction grating before it reaches the CD. The strong central maximum of the diffraction pattern is used to read the information in the track of pits. The two first-order side maxima are designed to fall on the flat surfaces on both sides of the information track and are used for steering. As long as both beams are reflecting from smooth, nonpitted surfaces, they are detected with constant high intensity. If the main beam wanders off the track, however, one of the side beams begins to strike pits on the information track and the reflected light diminishes. This change is used with an electronic circuit to guide the beam back to the desired location. Assume the laser light has a wavelength of 780 nm and the diffraction grating is positioned 6.90 μm from the disk. Assume the first-order beams are to fall on the CD 0.400 μm on either side of the information track. What should be the number of grooves per millimeter in the grating?

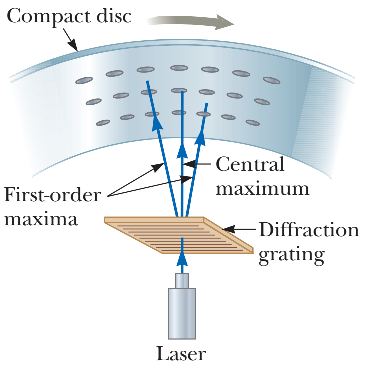


Figure P38.29

30. A grating with 250 grooves/mm is used with an incandescent light source. Assume the visible spectrum to range in wavelength from 400 nm to 700 nm. In how many orders can one see (a) the entire visible spectrum and (b) the short-wavelength region of the visible spectrum?

31. A diffraction grating has 4 200 rulings/cm. On a screen 2.00 m from the grating, it is found that for a particular order m, the maxima corresponding to two closely spaced wavelengths of sodium (589.0 nm and 589.6 nm) are separated by 1.54 mm. Determine the value of m.

32. M The hydrogen spectrum includes a red line at 656 nm and a blue-violet line at 434 nm. What are the angular separations between these two spectral lines for all visible orders obtained with a diffraction grating that has 4 500 grooves/cm?

33. W Light from an argon laser strikes a diffraction grating that has 5 310 grooves per centimeter. The central and first-order principal maxima are separated by 0.488 m on a wall 1.72 m from the grating. Determine the wavelength of the laser light.

34. Show that whenever white light is passed through a diffraction grating of any spacing size, the violet end of the spectrum in the third order on a screen always overlaps the red end of the spectrum in the second order.

35. Light of wavelength 500 nm is incident normally on a diffraction grating. If the third-order maximum of the diffraction pattern is observed at 32.0°, (a) what is the number of rulings per centimeter for the grating? (b) Determine the total number of primary maxima that can be observed in this situation.

36. A wide beam of laser light with a wavelength of 632.8 nm is directed through several narrow parallel slits, separated by 1.20 mm, and falls on a sheet of photographic film 1.40 m away. The exposure time is chosen so that the film stays unexposed everywhere except at the central region of each bright fringe. (a) Find the distance between these interference maxima. The film is printed as a transparency; it is opaque everywhere except at the exposed lines. Next, the same beam of laser light is directed through the transparency and allowed to fall on a screen 1.40 m beyond. (b) Argue that several narrow, parallel, bright regions, separated by 1.20 mm, appear on the screen as real images of the original slits. (A similar train of thought, at a soccer game, led Dennis Gabor to invent holography.)

37. A beam of bright red light of wavelength 654 nm passes through a diffraction grating. Enclosing the space beyond the grating is a large semicylindrical screen centered on the grating, with its axis parallel to the slits in the grating. Fifteen bright spots appear on the screen. Find (a) the maximum and (b) the minimum possible values for the slit separation in the diffraction grating.

Section 38.5 Diffraction of X-Rays by Crystals

38. M If the spacing between planes of atoms in a NaCl crystal is 0.281 nm, what is the predicted angle at which 0.140-nm x-rays are diffracted in a first-order maximum?

39. AMT M Potassium iodide (KI) has the same crystalline structure as NaCl, with atomic planes separated by 0.353 nm. A monochromatic x-ray beam shows a first-order diffraction maximum when the grazing angle is 7.60°. Calculate the x-ray wavelength.

40. Monochromatic x-rays (λ = 0.166 nm) from a nickel target are incident on a potassium chloride (KCl) crystal surface. The spacing between planes of atoms in KCl is 0.314 nm. At what angle (relative to the surface) should the beam be directed for a second-order maximum to be observed?

41. The first-order diffraction maximum is observed at 12.6° for a crystal having a spacing between planes of atoms of 0.250 nm. (a) What wavelength x-ray is used to observe this first-order pattern? (b) How many orders can be observed for this crystal at this wavelength?

Section 38.6 Polarization of Light Waves

Problem 62 in Chapter 34 can be assigned with this section.

42. *Why is the following situation impossible?* A technician is measuring the index of refraction of a solid material by observing the polarization of light reflected from its surface. She notices that when a light beam is projected from air onto the material surface, the reflected light is totally polarized parallel to the surface when the incident angle is 41.0°.

43. M Plane-polarized light is incident on a single polarizing disk with the direction of $\vec{\mathbf{E}}_0$ parallel to the direction of the transmission axis. Through what angle should the disk be rotated so that the intensity in the transmitted beam is reduced by a factor of (a) 3.00, (b) 5.00, and (c) 10.0?

44. AMT The angle of incidence of a light beam onto a reflecting surface is continuously variable. The reflected ray in air is completely polarized when the angle of incidence is 48.0°. What is the index of refraction of the reflecting material?

45. Unpolarized light passes through two ideal Polaroid sheets. The axis of the first is vertical, and the axis of the second is at 30.0° to the vertical. What fraction of the incident light is transmitted?

46. Two handheld radio transceivers with dipole antennas are separated by a large fixed distance. If the transmitting antenna is vertical, what fraction of the maximum received power will appear in the receiving antenna when it is inclined from the vertical by (a) 15.0°, (b) 45.0°, and (c) 90.0°?

47. You use a sequence of ideal polarizing filters, each with its axis making the same angle with the axis of the previous filter, to rotate the plane of polarization of a polarized light beam by a total of 45.0°. You wish to have an intensity reduction no larger than 10.0%. (a) How many polarizers do you need to achieve your goal? (b) What is the angle between adjacent polarizers?

48. An unpolarized beam of light is incident on a stack of ideal polarizing filters. The axis of the first filter is perpendicular to the axis of the last filter in the stack. Find the fraction by which the transmitted beam's intensity is reduced in the three following cases. (a) Three filters are in the stack, each with its transmission axis at 45.0° relative to the preceding filter. (b) Four filters are in the stack, each with its transmission axis at 30.0° relative to the preceding filter. (c) Seven filters are in the stack, each with its transmission axis at 15.0° relative to the preceding filter. (d) Comment on comparing the answers to parts (a), (b), and (c).

49. The critical angle for total internal reflection for sapphire surrounded by air is 34.4°. Calculate the polarizing angle for sapphire.

50. For a particular transparent medium surrounded by air, find the polarizing angle θ_p in terms of the critical angle for total internal reflection θ_c.

51. M Three polarizing plates whose planes are parallel are centered on a common axis. The directions of the transmission axes relative to the common vertical direction are shown in Figure P38.51. A linearly polarized beam of light with plane of polarization parallel to the vertical reference direction is incident from the left onto the first disk with intensity $I_i = 10.0$ units (arbitrary). Calculate the transmitted intensity I_f when $\theta_1 = 20.0°$, $\theta_2 = 40.0°$, and $\theta_3 = 60.0°$. *Hint:* Make repeated use of Malus's law.

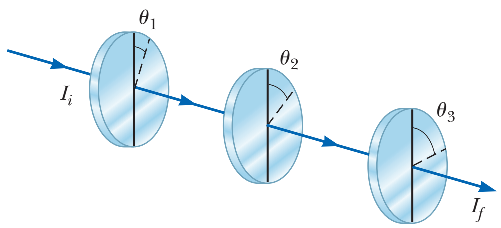


Figure P38.51

52. Two polarizing sheets are placed together with their transmission axes crossed so that no light is transmitted. A third sheet is inserted between them with its transmission axis at an angle of 45.0° with respect to each of the other axes. Find the fraction of incident unpolarized light intensity transmitted by the three-sheet combination. (Assume each polarizing sheet is ideal.)

Additional Problems

53. In a single-slit diffraction pattern, assuming each side maximum is halfway between the adjacent minima, find the ratio of the intensity of (a) the first-order side maximum and (b) the second-order side maximum to the intensity of the central maximum.

54. Laser light with a wavelength of 632.8 nm is directed through one slit or two slits and allowed to fall on a screen 2.60 m beyond. Figure P38.54 shows the pattern on the screen, with a centimeter ruler below it. (a) Did the light pass through one slit or two slits? Explain how you can determine the answer. (b) If one slit, find its width. If two slits, find the distance between their centers.

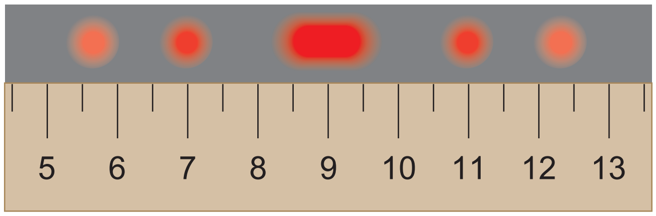


Figure P38.54

55. In water of uniform depth, a wide pier is supported on pilings in several parallel rows 2.80 m apart. Ocean waves of uniform wavelength roll in, moving in a direction that makes an angle of 80.0° with the rows of pilings. Find the three longest wavelengths of waves that are strongly reflected by the pilings.

56. The second-order dark fringe in a single-slit diffraction pattern is 1.40 mm from the center of the central maximum. Assuming the screen is 85.0 cm from a slit of width 0.800 mm and assuming monochromatic incident light, calculate the wavelength of the incident light.

57. Light from a helium–neon laser ($\lambda = 632.8$ nm) is incident on a single slit. What is the maximum width of the slit for which no diffraction minima are observed?

58. W Two motorcycles separated laterally by 2.30 m are approaching an observer wearing night-vision goggles sensitive to infrared light of wavelength 885 nm. (a) Assume the light propagates through perfectly steady and uniform air. What aperture diameter is required if the motorcycles' headlights are to be resolved at a distance of 12.0 km? (b) Comment on how realistic the assumption in part (a) is.

59. The *Very Large Array* (VLA) is a set of 27 radio telescope dishes in Catron and Socorro counties, New Mexico (Fig. P38.59). The antennas can be moved apart on railroad tracks, and their combined signals give the resolving power of a synthetic aperture 36.0 km in diameter. (a) If the detectors are tuned to a frequency of 1.40 GHz, what is the angular resolution of the VLA? (b) Clouds of interstellar hydrogen radiate at the frequency used in part (a). What must be the separation distance of two clouds at the center of the galaxy, 26 000 light-years away, if they are to be resolved? (c) **What If?** As the telescope looks up, a circling hawk looks down. Assume the hawk is most sensitive to green light having a wavelength of 500 nm and has a pupil of diameter 12.0 mm. Find the angular resolution of the hawk's eye. (d) A mouse is on the ground 30.0 m below. By what distance must the mouse's whiskers be separated if the hawk can resolve them?

Figure P38.59

60. Two wavelengths λ and $\lambda + \Delta\lambda$ (with $\Delta\lambda << \lambda$) are incident on a diffraction grating. Show that the angular separation between the spectral lines in the mth-order spectrum is

$$\Delta\theta = \frac{\Delta\lambda}{\sqrt{(d/m)^2 - \lambda^2}}$$

where d is the slit spacing and m is the order number.

61. **Review.** A beam of 541-nm light is incident on a diffraction grating that has 400 grooves/mm. (a) Determine the angle of the second-order ray. (b) **What If?** If the entire apparatus is immersed in water, what is the new second-order angle of diffraction? (c) Show that the two diffracted rays of parts (a) and (b) are related through the law of refraction.

62. *Why is the following situation impossible?* A technician is sending laser light of wavelength 632.8 nm through a pair of slits separated by 30.0 μm. Each slit is of width 2.00 μm. The screen on which he projects the pattern is not wide enough, so light from the $m = 15$ interference maximum misses the edge of the screen and passes into the next lab station, startling a coworker.

63. A 750-nm light beam in air hits the flat surface of a certain liquid, and the beam is split into a reflected ray and a refracted ray. If the reflected ray is completely polarized when it is at 36.0° with respect to the surface, what is the wavelength of the refracted ray?

64. Iridescent peacock feathers are shown in Figure P38.64a. The surface of one microscopic barbule is composed of transparent keratin that supports rods of dark brown melanin in a regular lattice, represented in Figure P38.64b. (Your fingernails are made of keratin, and melanin is the dark pigment giving color to human skin.) In a portion of the feather that can appear turquoise (blue-green), assume the melanin rods are uniformly separated by 0.25 μm, with air between them. (a) Explain how this structure can appear turquoise when it contains no blue or green pigment. (b) Explain how it can also appear violet if light falls on it in a different direction. (c) Explain how it can present different colors to your two eyes simultaneously, which is a characteristic of iridescence. (d) A compact disc can appear to be any color of the rainbow. Explain why the portion of the feather in Figure P38.64b cannot appear yellow or red. (e) What could be different about the array of melanin rods in a portion of the feather that does appear to be red?

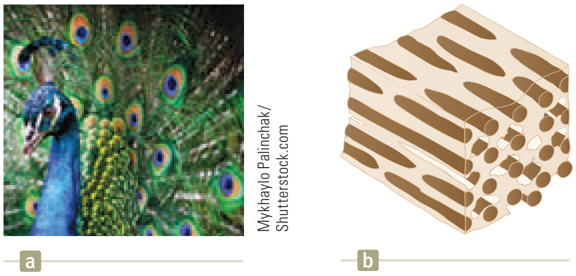


Figure P38.64

65. AMT Light in air strikes a water surface at the polarizing angle. The part of the beam refracted into the water strikes a submerged slab of material with refractive index $n = 1.62$ as shown in Figure P38.65. The light reflected from the upper surface of the slab is completely polarized. Find the angle θ between the water surface and the surface of the slab.

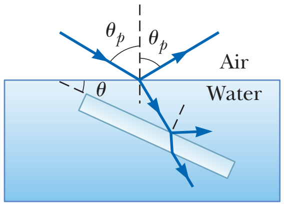


Figure P38.65
Problems 65 and 66.

66. Light in air (assume $n = 1$) strikes the surface of a liquid of index of refraction n_ℓ at the polarizing angle. The part of the beam refracted into the liquid strikes a submerged slab of material with refractive index n as shown in Figure P38.65. The light reflected from the upper surface of the slab is completely polarized. Find the angle θ between the water surface and the surface of the slab as a function of n and n_ℓ.

67. An American standard analog television picture (non-HDTV), also known as NTSC, is composed of approximately 485 visible horizontal lines of varying light intensity. Assume your ability to resolve the lines is limited only by the Rayleigh criterion, the pupils of your eyes are 5.00 mm in diameter, and the average wavelength of the light coming from the screen is 550 nm. Calculate the ratio of the minimum viewing distance to the vertical dimension of the picture such that you will not be able to resolve the lines.

68. A pinhole camera has a small circular aperture of diameter D. Light from distant objects passes through the aperture into an otherwise dark box, falling on a screen located a distance L away. If D is too large, the display on the screen will be fuzzy because a bright point in the field of view will send light onto a circle of diameter slightly larger than D. On the other hand, if D is too small, diffraction will blur the display on the screen. The screen shows a reasonably sharp image if the diameter of the central disk of the diffraction pattern, specified by Equation 38.6, is equal to D at the screen. (a) Show that for monochromatic light with plane wave fronts and $L >> D$, the condition for a sharp view is fulfilled if $D^2 = 2.44\lambda L$. (b) Find the optimum pinhole diameter for 500-nm light projected onto a screen 15.0 cm away.

69. The *scale* of a map is a number of kilometers per centimeter specifying the distance on the ground that any distance on the map represents. The scale of a spectrum is its *dispersion,* a number of nanometers per centimeter, specifying the change in wavelength that a distance across the spectrum represents. You must know the dispersion if you want to compare one spectrum with another or make a measurement of, for example, a Doppler shift. Let y represent the position relative to the center of a diffraction pattern projected onto a flat screen at distance L by a diffraction grating with slit spacing d. The dispersion is $d\lambda/dy$. (a) Prove that the dispersion is given by

$$\frac{d\lambda}{dy} = \frac{L^2 d}{m(L^2 + y^2)^{3/2}}$$

(b) A light with a mean wavelength of 550 nm is analyzed with a grating having 8 000 rulings/cm and projected onto a screen 2.40 m away. Calculate the dispersion in first order.

70. (a) Light traveling in a medium of index of refraction n_1 is incident at an angle θ on the surface of a medium of index n_2. The angle between reflected and refracted rays is β. Show that

$$\tan\theta = \frac{n_2 \sin\beta}{n_1 - n_2\cos\beta}$$

(b) **What If?** Show that this expression for $\tan\theta$ reduces to Brewster's law when $\beta = 90°$.

71. The intensity of light in a diffraction pattern of a single slit is described by the equation

$$I = I_{\max}\frac{\sin^2\phi}{\phi^2}$$

where $\phi = (\pi a \sin\theta)/\lambda$. The central maximum is at $\phi = 0$, and the side maxima are *approximately* at $\phi = (m + \frac{1}{2})\pi$ for $m = 1, 2, 3, \ldots$. Determine more precisely (a) the location of the first side maximum, where $m = 1$, and (b) the location of the second side maximum. *Suggestion:* Observe in Figure 38.6a that the graph of intensity versus ϕ has a horizontal tangent at maxima and also at minima.

72. How much diffraction spreading does a light beam undergo? One quantitative answer is the *full width at half maximum* of the central maximum of the single-slit Fraunhofer diffraction pattern. You can evaluate this angle of spreading in this problem. (a) In Equation 38.2, define $\phi = \pi a \sin\theta/\lambda$ and show that at the point where $I = 0.5I_{\max}$ we must have $\phi = \sqrt{2}\sin\phi$. (b) Let $y_1 = \sin\phi$ and $y_2 = \phi/\sqrt{2}$. Plot y_1 and y_2 on the same set of axes over a range from $\phi = 1$ rad to $\phi = \pi/2$ rad. Determine ϕ from the point of intersection of the two curves. (c) Then show that if the fraction λ/a is not large, the angular full width at half maximum of the central diffraction maximum is $\theta = 0.885\lambda/a$. (d) **What If?** Another method to solve the transcendental equation $\phi = \sqrt{2}\sin\phi$ in part (a) is to guess a first value of ϕ, use a computer or calculator to see how nearly it fits, and continue to update your estimate until the equation balances. How many steps (iterations) does this process take?

73. Two closely spaced wavelengths of light are incident on a diffraction grating. (a) Starting with Equation 38.7, show that the angular dispersion of the grating is given by

$$\frac{d\theta}{d\lambda} = \frac{m}{d\cos\theta}$$

(b) A square grating 2.00 cm on each side containing 8 000 equally spaced slits is used to analyze the spectrum of mercury. Two closely spaced lines emitted by this element have wavelengths of 579.065 nm and 576.959 nm. What is the angular separation of these two wavelengths in the second-order spectrum?

74. Light of wavelength 632.8 nm illuminates a single slit, and a diffraction pattern is formed on a screen 1.00 m from the slit. (a) Using the data in the following table, plot relative intensity versus position. Choose an appropriate value for the slit width a and, on the same graph used for the experimental data, plot the theoretical expression for the relative intensity

$$\frac{I}{I_{\max}} = \frac{\sin^2\phi}{\phi^2}$$

where $\phi = (\pi a \sin\theta)/\lambda$. (b) What value of a gives the best fit of theory and experiment?

Position Relative to Central Maximum (mm)	Relative Intensity
0	1.00
0.8	0.95
1.6	0.80
3.2	0.39
4.8	0.079
6.5	0.003
8.1	0.036
9.7	0.043
11.3	0.013
12.9	0.000 3
14.5	0.012
16.1	0.015
17.7	0.004 4
19.3	0.000 3

Challenge Problems

75. Figure P38.75a is a three-dimensional sketch of a birefringent crystal. The dotted lines illustrate how a thin, parallel-faced slab of material could be cut from the larger specimen with the crystal's optic axis parallel to the faces of the plate. A section cut from the crystal in this manner is known as a *retardation plate*. When a beam of light is incident on the plate perpendicular to the direction of the optic axis as shown in Figure P38.75b, the O ray and the E ray travel along a single straight line, but with different speeds. The figure shows the wave fronts for the two rays. (a) Let the thickness of the plate be d. Show that the phase difference between the O ray and the E ray after traveling the thickness of the plate is

$$\theta = \frac{2\pi d}{\lambda}|n_O - n_E|$$

where λ is the wavelength in air. (b) In a particular case, the incident light has a wavelength of 550 nm. Find the minimum value of d for a quartz plate for which $\theta = \pi/2$. Such a plate is called a *quarter-wave plate*. Use values of n_O and n_E from Table 38.1.

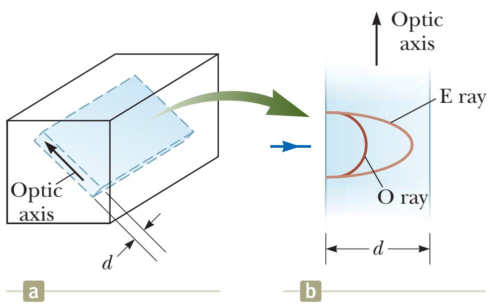


Figure P38.75

76. A spy satellite can consist of a large-diameter concave mirror forming an image on a digital-camera detector and sending the picture to a ground receiver by radio waves. In effect, it is an astronomical telescope in orbit, looking down instead of up. (a) Can a spy satellite read a license plate? (b) Can it read the date on a dime? Argue for your answers by making an order-of-magnitude calculation, specifying the data you estimate.

$\boxed{\textbf{77.}}$ Suppose the single slit in Figure 38.4 is 6.00 cm wide and in front of a microwave source operating at 7.50 GHz. (a) Calculate the angle for the first minimum in the diffraction pattern. (b) What is the relative intensity I/I_{max} at $\theta = 15.0°$? (c) Assume two such sources, separated laterally by 20.0 cm, are behind the slit. What must be the maximum distance between the plane of the sources and the slit if the diffraction patterns are to be resolved? In this case, the approximation $\sin\theta \approx \tan\theta$ is not valid because of the relatively small value of a/λ.

78. In Figure P38.78, suppose the transmission axes of the left and right polarizing disks are perpendicular to each other. Also, let the center disk be rotated on the common axis with an angular speed ω. Show that if unpolarized light is incident on the left disk with an intensity I_{max}, the intensity of the beam emerging from the right disk is

$$I = \tfrac{1}{16}I_{max}(1 - \cos 4\omega t)$$

This result means that the intensity of the emerging beam is modulated at a rate four times the rate of rotation of the center disk. *Suggestion:* Use the trigonometric identities $\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$ and $\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$.

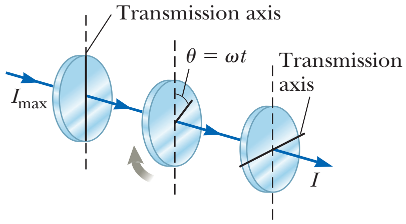


Figure P38.78

79. Consider a light wave passing through a slit and propagating toward a distant screen. Figure P38.79 shows the intensity variation for the pattern on the screen. Give a mathematical argument that more than 90% of the transmitted energy is in the central maximum of the diffraction pattern. *Suggestion:* You are not expected to calculate the precise percentage, but explain the steps of your reasoning. You may use the identification

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$$

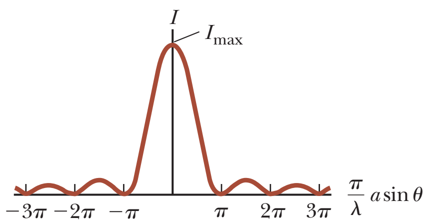


Figure P38.79