CHAPTER

37

Wave Optics

The colors in many of a hummingbird's feathers are not due to pigment. The *iridescence* that makes the brilliant colors that often appear on the bird's throat and belly is due to an interference effect caused by structures in the feathers. The colors will vary with the viewing angle. *(Dec Hogan/ Shutterstock.com)*

In Chapter 36, we studied light rays passing through a lens or reflecting from a mirror to describe the formation of images. This discussion completed our study of *ray optics.* In this chapter and in Chapter 38, we are concerned with *wave optics,* sometimes called *physical optics,* the study of interference, diffraction, and polarization of light. These phenomena cannot be adequately explained with the ray optics used in Chapters 35 and 36. We now learn how treating light as waves rather than as rays leads to a satisfying description of such phenomena.

37.1 Young's Double-Slit Experiment

In Chapter 18, we studied the waves in interference model and found that the superposition of two mechanical waves can be constructive or destructive. In constructive interference, the amplitude of the resultant wave is greater than that of either individual wave, whereas in destructive interference, the resultant amplitude is less than that of the larger wave. Light waves also interfere with one another. Fundamentally, all interference associated with light waves arises when the electromagnetic fields that constitute the individual waves combine.

Interference in light waves from two sources was first demonstrated by Thomas Young in 1801. A schematic diagram of the apparatus Young used is shown in Figure 37.1a. Plane light waves arrive at a barrier that contains two slits S_1 and S_2. The light from S_1 and S_2 produces on a viewing screen a visible pattern of bright and dark parallel bands called **fringes** (Fig. 37.1b). When the light from S_1 and that from S_2 both arrive at a point on the screen such that constructive interference occurs at

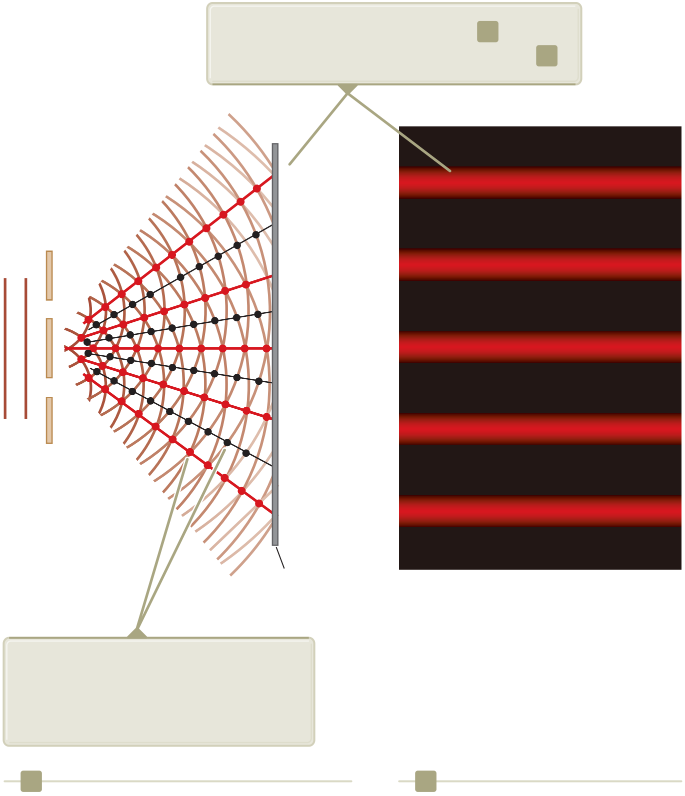


Figure 37.1 (a) Schematic dia gram of Young's double-slit experi ment. Slits S and S behave as coherent sources of light waves that produce an interference pattern on the viewing screen (drawing not to scale). (b) A simulation of an enlargement of the center of a fringe pattern formed on the view ing screen.

that location, a bright fringe appears. When the light from the two slits combines destructively at any location on the screen, a dark fringe results.

Figure 37.2 is a photograph looking down on an interference pattern produced on the surface of a water tank by two vibrating sources. The linear regions of con structive interference, such as at , and destructive interference, such as at , radi ating from the area between the sources are analogous to the red and black lines in Figure 37.1a.

Figure 37.3 on page 1136 shows some of the ways in which two waves can com bine at the screen. In Figure 37.3a, the two waves, which leave the two slits in phase, strike the screen at the central point Because both waves travel the same distance, they arrive at in phase. As a result, constructive interference occurs at this location and a bright fringe is observed. In Figure 37.3b, the two waves also start in phase, but here the lower wave has to travel one wavelength farther than the upper wave to reach point Because the lower wave falls behind

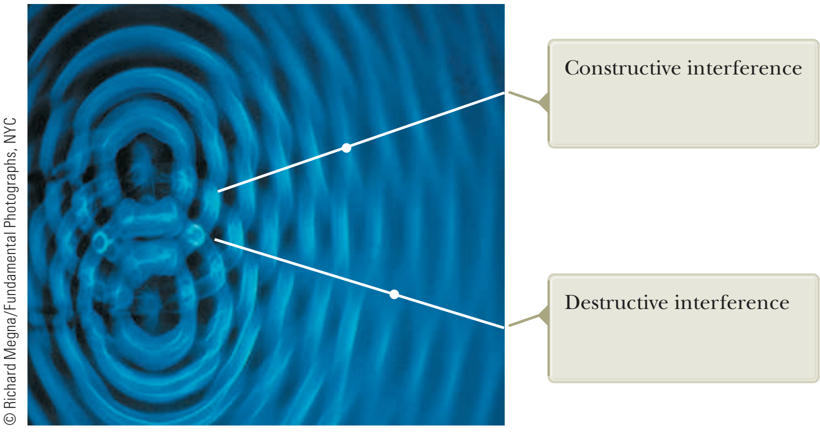


Figure 37.2 An interference pattern involving water waves is produced by two vibrating sources at the water's surface.

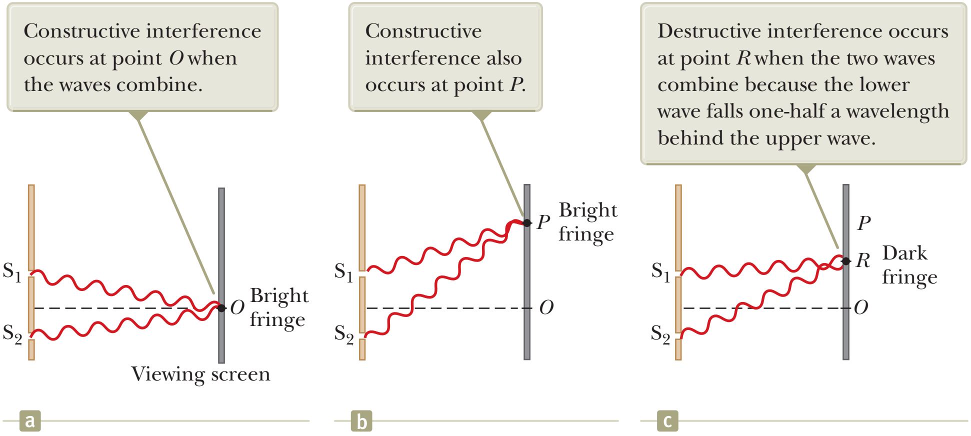


Figure 37.3 Waves leave the slits and combine at various points on the viewing screen. (All figures not to scale.)

the upper one by exactly one wavelength, they still arrive in phase at P and a second bright fringe appears at this location. At point R in Figure 37.3c, however, between points O and P, the lower wave has fallen half a wavelength behind the upper wave and a crest of the upper wave overlaps a trough of the lower wave, giving rise to destructive interference at point R. A dark fringe is therefore observed at this location.

If two lightbulbs are placed side by side so that light from both bulbs combines, no interference effects are observed because the light waves from one bulb are emitted independently of those from the other bulb. The emissions from the two lightbulbs do not maintain a constant phase relationship with each other over time. Light waves from an ordinary source such as a lightbulb undergo random phase changes in time intervals of less than a nanosecond. Therefore, the conditions for constructive interference, destructive interference, or some intermediate state are maintained only for such short time intervals. Because the eye cannot follow such rapid changes, no interference effects are observed. Such light sources are said to be **incoherent.**

To observe interference of waves from two sources, the following conditions must be met:

Conditions for interference ▶

- The sources must be **coherent;** that is, they must maintain a constant phase with respect to each other.
- The sources should be **monochromatic;** that is, they should be of a single wavelength.

As an example, single-frequency sound waves emitted by two side-by-side loudspeakers driven by a single amplifier can interfere with each other because the two speakers are coherent. In other words, they respond to the amplifier in the same way at the same time.

A common method for producing two coherent light sources is to use a monochromatic source to illuminate a barrier containing two small openings, usually in the shape of slits, as in the case of Young's experiment illustrated in Figure 37.1. The light emerging from the two slits is coherent because a single source produces the original light beam and the two slits serve only to separate the original beam into two parts (which, after all, is what is done to the sound signal from two side-by-side loudspeakers). Any random change in the light emitted by the source occurs in both beams at the same time. As a result, interference effects can be observed when the light from the two slits arrives at a viewing screen.

If the light traveled only in its original direction after passing through the slits as shown in Figure 37.4a, the waves would not overlap and no interference pattern would be seen. Instead, as we have discussed in our treatment of Huygens's principle (Section 35.6), the waves spread out from the slits as shown in Figure 37.4b. In other words, the light deviates from a straight-line path and enters the region that

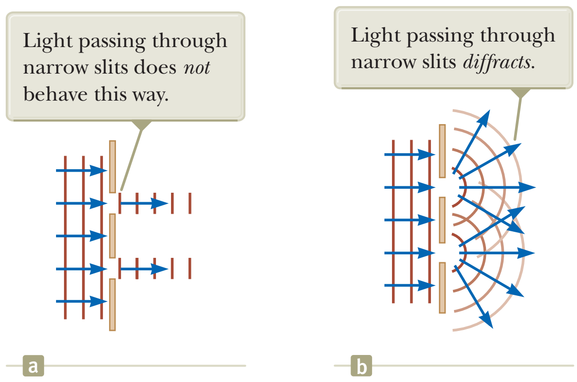


Figure 37.4 (a) If light waves did not spread out after passing through the slits, no interference would occur. (b) The light waves from the two slits overlap as they spread out, filling what we expect to be shadowed regions with light and producing interference fringes on a screen placed to the right of the slits.

would otherwise be shadowed. As noted in Section 35.3, this divergence of light from its initial line of travel is called **diffraction.**

37.2 Analysis Model: Waves in Interference

We discussed the superposition principle for waves on strings in Section 18.1, leading to a one-dimensional version of the waves in interference analysis model. In Example 18.1 on page 537, we briefly discussed a two-dimensional interference phenomenon for sound from two loudspeakers. In walking from point O to point P in Figure 18.5, the listener experienced a maximum in sound intensity at O and a minimum at P. This experience is exactly analogous to an observer looking at point O in Figure 37.3 and seeing a bright fringe and then sweeping his eyes upward to point R, where there is a minimum in light intensity.

Let's look in more detail at the two-dimensional nature of Young's experiment with the help of Figure 37.5. The viewing screen is located a perpendicular distance L from the barrier containing two slits, S_1 and S_2 (Fig. 37.5a). These slits are separated by a distance d, and the source is monochromatic. To reach any arbitrary point P in the upper half of the screen, a wave from the lower slit must travel farther than a wave from the upper slit. The extra distance traveled from the lower slit is the **path difference** δ (Greek letter delta). If we assume the rays labeled r_1 and r_2 are parallel (Fig. 37.5b), which is approximately true if L is much greater than d, then δ is given by

$$\delta = r_2 - r_1 = d \sin \theta \quad \textbf{(37.1)}$$

The value of δ determines whether the two waves are in phase when they arrive at point P. If δ is either zero or some integer multiple of the wavelength, the two waves

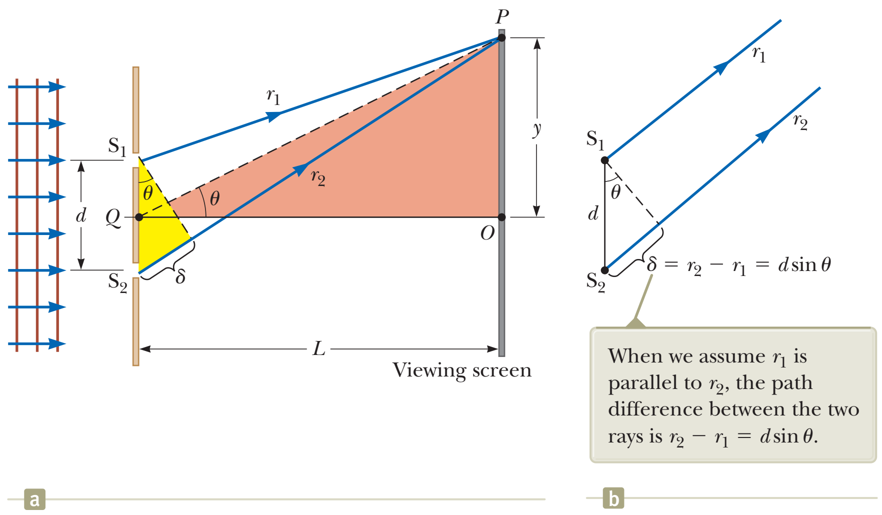


Figure 37.5 (a) Geometric construction for describing Young's double-slit experiment (not to scale). (b) The slits are represented as sources, and the outgoing light rays are assumed to be parallel as they travel to P. To achieve that in practice, it is essential that $L >> d$.

are in phase at point P and constructive interference results. Therefore, the condition for bright fringes, or **constructive interference,** at point P is

Condition for constructive interference

$$d \sin \theta_{\text{bright}} = m\lambda \qquad m = 0, \pm 1, \pm 2, \ldots \tag{37.2}$$

The number m is called the **order number.** For constructive interference, the order number is the same as the number of wavelengths that represents the path difference between the waves from the two slits. The central bright fringe at $\theta_{\text{bright}} = 0$ is called the *zeroth-order maximum*. The first maximum on either side, where $m = \pm 1$, is called the *first-order maximum,* and so forth.

When δ is an odd multiple of $\lambda/2$, the two waves arriving at point P are 180° out of phase and give rise to destructive interference. Therefore, the condition for dark fringes, or **destructive interference,** at point P is

Condition for destructive interference

$$d \sin \theta_{\text{dark}} = (m + \tfrac{1}{2})\lambda \qquad m = 0, \pm 1, \pm 2, \ldots \tag{37.3}$$

These equations provide the *angular* positions of the fringes. It is also useful to obtain expressions for the *linear* positions measured along the screen from O to P. From the triangle OPQ in Figure 37.5a, we see that

$$\tan \theta = \frac{y}{L} \tag{37.4}$$

Using this result, the linear positions of bright and dark fringes are given by

$$y_{\text{bright}} = L \tan \theta_{\text{bright}} \tag{37.5}$$

$$y_{\text{dark}} = L \tan \theta_{\text{dark}} \tag{37.6}$$

where θ_{bright} and θ_{dark} are given by Equations 37.2 and 37.3.

When the angles to the fringes are small, the positions of the fringes are linear near the center of the pattern. That can be verified by noting that for small angles, $\tan \theta \approx \sin \theta$, so Equation 37.5 gives the positions of the bright fringes as $y_{\text{bright}} = L \sin \theta_{\text{bright}}$. Incorporating Equation 37.2 gives

$$y_{\text{bright}} = L \frac{m\lambda}{d} \quad \text{(small angles)} \tag{37.7}$$

This result shows that y_{bright} is linear in the order number m, so the fringes are equally spaced for small angles. Similarly, for dark fringes,

$$y_{\text{dark}} = L \frac{(m + \frac{1}{2})\lambda}{d} \quad \text{(small angles)} \tag{37.8}$$

As demonstrated in Example 37.1, Young's double-slit experiment provides a method for measuring the wavelength of light. In fact, Young used this technique to do precisely that. In addition, his experiment gave the wave model of light a great deal of credibility. It was inconceivable that particles of light coming through the slits could cancel one another in a way that would explain the dark fringes.

The principles discussed in this section are the basis of the **waves in interference** analysis model. This model was applied to mechanical waves in one dimension in Chapter 18. Here we see the details of applying this model in three dimensions to light.

Quick Quiz 37.1 Which of the following causes the fringes in a two-slit interference pattern to move farther apart? **(a)** decreasing the wavelength of the light **(b)** decreasing the screen distance L **(c)** decreasing the slit spacing d **(d)** immersing the entire apparatus in water

Analysis Model — Waves in Interference

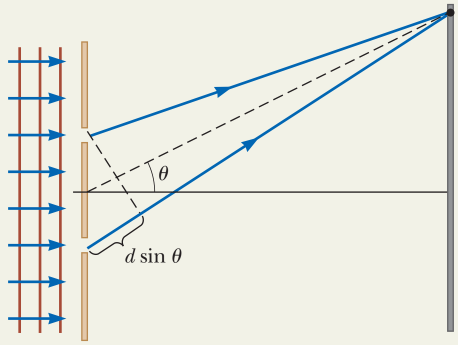
Imagine a broad beam of light that illuminates a double slit in an otherwise opaque material. An interference pattern of bright and dark fringes is created on a distant screen. The condition for bright fringes **(constructive interference)** is

$$d \sin \theta_{\text{bright}} = m\lambda \quad m = 0, \pm 1, \pm 2, \ldots \qquad \textbf{(37.2)}$$

The condition for dark fringes **(destructive interference)** is

$$d \sin \theta_{\text{dark}} = (m + \tfrac{1}{2})\lambda \quad m = 0, \pm 1, \pm 2, \ldots \qquad \textbf{(37.3)}$$

The number m is called the **order number** of the fringe.



Examples:

- a thin film of oil on top of water shows swirls of color (Section 37.5)
- x-rays passing through a crystalline solid combine to form a Laue pattern (Chapter 38)
- a Michelson interferometer (Section 37.6) is used to search for the ether representing the medium through which light travels (Chapter 39)
- electrons exhibit interference just like light waves when they pass through a double slit (Chapter 40)

Example 37.1 Measuring the Wavelength of a Light Source AM

A viewing screen is separated from a double slit by 4.80 m. The distance between the two slits is 0.030 0 mm. Monochromatic light is directed toward the double slit and forms an interference pattern on the screen. The first dark fringe is 4.50 cm from the center line on the screen.

(A) Determine the wavelength of the light.

SOLUTION

Conceptualize Study Figure 37.5 to be sure you understand the phenomenon of interference of light waves. The distance of 4.50 cm is y in Figure 37.5. Because $L >> y$, the angles for the fringes are small.

Categorize This problem is a simple application of the *waves in interference* model.

Analyze

Solve Equation 37.8 for the wavelength and substitute numerical values, taking $m = 0$ for the first dark fringe:

$$\lambda = \frac{y_{\text{dark}} d}{(m + \frac{1}{2})L} = \frac{(4.50 \times 10^{-2}\ \text{m})(3.00 \times 10^{-5}\ \text{m})}{(0 + \frac{1}{2})(4.80\ \text{m})}$$

$$= 5.62 \times 10^{-7}\ \text{m} = \boxed{562\ \text{nm}}$$

(B) Calculate the distance between adjacent bright fringes.

SOLUTION

Find the distance between adjacent bright fringes from Equation 37.7 and the results of part (A):

$$y_{m+1} - y_m = L\frac{(m+1)\lambda}{d} - L\frac{m\lambda}{d}$$

$$= L\frac{\lambda}{d} = 4.80\ \text{m}\left(\frac{5.62 \times 10^{-7}\ \text{m}}{3.00 \times 10^{-5}\ \text{m}}\right)$$

$$= 9.00 \times 10^{-2}\ \text{m} = \boxed{9.00\ \text{cm}}$$

Finalize For practice, find the wavelength of the sound in Example 18.1 using the procedure in part (A) of this example.

Example 37.2 **Separating Double-Slit Fringes of Two Wavelengths** AM

A light source emits visible light of two wavelengths: $\lambda = 430$ nm and $\lambda' = 510$ nm. The source is used in a double-slit interference experiment in which $L = 1.50$ m and $d = 0.025\,0$ mm. Find the separation distance between the third-order bright fringes for the two wavelengths.

SOLUTION

Conceptualize In Figure 37.5a, imagine light of two wavelengths incident on the slits and forming two interference patterns on the screen. At some points, the fringes of the two colors might overlap, but at most points, they will not.

Categorize This problem is an application of the mathematical representation of the *waves in interference* analysis model.

Analyze

Use Equation 37.7 to find the fringe positions corresponding to these two wavelengths and subtract them:

$$\Delta y = y'_{\text{bright}} - y_{\text{bright}} = L\frac{m\lambda'}{d} - L\frac{m\lambda}{d} = \frac{Lm}{d}(\lambda' - \lambda)$$

Substitute numerical values:

$$\Delta y = \frac{(1.50\text{ m})(3)}{0.025\,0 \times 10^{-3}\text{ m}}(510 \times 10^{-9}\text{ m} - 430 \times 10^{-9}\text{ m})$$

$$= 0.014\,4\text{ m} = 1.44\text{ cm}$$

Finalize Let's explore further details of the interference pattern in the following **What If?**

WHAT IF? What if we examine the entire interference pattern due to the two wavelengths and look for overlapping fringes? Are there any locations on the screen where the bright fringes from the two wavelengths overlap exactly?

Answer Find such a location by setting the location of any bright fringe due to λ equal to one due to λ', using Equation 37.7:

$$L\frac{m\lambda}{d} = L\frac{m'\lambda'}{d} \rightarrow \frac{m'}{m} = \frac{\lambda}{\lambda'}$$

Substitute the wavelengths:

$$\frac{m'}{m} = \frac{430\text{ nm}}{510\text{ nm}} = \frac{43}{51}$$

Therefore, the 51st fringe of the 430-nm light overlaps with the 43rd fringe of the 510-nm light.

Use Equation 37.7 to find the value of y for these fringes:

$$y = (1.50\text{ m})\left[\frac{51(430 \times 10^{-9}\text{ m})}{0.025\,0 \times 10^{-3}\text{ m}}\right] = 1.32\text{ m}$$

This value of y is comparable to L, so the small-angle approximation used for Equation 37.7 is *not* valid. This conclusion suggests we should not expect Equation 37.7 to give us the correct result. If you use Equation 37.5, you can show that the bright fringes do indeed overlap when the same condition, $m'/m = \lambda/\lambda'$, is met (see Problem 48). Therefore, the 51st fringe of the 430-nm light does overlap with the 43rd fringe of the 510-nm light, but not at the location of 1.32 m. You are asked to find the correct location as part of Problem 48.

37.3 Intensity Distribution of the Double-Slit Interference Pattern

Notice that the edges of the bright fringes in Figure 37.1b are not sharp; rather, there is a gradual change from bright to dark. So far, we have discussed the locations of only the centers of the bright and dark fringes on a distant screen. Let's now direct our attention to the intensity of the light at other points between the positions of maximum constructive and destructive interference. In other words, we now calculate the distribution of light intensity associated with the double-slit interference pattern.

Again, suppose the two slits represent coherent sources of sinusoidal waves such that the two waves from the slits have the same angular frequency ω and are in phase. The total magnitude of the electric field at point P on the screen in Figure 37.5 is the superposition of the two waves. Assuming the two waves have the same amplitude E_0, we can write the magnitude of the electric field at point P due to each wave separately as

$$E_1 = E_0 \sin \omega t \quad \text{and} \quad E_2 = E_0 \sin (\omega t + \phi) \tag{37.9}$$

Although the waves are in phase at the slits, their phase difference ϕ at P depends on the path difference $\delta = r_2 - r_1 = d \sin \theta$. A path difference of λ (for constructive interference) corresponds to a phase difference of 2π rad. A path difference of δ is the same fraction of λ as the phase difference ϕ is of 2π. We can describe this fraction mathematically with the ratio

$$\frac{\delta}{\lambda} = \frac{\phi}{2\pi}$$

which gives

$$\phi = \frac{2\pi}{\lambda}\delta = \frac{2\pi}{\lambda} d \sin \theta \tag{37.10}$$

Phase difference

This equation shows how the phase difference ϕ depends on the angle θ in Figure 37.5.

Using the superposition principle and Equation 37.9, we obtain the following expression for the magnitude of the resultant electric field at point P:

$$E_P = E_1 + E_2 = E_0[\sin \omega t + \sin (\omega t + \phi)] \tag{37.11}$$

We can simplify this expression by using the trigonometric identity

$$\sin A + \sin B = 2 \sin\left(\frac{A + B}{2}\right) \cos\left(\frac{A - B}{2}\right)$$

Taking $A = \omega t + \phi$ and $B = \omega t$, Equation 37.11 becomes

$$E_P = 2E_0 \cos\left(\frac{\phi}{2}\right) \sin\left(\omega t + \frac{\phi}{2}\right) \tag{37.12}$$

This result indicates that the electric field at point P has the same frequency ω as the light at the slits but that the amplitude of the field is multiplied by the factor $2 \cos (\phi/2)$. To check the consistency of this result, note that if $\phi = 0, 2\pi, 4\pi, \ldots$, the magnitude of the electric field at point P is $2E_0$, corresponding to the condition for maximum constructive interference. These values of ϕ are consistent with Equation 37.2 for constructive interference. Likewise, if $\phi = \pi, 3\pi, 5\pi, \ldots$, the magnitude of the electric field at point P is zero, which is consistent with Equation 37.3 for total destructive interference.

Finally, to obtain an expression for the light intensity at point P, recall from Section 34.4 that the intensity of a wave is proportional to the square of the resultant electric field magnitude at that point (Eq. 34.24). Using Equation 37.12, we can therefore express the light intensity at point P as

$$I \propto E_P^2 = 4E_0^2 \cos^2\left(\frac{\phi}{2}\right) \sin^2\left(\omega t + \frac{\phi}{2}\right)$$

Most light-detecting instruments measure time-averaged light intensity, and the time-averaged value of $\sin^2 (\omega t + \phi/2)$ over one cycle is $\frac{1}{2}$. (See Fig. 33.5.) Therefore, we can write the average light intensity at point P as

$$I = I_{\text{max}} \cos^2\left(\frac{\phi}{2}\right) \tag{37.13}$$

where I_{max} is the maximum intensity on the screen and the expression represents the time average. Substituting the value for ϕ given by Equation 37.10 into this expression gives

$$I = I_{max} \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right) \qquad (37.14)$$

Alternatively, because $\sin\theta \approx y/L$ for small values of θ in Figure 37.5, we can write Equation 37.14 in the form

$$I = I_{max} \cos^2\left(\frac{\pi d}{\lambda L} y\right) \quad \text{(small angles)} \qquad (37.15)$$

Constructive interference, which produces light intensity maxima, occurs when the quantity $\pi dy/\lambda L$ is an integral multiple of π, corresponding to $y = (\lambda L/d)m$, where m is the order number. This result is consistent with Equation 37.7.

A plot of light intensity versus $d \sin\theta$ is given in Figure 37.6. The interference pattern consists of equally spaced fringes of equal intensity.

Figure 37.7 shows similar plots of light intensity versus $d \sin\theta$ for light passing through multiple slits. For more than two slits, we would add together more electric field magnitudes than the two in Equation 37.9. In this case, the pattern contains primary and secondary maxima. For three slits, notice that the primary maxima are nine times more intense than the secondary maxima as measured by the height of the curve because the intensity varies as E^2. For N slits, the intensity of the primary maxima is N^2 times greater than that for the secondary maxima. As the number of slits increases, the primary maxima increase in intensity and become narrower, while the secondary maxima decrease in intensity relative to the primary maxima. Figure 37.7 also shows that as the number of slits increases, the number of secondary maxima also increases. In fact, the number of secondary maxima is always $N - 2$, where N is the number of slits. In Section 38.4, we shall investigate the pattern for a very large number of slits in a device called a *diffraction grating*.

Quick Quiz 37.2 Using Figure 37.7 as a model, sketch the interference pattern from six slits.

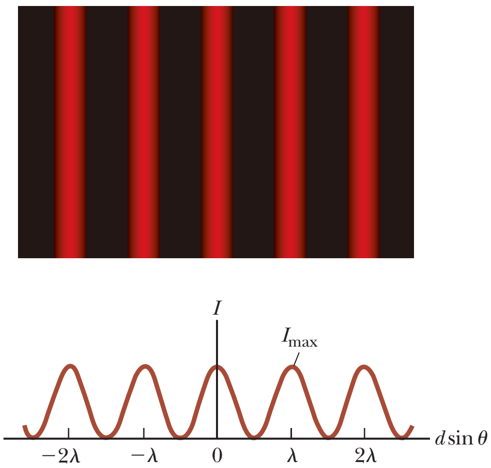


Figure 37.6 Light intensity versus $d \sin\theta$ for a double-slit interference pattern when the screen is far from the two slits ($L >> d$).

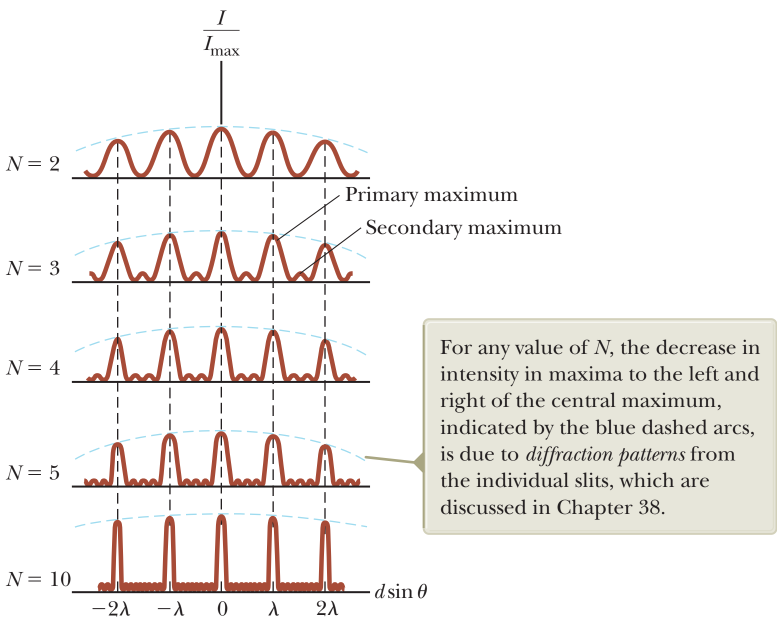


Figure 37.7 Multiple-slit interference patterns. As N, the number of slits, is increased, the primary maxima (the tallest peaks in each graph) become narrower but remain fixed in position and the number of secondary maxima increases.

37.4 Change of Phase Due to Reflection

Young's method for producing two coherent light sources involves illuminating a pair of slits with a single source. Another simple, yet ingenious, arrangement for producing an interference pattern with a single light source is known as *Lloyd's mirror*[1] (Fig. 37.8). A point light source S is placed close to a mirror, and a viewing screen is positioned some distance away and perpendicular to the mirror. Light waves can reach point P on the screen either directly from S to P or by the path involving reflection from the mirror. The reflected ray can be treated as a ray originating from a virtual source S′. As a result, we can think of this arrangement as a double-slit source where the distance d between sources S and S′ in Figure 37.8 is analogous to length d in Figure 37.5. Hence, at observation points far from the source ($L >> d$), we expect waves from S and S′ to form an interference pattern exactly like the one formed by two real coherent sources. An interference pattern is indeed observed. The positions of the dark and bright fringes, however, are reversed relative to the pattern created by two real coherent sources (Young's experiment). Such a reversal can only occur if the coherent sources S and S′ differ in phase by 180°.

To illustrate further, consider point P', the point where the mirror intersects the screen. This point is equidistant from sources S and S′. If path difference alone were responsible for the phase difference, we would see a bright fringe at P' (because the path difference is zero for this point), corresponding to the central bright fringe of the two-slit interference pattern. Instead, a dark fringe is observed at P'. We therefore conclude that a 180° phase change must be produced by reflection from the mirror. In general, an electromagnetic wave undergoes a phase change of 180° upon reflection from a medium that has a higher index of refraction than the one in which the wave is traveling.

It is useful to draw an analogy between reflected light waves and the reflections of a transverse pulse on a stretched string (Section 16.4). The reflected pulse on a string undergoes a phase change of 180° when reflected from the boundary of a denser string or a rigid support, but no phase change occurs when the pulse is reflected from the boundary of a less dense string or a freely-supported end. Similarly, an electromagnetic wave undergoes a 180° phase change when reflected from a boundary leading to an optically denser medium (defined as a medium with a higher index of refraction), but no phase change occurs when the wave is reflected from a boundary leading to a less dense medium. These rules, summarized in Figure 37.9, can be deduced from Maxwell's equations, but the treatment is beyond the scope of this text.

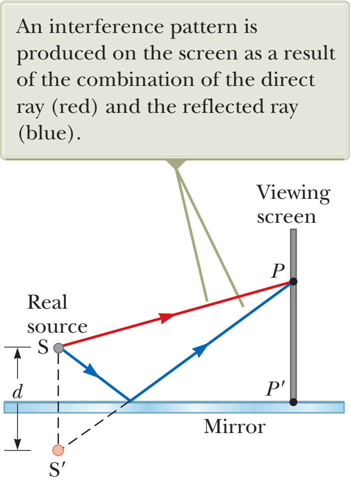


Figure 37.8 Lloyd's mirror. The reflected ray undergoes a phase change of 180°.

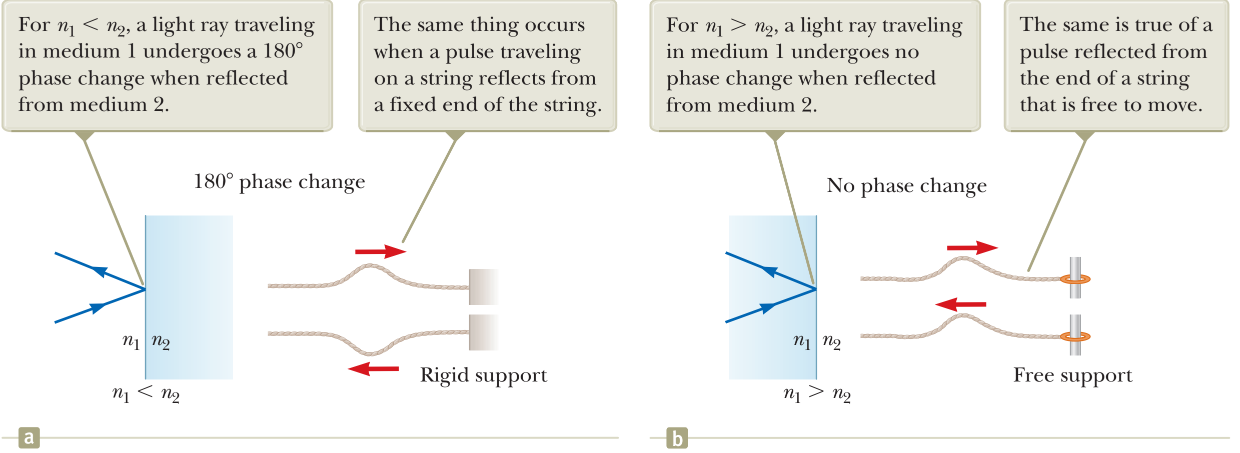


Figure 37.9 Comparisons of reflections of light waves and waves on strings.

[1]Developed in 1834 by Humphrey Lloyd (1800–1881), Professor of Natural and Experimental Philosophy, Trinity College, Dublin.

37.5 Interference in Thin Films

Interference effects are commonly observed in thin films, such as thin layers of oil on water or the thin surface of a soap bubble. The varied colors observed when white light is incident on such films result from the interference of waves reflected from the two surfaces of the film.

Consider a film of uniform thickness t and index of refraction n. The wavelength of light λ_n in the film (see Section 35.5) is

$$\lambda_n = \frac{\lambda}{n}$$

where λ is the wavelength of the light in free space and n is the index of refraction of the film material. Let's assume light rays traveling in air are nearly normal to the two surfaces of the film as shown in Figure 37.10.

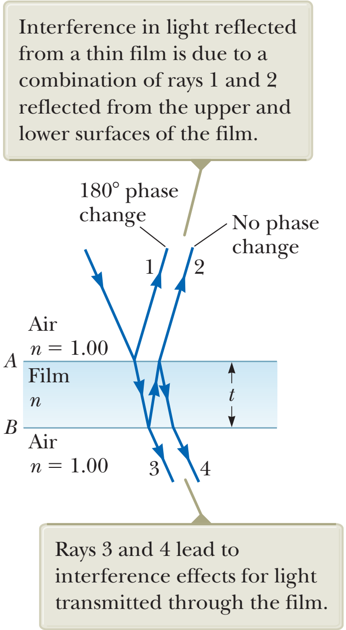


Figure 37.10 Light paths through a thin film.

Reflected ray 1, which is reflected from the upper surface (A) in Figure 37.10, undergoes a phase change of 180° with respect to the incident wave. Reflected ray 2, which is reflected from the lower film surface (B), undergoes no phase change because it is reflected from a medium (air) that has a lower index of refraction. Therefore, ray 1 is 180° out of phase with ray 2, which is equivalent to a path difference of $\lambda_n/2$. We must also consider, however, that ray 2 travels an extra distance $2t$ before the waves recombine in the air above surface A. (Remember that we are considering light rays that are close to normal to the surface. If the rays are not close to normal, the path difference is larger than $2t$.) If $2t = \lambda_n/2$, rays 1 and 2 recombine in phase and the result is constructive interference. In general, the condition for *constructive* interference in thin films is[2]

$$2t = (m + \tfrac{1}{2})\lambda_n \quad m = 0, 1, 2, \ldots \tag{37.16}$$

This condition takes into account two factors: (1) the difference in path length for the two rays (the term $m\lambda_n$) and (2) the 180° phase change upon reflection (the term $\frac{1}{2}\lambda_n$). Because $\lambda_n = \lambda/n$, we can write Equation 37.16 as

$$2nt = (m + \tfrac{1}{2})\lambda \quad m = 0, 1, 2, \ldots \tag{37.17}$$

If the extra distance $2t$ traveled by ray 2 corresponds to a multiple of λ_n, the two waves combine out of phase and the result is destructive interference. The general equation for *destructive* interference in thin films is

$$2nt = m\lambda \quad m = 0, 1, 2, \ldots \tag{37.18}$$

The foregoing conditions for constructive and destructive interference are valid when the medium above the top surface of the film is the same as the medium below the bottom surface or, if there are different media above and below the film, the index of refraction of both is less than n. If the film is placed between two different media, one with $n < n_{\text{film}}$ and the other with $n > n_{\text{film}}$, the conditions for constructive and destructive interference are reversed. In that case, either there is a phase change of 180° for both ray 1 reflecting from surface A and ray 2 reflecting from surface B or there is no phase change for either ray; hence, the net change in relative phase due to the reflections is zero.

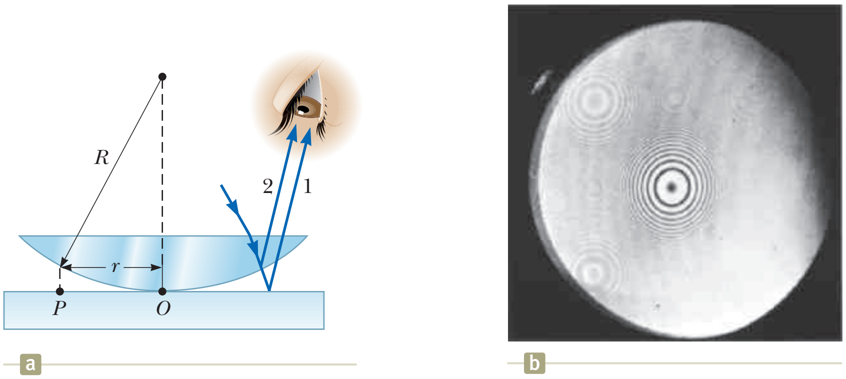
Rays 3 and 4 in Figure 37.10 lead to interference effects in the light transmitted through the thin film. The analysis of these effects is similar to that of the reflected light. You are asked to explore the transmitted light in Problems 35, 36, and 38.

Pitfall Prevention 37.1

Be Careful with Thin Films Be sure to include *both* effects—path length and phase change—when analyzing an interference pattern resulting from a thin film. The possible phase change is a new feature we did not need to consider for double-slit interference. Also think carefully about the material on either side of the film. If there are different materials on either side of the film, you may have a situation in which there is a 180° phase change at *both* surfaces or at *neither* surface.

Quick Quiz 37.3 One microscope slide is placed on top of another with their left edges in contact and a human hair under the right edge of the upper slide. As a result, a wedge of air exists between the slides. An interference pattern results when monochromatic light is incident on the wedge. What is at the left edges of the slides? **(a)** a dark fringe **(b)** a bright fringe **(c)** impossible to determine

[2]The full interference effect in a thin film requires an analysis of an infinite number of reflections back and forth between the top and bottom surfaces of the film. We focus here only on a single reflection from the bottom of the film, which provides the largest contribution to the interference effect.



Courtesy of Bausch and Lomb

Figure 37.11 (a) The combination of rays reflected from the flat plate and the curved lens surface gives rise to an interference pattern known as Newton's rings. (b) Photograph of Newton's rings.

Newton's Rings

Another method for observing interference in light waves is to place a plano-convex lens on top of a flat glass surface as shown in Figure 37.11a. With this arrangement, the air film between the glass surfaces varies in thickness from zero at the point of contact to some nonzero value at point P. If the radius of curvature R of the lens is much greater than the distance r and the system is viewed from above, a pattern of light and dark rings is observed as shown in Figure 37.11b. These circular fringes, discovered by Newton, are called **Newton's rings.**

The interference effect is due to the combination of ray 1, reflected from the flat plate, with ray 2, reflected from the curved surface of the lens. Ray 1 undergoes a phase change of 180° upon reflection (because it is reflected from a medium of higher index of refraction), whereas ray 2 undergoes no phase change (because it is reflected from a medium of lower index of refraction). Hence, the conditions for constructive and destructive interference are given by Equations 37.17 and 37.18, respectively, with $n = 1$ because the film is air. Because there is no path difference and the total phase change is due only to the 180° phase change upon reflection, the contact point at O is dark as seen in Figure 37.11b.

Using the geometry shown in Figure 37.11a, we can obtain expressions for the radii of the bright and dark bands in terms of the radius of curvature R and wavelength λ. For example, the dark rings have radii given by the expression $r \approx \sqrt{m\lambda R/n}$. The details are left as a problem (see Problem 66). We can obtain the wavelength of the light causing the interference pattern by measuring the radii of the rings, provided R is known. Conversely, we can use a known wavelength to obtain R.

One important use of Newton's rings is in the testing of optical lenses. A circular pattern like that pictured in Figure 37.11b is obtained only when the lens is ground to a perfectly symmetric curvature. Variations from such symmetry produce a pattern with fringes that vary from a smooth, circular shape. These variations indicate how the lens must be reground and repolished to remove imperfections.

Peter Aprahamian/Photo Researchers, Inc.

Dr. Jeremy Burgess/Science Photo Library/Photo Researchers, Inc.

(a) A thin film of oil floating on water displays interference, shown by the pattern of colors when white light is incident on the film. Variations in film thickness produce the interesting color pattern. The razor blade gives you an idea of the size of the colored bands. (b) Interference in soap bubbles. The colors are due to interference between light rays reflected from the inner and outer surfaces of the thin film of soap making up the bubble. The color depends on the thickness of the film, ranging from black, where the film is thinnest, to magenta, where it is thickest.

Problem-Solving Strategy Thin-Film Interference

The following features should be kept in mind when working thin-film interference problems.

1. Conceptualize. Think about what is going on physically in the problem. Identify the light source and the location of the observer.

2. Categorize. Confirm that you should use the techniques for thin-film interference by identifying the thin film causing the interference.

3. Analyze. The type of interference that occurs is determined by the phase relationship between the portion of the wave reflected at the upper surface of the film and the portion reflected at the lower surface. Phase differences between the two portions of the wave have two causes: differences in the distances traveled by the two portions and phase changes occurring on reflection. *Both* causes must be considered when determining which type of interference occurs. If the media above and below the film both have index of refraction larger than that of the film or if both indices are smaller, use Equation 37.17 for constructive interference and Equation 37.18 for destructive interference. If the film is located between two different media, one with $n < n_{\text{film}}$ and the other with $n > n_{\text{film}}$, reverse these two equations for constructive and destructive interference.

4. Finalize. Inspect your final results to see if they make sense physically and are of an appropriate size.

Example 37.3 Interference in a Soap Film

Calculate the minimum thickness of a soap-bubble film that results in constructive interference in the reflected light if the film is illuminated with light whose wavelength in free space is $\lambda = 600$ nm. The index of refraction of the soap film is 1.33.

SOLUTION

Conceptualize Imagine that the film in Figure 37.10 is soap, with air on both sides.

Categorize We determine the result using an equation from this section, so we categorize this example as a substitution problem.

The minimum film thickness for constructive interference in the reflected light corresponds to $m = 0$ in Equation 37.17. Solve this equation for t and substitute numerical values:

$$t = \frac{(0 + \frac{1}{2})\lambda}{2n} = \frac{\lambda}{4n} = \frac{(600 \text{ nm})}{4(1.33)} = \boxed{113 \text{ nm}}$$

WHAT IF? What if the film is twice as thick? Does this situation produce constructive interference?

Answer Using Equation 37.17, we can solve for the thicknesses at which constructive interference occurs:

$$t = (m + \tfrac{1}{2})\frac{\lambda}{2n} = (2m + 1)\frac{\lambda}{4n} \quad m = 0, 1, 2, \ldots$$

The allowed values of m show that constructive interference occurs for *odd* multiples of the thickness corresponding to $m = 0$, $t = 113$ nm. Therefore, constructive interference does *not* occur for a film that is twice as thick.

Example 37.4 Nonreflective Coatings for Solar Cells

Solar cells—devices that generate electricity when exposed to sunlight—are often coated with a transparent, thin film of silicon monoxide (SiO, $n = 1.45$) to minimize reflective losses from the surface. Suppose a silicon solar cell ($n = 3.5$) is coated with a thin film of silicon monoxide for this purpose (Fig. 37.12a). Determine the minimum film thickness that produces the least reflection at a wavelength of 550 nm, near the center of the visible spectrum.

▶ **37.4** continued

SOLUTION

Conceptualize Figure 37.12a helps us visualize the path of the rays in the SiO film that result in interference in the reflected light.

Categorize Based on the geometry of the SiO layer, we categorize this example as a thin-film interference problem.

Analyze The reflected light is a minimum when rays 1 and 2 in Figure 37.12a meet the condition of destructive interference. In this situation, *both* rays undergo a 180° phase change upon reflection: ray 1 from the upper SiO surface and ray 2 from the lower SiO surface. The net change in phase due to reflection is therefore zero, and the condition for a reflection minimum requires a path difference of $\lambda_n/2$, where λ_n is the wavelength of the light in SiO. Hence, $2nt = \lambda/2$, where λ is the wavelength in air and n is the index of refraction of SiO.

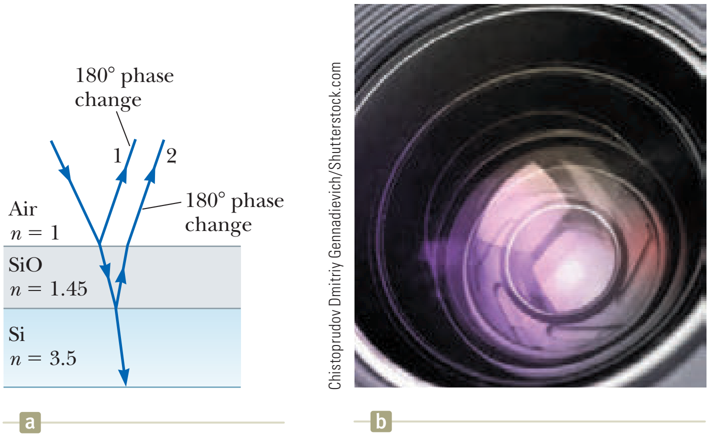


Figure 37.12 (Example 37.4) (a) Reflective losses from a silicon solar cell are minimized by coating the surface of the cell with a thin film of silicon monoxide. (b) The reflected light from a coated camera lens often has a reddish-violet appearance.

Solve the equation $2nt = \lambda/2$ for t and substitute numerical values:

$$t = \frac{\lambda}{4n} = \frac{550 \text{ nm}}{4(1.45)} = \boxed{94.8 \text{ nm}}$$

Finalize A typical uncoated solar cell has reflective losses as high as 30%, but a coating of SiO can reduce this value to about 10%. This significant decrease in reflective losses increases the cell's efficiency because less reflection means that more sunlight enters the silicon to create charge carriers in the cell. No coating can ever be made perfectly nonreflecting because the required thickness is wavelength-dependent and the incident light covers a wide range of wavelengths.

Glass lenses used in cameras and other optical instruments are usually coated with a transparent thin film to reduce or eliminate unwanted reflection and to enhance the transmission of light through the lenses. The camera lens in Figure 37.12b has several coatings (of different thicknesses) to minimize reflection of light waves having wavelengths near the center of the visible spectrum. As a result, the small amount of light that is reflected by the lens has a greater proportion of the far ends of the spectrum and often appears reddish violet.

37.6 The Michelson Interferometer

The **interferometer,** invented by American physicist A. A. Michelson (1852–1931), splits a light beam into two parts and then recombines the parts to form an interference pattern. The device can be used to measure wavelengths or other lengths with great precision because a large and precisely measurable displacement of one of the mirrors is related to an exactly countable number of wavelengths of light.

A schematic diagram of the interferometer is shown in Figure 37.13 (page 1148). A ray of light from a monochromatic source is split into two rays by mirror M_0, which is inclined at 45° to the incident light beam. Mirror M_0, called a *beam splitter,* transmits half the light incident on it and reflects the rest. One ray is reflected from M_0 to the right toward mirror M_1, and the second ray is transmitted vertically through M_0 toward mirror M_2. Hence, the two rays travel separate paths L_1 and L_2. After reflecting from M_1 and M_2, the two rays eventually recombine at M_0 to produce an interference pattern, which can be viewed through a telescope.

The interference condition for the two rays is determined by the difference in their path length. When the two mirrors are exactly perpendicular to each other, the interference pattern is a target pattern of bright and dark circular fringes. As M_1 is moved, the fringe pattern collapses or expands, depending on the direction in which M_1 is moved. For example, if a dark circle appears at the center of the

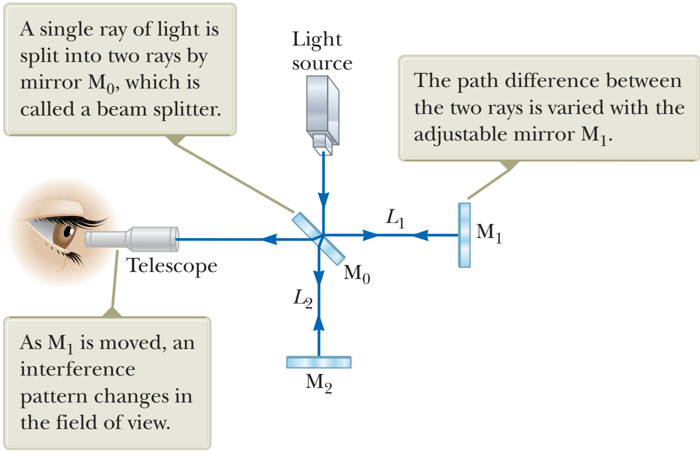


Figure 37.13 Diagram of the Michelson interferometer.

target pattern (corresponding to destructive interference) and M_1 is then moved a distance $\lambda/4$ toward M_0, the path difference changes by $\lambda/2$. What was a dark circle at the center now becomes a bright circle. As M_1 is moved an additional distance $\lambda/4$ toward M_0, the bright circle becomes a dark circle again. Therefore, the fringe pattern shifts by one-half fringe each time M_1 is moved a distance $\lambda/4$. The wavelength of light is then measured by counting the number of fringe shifts for a given displacement of M_1. If the wavelength is accurately known, mirror displacements can be measured to within a fraction of the wavelength.

We will see an important historical use of the Michelson interferometer in our discussion of relativity in Chapter 39. Modern uses include the following two applications, Fourier transform infrared spectroscopy and the laser interferometer gravitational-wave observatory.

Fourier Transform Infrared Spectroscopy

Spectroscopy is the study of the wavelength distribution of radiation from a sample that can be used to identify the characteristics of atoms or molecules in the sample. Infrared spectroscopy is particularly important to organic chemists when analyzing organic molecules. Traditional spectroscopy involves the use of an optical element, such as a prism (Section 35.5) or a diffraction grating (Section 38.4), which spreads out various wavelengths in a complex optical signal from the sample into different angles. In this way, the various wavelengths of radiation and their intensities in the signal can be determined. These types of devices are limited in their resolution and effectiveness because they must be scanned through the various angular deviations of the radiation.

The technique of *Fourier transform infrared* (FTIR) *spectroscopy* is used to create a higher-resolution spectrum in a time interval of 1 second that may have required 30 minutes with a standard spectrometer. In this technique, the radiation from a sample enters a Michelson interferometer. The movable mirror is swept through the zero-path-difference condition, and the intensity of radiation at the viewing position is recorded. The result is a complex set of data relating light intensity as a function of mirror position, called an *interferogram*. Because there is a relationship between mirror position and light intensity for a given wavelength, the interferogram contains information about all wavelengths in the signal.

In Section 18.8, we discussed Fourier analysis of a waveform. The waveform is a function that contains information about all the individual frequency components that make up the waveform.[3] Equation 18.13 shows how the waveform is generated from the individual frequency components. Similarly, the interferogram can be

[3]In acoustics, it is common to talk about the components of a complex signal in terms of frequency. In optics, it is more common to identify the components by wavelength.

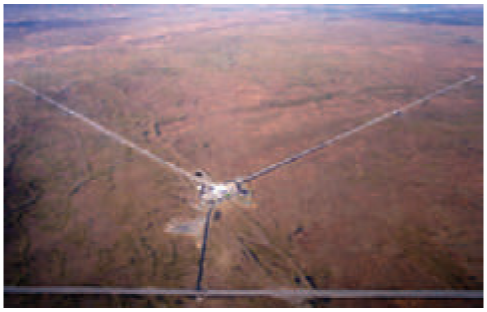


Figure 37.14 The Laser Interferometer Gravitational-Wave Observatory (LIGO) near Richland, Washington. Notice the two perpendicular arms of the Michelson interferometer.

analyzed by computer, in a process called a *Fourier transform,* to provide all the wavelength components. This information is the same as that generated by traditional spectroscopy, but the resolution of FTIR spectroscopy is much higher.

Laser Interferometer Gravitational-Wave Observatory

Einstein's general theory of relativity (Section 39.9) predicts the existence of *gravitational waves.* These waves propagate from the site of any gravitational disturbance, which could be periodic and predictable, such as the rotation of a double star around a center of mass, or unpredictable, such as the supernova explosion of a massive star.

In Einstein's theory, gravitation is equivalent to a distortion of space. Therefore, a gravitational disturbance causes an additional distortion that propagates through space in a manner similar to mechanical or electromagnetic waves. When gravitational waves from a disturbance pass by the Earth, they create a distortion of the local space. The laser interferometer gravitational-wave observatory (LIGO) apparatus is designed to detect this distortion. The apparatus employs a Michelson interferometer that uses laser beams with an effective path length of several kilometers. At the end of an arm of the interferometer, a mirror is mounted on a massive pendulum. When a gravitational wave passes by, the pendulum and the attached mirror move and the interference pattern due to the laser beams from the two arms changes.

Two sites for interferometers have been developed in the United States—in Richland, Washington, and in Livingston, Louisiana—to allow coincidence studies of gravitational waves. Figure 37.14 shows the Washington site. The two arms of the Michelson interferometer are evident in the photograph. Six data runs have been performed as of 2010. These runs have been coordinated with other gravitational wave detectors, such as GEO in Hannover, Germany, TAMA in Mitaka, Japan, and VIRGO in Cascina, Italy. So far, gravitational waves have not yet been detected, but the data runs have provided critical information for modifications and design features for the next generation of detectors. The original detectors are currently being dismantled, in preparation for the installation of Advanced LIGO, an upgrade that should increase the sensitivity of the observatory by a factor of 10. The target date for the beginning of scientific operation of Advanced LIGO is 2014.

Summary

Concepts and Principles

Interference in light waves occurs whenever two or more waves overlap at a given point. An interference pattern is observed if (1) the sources are coherent and (2) the sources have identical wavelengths.

The **intensity** at a point in a double-slit interference pattern is

$$I = I_{\max} \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right) \quad \textbf{(37.14)}$$

where $I_{\max}$ is the maximum intensity on the screen and the expression represents the time average.

continued

A wave traveling from a medium of index of refraction toward a medium of index of refraction undergoes a 180° phase change upon reflection when and undergoes no phase change when

The condition for constructive interference in a film of thickness and index of refraction surrounded by air is

$$nt \quad - \quad 0, 1, 2, \qquad \textbf{(37.17)}$$

where is the wavelength of the light in free space.

Similarly, the condition for destructive interference in a thin film surrounded by air is

$$0, 1, 2, \qquad \textbf{(37.18)}$$

Analysis Models for Problem Solving

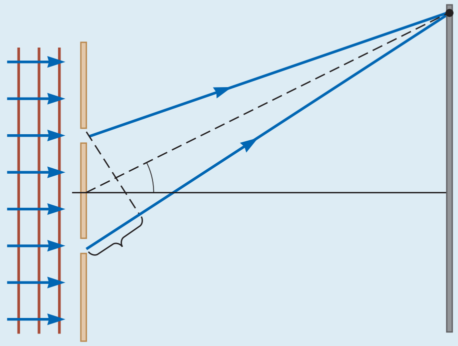
Waves in Interference. Young's double-slit experiment serves as a pro totype for interference phenomena involving electromagnetic radiation. In this experiment, two slits separated by a distance are illuminated by a single-wavelength light source. The condition for bright fringes **(constructive interference)**

$$\sin \ _{\text{bright}} \quad 0, \quad 1, \quad 2, \qquad \textbf{(37.2)}$$

The condition for dark fringes **(destructive interference)**

$$\sin \ _{\text{dark}} \quad - \quad 0, \quad 1, \quad 2, \qquad \textbf{(37.3)}$$

The number is called the **order number** of the fringe.



Objective Questions

While using a Michelson interferometer (shown in Fig. 37.13), you see a dark circle at the center of the inter ference pattern. **(i)** As you gradually move the light source toward the central mirror M , through a dis tance /2, what do you see? (a) There is no change in the pattern. (b) The dark circle changes into a bright circle. (c) The dark circle changes into a bright circle and then back into a dark circle. (d) The dark circle changes into a bright circle, then into a dark circle, and then into a bright circle. **(ii)** As you gradually move the moving mirror toward the central mirror M , through a distance /2, what do you see? Choose from the same possibilities.

2. Four trials of Young's double-slit experiment are con ducted. (a) In the first trial, blue light passes through two fine slits 400 m apart and forms an interference pattern on a screen 4 m away. (b) In a second trial, red light passes through the same slits and falls on the same screen. (c) A third trial is performed with red light and the same screen, but with slits 800 apart. (d) A final trial is performed with red light, slits 800 m apart, and a screen 8 m away. **(i)** Rank the trials (a) through (d) from the largest to the smallest value of the angle between the central maximum and the first-order side maximum. In your ranking, note any cases of equality. **(ii)** Rank the same trials accord ing to the distance between the central maximum and the first-order side maximum on the screen.

3. Suppose Young's double-slit experiment is performed in air using red light and then the apparatus is immersed in water. What happens to the interference pattern on the screen? (a) It disappears. (b) The bright and dark fringes stay in the same locations, but the contrast is reduced. (c) The bright fringes are closer together. (d) The bright fringes are farther apart. (e) No change happens in the interference pattern.

4. Green light has a wavelength of 500 nm in air. **(i)** Assume green light is reflected from a mirror with angle of incidence 0°. The incident and reflected waves together constitute a standing wave with what distance from one node to the next node? (a) 1 000 nm (b) 500 nm (c) 250 nm (d) 125 nm (e) 62.5 nm **(ii).** The green light is sent into a Michelson interferometer that is adjusted to produce a central bright circle. How far must the interferometer's moving mirror be shifted to change the center of the pattern into a dark circle? Choose from the same possibilities as in part (i). **(iii).** The green light is reflected perpendicularly from a thin film of a plastic with an index of refraction 2.00. The film appears bright in the reflected light. How much addi tional thickness would make the film appear dark?

5. A thin layer of oil (1.25) is floating on water (1.33). What is the minimum nonzero thickness of the oil in the region that strongly reflects green light (530 nm)? (a) 500 nm (b) 313 nm (c) 404 nm (d) 212 nm (e) 285 nm

6. A monochromatic beam of light of wavelength 500 nm illuminates a double slit having a slit separation of 2.00 m. What is the angle of the second-order bright fringe? (a) 0.050 0 rad (b) 0.025 0 rad (c) 0.100 rad (d) 0.250 rad (e) 0.010 0 rad

7. According to Table 35.1, the index of refraction of flint glass is 1.66 and the index of refraction of crown glass is 1.52. **(i)** A film formed by one drop of sassafras oil, on a horizontal surface of a flint glass block, is viewed by reflected light. The film appears brightest at its outer margin, where it is thinnest. A film of the same oil on crown glass appears dark at its outer margin. What can you say about the index of refraction of the oil? (a) It must be less than 1.52. (b) It must be between 1.52 and 1.66. (c) It must be greater than 1.66. (d) None of those statements is necessarily true. **(ii)** Could a very thin film of some other liquid appear bright by reflected light on both of the glass blocks? **(iii)** Could it appear dark on both? **(iv)** Could it appear dark on crown glass and bright on flint glass? Experiments described by Thomas Young suggested this question.

8. Suppose you perform Young's double-slit experiment with the slit separation slightly smaller than the wave length of the light. As a screen, you use a large half-cylinder with its axis along the midline between the slits. What interference pattern will you see on the inte rior surface of the cylinder? (a) bright and dark fringes so closely spaced as to be indistinguishable (b) one central bright fringe and two dark fringes only (c) a completely bright screen with no dark fringes (d) one central dark fringe and two bright fringes only (e) a completely dark screen with no bright fringes

A plane monochromatic light wave is incident on a dou ble slit as illustrated in Figure 37.1. **(i)** As the viewing screen is moved away from the double slit, what happens to the separation between the interference fringes on the screen? (a) It increases. (b) It decreases. (c) It remains the same. (d) It may increase or decrease, depending on the wavelength of the light. (e) More information is required. **(ii)** As the slit separation increases, what hap pens to the separation between the interference fringes on the screen? Select from the same choices.

10. A film of oil on a puddle in a parking lot shows a vari ety of bright colors in swirled patches. What can you say about the thickness of the oil film? (a) It is much less than the wavelength of visible light. (b) It is on the same order of magnitude as the wavelength of vis ible light. (c) It is much greater than the wavelength of visible light. (d) It might have any relationship to the wavelength of visible light.

Conceptual Questions

Why is the lens on a good-quality camera coated with a thin film?

2. A soap film is held vertically in air and is viewed in reflected light as in Figure CQ37.2. Explain why the film appears to be dark at the top.

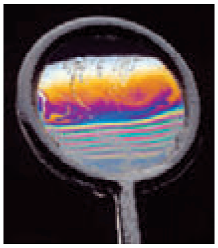


Figure CQ37.2 Conceptual Question 2 and Problem 70.

3. Explain why two flashlights held close together do not pro duce an interference pattern on a distant screen.

4. A lens with outer radius of cur vature and index of refrac tion rests on a flat glass plate. The combination is illuminated with white light from above and observed from above. (a) Is there a dark spot or a light spot at the center of the lens? (b) What does it mean if the observed rings are noncircular?

5. Consider a dark fringe in a double-slit interference pat tern at which almost no light energy is arriving. Light from both slits is arriving at the location of the dark fringe, but the waves cancel. Where does the energy at the positions of dark fringes go?

6. (a) In Young's double-slit experiment, why do we use monochromatic light? (b) If white light is used, how would the pattern change?

7. What is the necessary condition on the path length dif ference between two waves that interfere (a) construc tively and (b) destructively?

8. In a laboratory accident, you spill two liquids onto dif ferent parts of a water surface. Neither of the liquids mixes with the water. Both liquids form thin films on the water surface. As the films spread and become very thin, you notice that one film becomes brighter and the other darker in reflected light. Why?

9. A theatrical smoke machine fills the space between the barrier and the viewing screen in the Young's double-slit experiment shown in Figure CQ37.9. Would the smoke show evidence of interference within this space? Explain your answer.

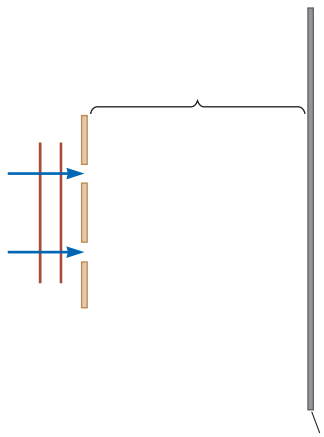


Figure CQ37.9

Problems

ENHANCED WebAssign The problems found in this chapter may be assigned online in Enhanced WebAssign

1. straightforward; **2.** intermediate; **3.** challenging

1. full solution available in the *Student Solutions Manual/Study Guide*

AMT Analysis Model tutorial available in Enhanced WebAssign

GP Guided Problem

M Master It tutorial available in Enhanced WebAssign

W Watch It video solution available in Enhanced WebAssign

Section 37.1 Young's Double-Slit Experiment

Section 37.2 Analysis Model: Waves in Interference

Problems 3, 5, 8, 10, and 13 in Chapter 18 can be assigned with this section.

1. Two slits are separated by 0.320 mm. A beam of 500-nm light strikes the slits, producing an interference pattern. Determine the number of maxima observed in the angular range $-30.0° < \theta < 30.0°$.

2. Light of wavelength 530 nm illuminates a pair of slits separated by 0.300 mm. If a screen is placed 2.00 m from the slits, determine the distance between the first and second dark fringes.

3. A laser beam is incident on two slits with a separation of 0.200 mm, and a screen is placed 5.00 m from the slits. An interference pattern appears on the screen. If the angle from the center fringe to the first bright fringe to the side is 0.181°, what is the wavelength of the laser light?

4. W A Young's interference experiment is performed with blue-green argon laser light. The separation between the slits is 0.500 mm, and the screen is located 3.30 m from the slits. The first bright fringe is located 3.40 mm from the center of the interference pattern. What is the wavelength of the argon laser light?

5. W Young's double-slit experiment is performed with 589-nm light and a distance of 2.00 m between the slits and the screen. The tenth interference minimum is observed 7.26 mm from the central maximum. Determine the spacing of the slits.

6. *Why is the following situation impossible?* Two narrow slits are separated by 8.00 mm in a piece of metal. A beam of microwaves strikes the metal perpendicularly, passes through the two slits, and then proceeds toward a wall some distance away. You know that the wavelength of the radiation is 1.00 cm ±5%, but you wish to measure it more precisely. Moving a microwave detector along the wall to study the interference pattern, you measure the position of the $m = 1$ bright fringe, which leads to a successful measurement of the wavelength of the radiation.

7. Light of wavelength 620 nm falls on a double slit, and the first bright fringe of the interference pattern is seen at an angle of 15.0° with the horizontal. Find the separation between the slits.

8. In a Young's double-slit experiment, two parallel slits with a slit separation of 0.100 mm are illuminated by light of wavelength 589 nm, and the interference pattern is observed on a screen located 4.00 m from the slits. (a) What is the difference in path lengths from each of the slits to the location of the center of a third-order bright fringe on the screen? (b) What is the difference in path lengths from the two slits to the location of the center of the third dark fringe away from the center of the pattern?

9. AMT M A pair of narrow, parallel slits separated by 0.250 mm is illuminated by green light ($\lambda = 546.1$ nm). The interference pattern is observed on a screen 1.20 m away from the plane of the parallel slits. Calculate the distance (a) from the central maximum to the first bright region on either side of the central maximum and (b) between the first and second dark bands in the interference pattern.

10. Light with wavelength 442 nm passes through a double-slit system that has a slit separation $d = 0.400$ mm. Determine how far away a screen must be placed so that dark fringes appear directly opposite both slits, with only one bright fringe between them.

11. AMT M The two speakers of a boom box are 35.0 cm apart. A single oscillator makes the speakers vibrate in phase at a frequency of 2.00 kHz. At what angles, measured from the perpendicular bisector of the line joining the speakers, would a distant observer hear maximum sound intensity? Minimum sound intensity? (Take the speed of sound as 340 m/s.)

12. In a location where the speed of sound is 343 m/s, a 2 000-Hz sound wave impinges on two slits 30.0 cm apart. (a) At what angle is the first maximum of sound intensity located? (b) **What If?** If the sound wave is replaced by 3.00-cm microwaves, what slit separation gives the same angle for the first maximum of microwave intensity? (c) **What If?** If the slit separation is 1.00 μm, what frequency of light gives the same angle to the first maximum of light intensity?

13. AMT M Two radio antennas separated by $d = 300$ m as shown in Figure P37.13 simultaneously broadcast identical signals at the same wavelength. A car travels due north along a straight line at position $x = 1\,000$ m from the center point between the antennas, and its radio receives the signals. (a) If the car is at the position of the second maximum after that at point O when it has traveled a distance $y = 400$ m northward, what is the wavelength of the signals? (b) How much farther must the car travel from this position to encounter the next minimum in reception? *Note:* Do not use the small-angle approximation in this problem.

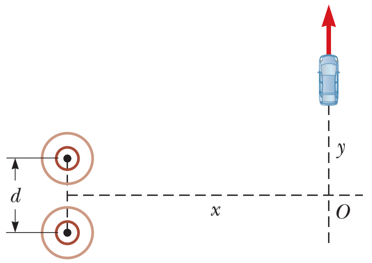


Figure P37.13

14. W A riverside warehouse has several small doors facing the river. Two of these doors are open as shown in Figure P37.14. The walls of the warehouse are lined with sound-absorbing material. Two people stand at a distance $L = 150$ m from the wall with the open doors. Person A stands along a line passing through the midpoint between the open doors, and person B stands a distance $y = 20$ m to his side. A boat on the river sounds its horn. To person A, the sound is loud and clear. To person B, the sound is barely audible. The principal wavelength of the sound waves is 3.00 m. Assuming person B is at the position of the first minimum, determine the distance d between the doors, center to center.

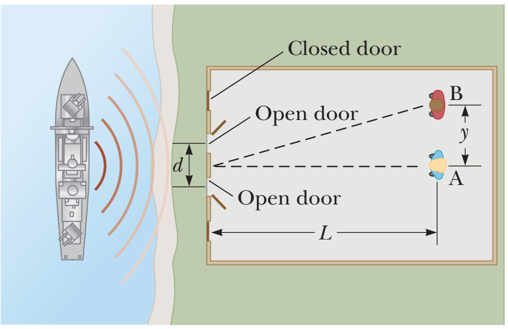


Figure P37.14

15. A student holds a laser that emits light of wavelength 632.8 nm. The laser beam passes though a pair of slits separated by 0.300 mm, in a glass plate attached to the front of the laser. The beam then falls perpendicularly on a screen, creating an interference pattern on it. The student begins to walk directly toward the screen at 3.00 m/s. The central maximum on the screen is stationary. Find the speed of the 50th-order maxima on the screen.

16. A student holds a laser that emits light of wavelength λ. The laser beam passes though a pair of slits separated by a distance d, in a glass plate attached to the front of the laser. The beam then falls perpendicularly on a screen, creating an interference pattern on it. The student begins to walk directly toward the screen at speed v. The central maximum on the screen is stationary. Find the speed of the mth-order maxima on the screen, where m can be very large.

17. Radio waves of wavelength 125 m from a galaxy reach a radio telescope by two separate paths as shown in Figure P37.17. One is a direct path to the receiver, which is situated on the edge of a tall cliff by the ocean, and the second is by reflection off the water. As the galaxy rises in the east over the water, the first minimum of destructive interference occurs when the galaxy is $\theta = 25.0°$ above the horizon. Find the height of the radio telescope dish above the water.

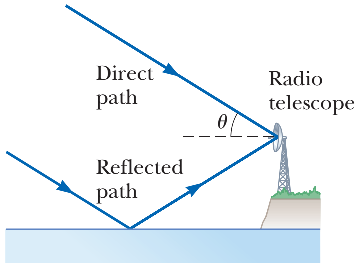


Figure P37.17 Problems 17 and 69.

18. M In Figure P37.18 (not to scale), let $L = 1.20$ m and $d = 0.120$ mm and assume the slit system is illuminated with monochromatic 500-nm light. Calculate the phase difference between the two wave fronts arriving at P when (a) $\theta = 0.500°$ and (b) $y = 5.00$ mm. (c) What is the value of θ for which the phase difference is 0.333 rad? (d) What is the value of θ for which the path difference is $\lambda/4$?

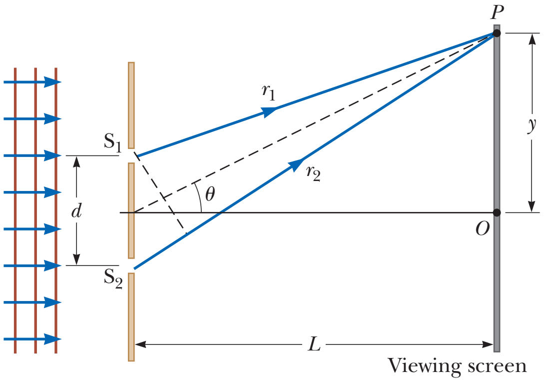


Figure P37.18 Problems 18 and 25.

19. Coherent light rays of wavelength λ strike a pair of slits separated by distance d at an angle θ_1 with respect to the normal to the plane containing the slits as shown in Figure P37.19. The rays leaving the slits make an

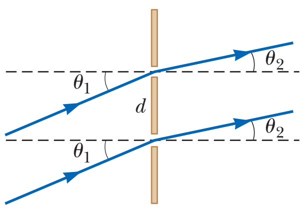


Figure P37.19

angle θ_2 with respect to the normal, and an interference maximum is formed by those rays on a screen that is a great distance from the slits. Show that the angle θ_2 is given by

$$\theta_2 = \sin^{-1}\left(\sin\theta_1 - \frac{m\lambda}{d}\right)$$

where m is an integer.

20. GP Monochromatic light of wavelength λ is incident on a pair of slits separated by 2.40×10^{-4} m and forms an interference pattern on a screen placed 1.80 m from the slits. The first-order bright fringe is at a position $y_{\text{bright}} = 4.52$ mm measured from the center of the central maximum. From this information, we wish to predict where the fringe for $n = 50$ would be located. (a) Assuming the fringes are laid out linearly along the screen, find the position of the $n = 50$ fringe by multiplying the position of the $n = 1$ fringe by 50.0. (b) Find the tangent of the angle the first-order bright fringe makes with respect to the line extending from the point midway between the slits to the center of the central maximum. (c) Using the result of part (b) and Equation 37.2, calculate the wavelength of the light. (d) Compute the angle for the 50th-order bright fringe from Equation 37.2. (e) Find the position of the 50th-order bright fringe on the screen from Equation 37.5. (f) Comment on the agreement between the answers to parts (a) and (e).

21. W In the double-slit arrangement of Figure P37.21, $d = 0.150$ mm, $L = 140$ cm, $\lambda = 643$ nm, and $y = 1.80$ cm. (a) What is the path difference δ for the rays from the two slits arriving at P? (b) Express this path difference in terms of λ. (c) Does P correspond to a maximum, a minimum, or an intermediate condition? Give evidence for your answer.

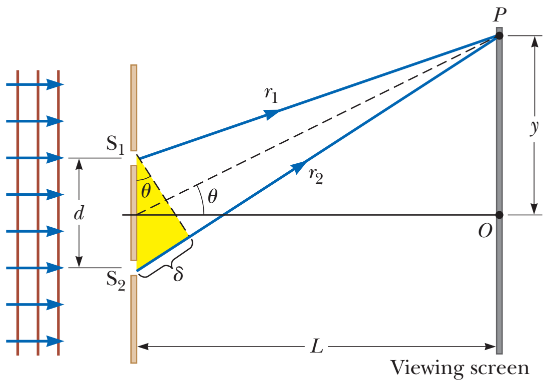


Figure P37.21

22. Young's double-slit experiment underlies the *instrument landing system* used to guide aircraft to safe landings at some airports when the visibility is poor. Although real systems are more complicated than the example described here, they operate on the same principles. A pilot is trying to align her plane with a runway as suggested in Figure P37.22. Two radio antennas (the black dots in the figure) are positioned adjacent to the runway, separated by $d = 40.0$ m. The antennas broadcast unmodulated coherent radio waves at 30.0 MHz. The red lines in Figure P37.22 represent paths along which maxima in the interference pattern of the radio waves exist. (a) Find the wavelength of the waves. The pilot "locks onto" the strong signal radiated along an interference maximum and steers the plane to keep the received signal strong. If she has found the central maximum, the plane will have precisely the correct heading to land when it reaches the runway as exhibited by plane A. (b) **What If?** Suppose the plane is flying along the first side maximum instead as is the case for plane B. How far to the side of the runway centerline will the plane be when it is 2.00 km from the antennas, measured along its direction of travel? (c) It is possible to tell the pilot that she is on the wrong maximum by sending out two signals from each antenna and equipping the aircraft with a two-channel receiver. The ratio of the two frequencies must not be the ratio of small integers (such as $\frac{3}{4}$). Explain how this two-frequency system would work and why it would not necessarily work if the frequencies were related by an integer ratio.

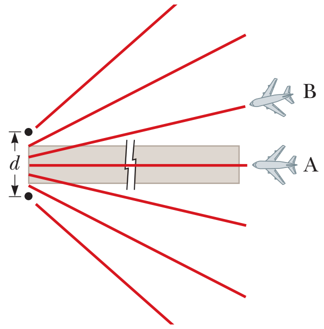


Figure P37.22

Section 37.3 Intensity Distribution of the Double-Slit Interference Pattern

23. Two slits are separated by 0.180 mm. An interference pattern is formed on a screen 80.0 cm away by 656.3-nm light. Calculate the fraction of the maximum intensity a distance $y = 0.600$ cm away from the central maximum.

24. Show that the two waves with wave functions given by $E_1 = 6.00 \sin(100\pi t)$ and $E_2 = 8.00 \sin(100\pi t + \pi/2)$ add to give a wave with the wave function $E_R \sin(100\pi t + \phi)$. Find the required values for E_R and ϕ.

25. M In Figure P37.18, let $L = 120$ cm and $d = 0.250$ cm. The slits are illuminated with coherent 600-nm light. Calculate the distance y from the central maximum for which the average intensity on the screen is 75.0% of the maximum.

26. Monochromatic coherent light of amplitude E_0 and angular frequency ω passes through three parallel slits, each separated by a distance d from its neighbor. (a) Show that the time-averaged intensity as a function of the angle θ is

$$I(\theta) = I_{\max}\left[1 + 2\cos\left(\frac{2\pi d \sin\theta}{\lambda}\right)\right]^2$$

(b) Explain how this expression describes both the primary and the secondary maxima. (c) Determine the ratio of the intensities of the primary and secondary maxima.

27. The intensity on the screen at a certain point in a double-slit interference pattern is 64.0% of the maximum value. (a) What minimum phase difference (in radians) between sources produces this result? (b) Express this phase difference as a path difference for 486.1-nm light.

28. Green light (λ = 546 nm) illuminates a pair of narrow, parallel slits separated by 0.250 mm. Make a graph of I/I_{max} as a function of θ for the interference pattern observed on a screen 1.20 m away from the plane of the parallel slits. Let θ range over the interval from $-0.3°$ to $+0.3°$.

29. W Two narrow, parallel slits separated by 0.850 mm are illuminated by 600-nm light, and the viewing screen is 2.80 m away from the slits. (a) What is the phase difference between the two interfering waves on a screen at a point 2.50 mm from the central bright fringe? (b) What is the ratio of the intensity at this point to the intensity at the center of a bright fringe?

Section 37.4 Change of Phase Due to Reflection

Section 37.5 Interference in Thin Films

30. A soap bubble (n = 1.33) floating in air has the shape of a spherical shell with a wall thickness of 120 nm. (a) What is the wavelength of the visible light that is most strongly reflected? (b) Explain how a bubble of different thickness could also strongly reflect light of this same wavelength. (c) Find the two smallest film thicknesses larger than 120 nm that can produce strongly reflected light of the same wavelength.

31. W A thin film of oil (n = 1.25) is located on smooth, wet pavement. When viewed perpendicular to the pavement, the film reflects most strongly red light at 640 nm and reflects no green light at 512 nm. How thick is the oil film?

32. A material having an index of refraction of 1.30 is used as an antireflective coating on a piece of glass (n = 1.50). What should the minimum thickness of this film be to minimize reflection of 500-nm light?

33. M A possible means for making an airplane invisible to radar is to coat the plane with an antireflective polymer. If radar waves have a wavelength of 3.00 cm and the index of refraction of the polymer is n = 1.50, how thick would you make the coating?

34. A film of MgF_2 (n = 1.38) having thickness 1.00×10^{-5} cm is used to coat a camera lens. (a) What are the three longest wavelengths that are intensified in the reflected light? (b) Are any of these wavelengths in the visible spectrum?

35. W A beam of 580-nm light passes through two closely spaced glass plates at close to normal incidence as shown in Figure P37.35. For what minimum nonzero value of the plate separation d is the transmitted light bright?

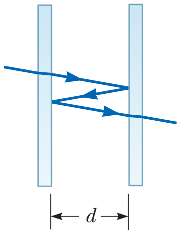


Figure P37.35

36. M An oil film (n = 1.45) floating on water is illuminated by white light at normal incidence. The film is 280 nm thick. Find (a) the wavelength and color of the light in the visible spectrum most strongly reflected and (b) the wavelength and color of the light in the spectrum most strongly transmitted. Explain your reasoning.

37. M An air wedge is formed between two glass plates separated at one edge by a very fine wire of circular cross section as shown in Figure P37.37. When the wedge is illuminated from above by 600-nm light and viewed from above, 30 dark fringes are observed. Calculate the diameter d of the wire.

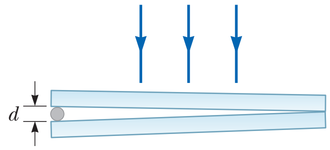


Figure P37.37 Problems 37, 41, 49, and 59.

38. Astronomers observe the chromosphere of the Sun with a filter that passes the red hydrogen spectral line of wavelength 656.3 nm, called the H_α line. The filter consists of a transparent dielectric of thickness d held between two partially aluminized glass plates. The filter is held at a constant temperature. (a) Find the minimum value of d that produces maximum transmission of perpendicular H_α light if the dielectric has an index of refraction of 1.378. (b) **What If?** If the temperature of the filter increases above the normal value, increasing its thickness, what happens to the transmitted wavelength? (c) The dielectric will also pass what near-visible wavelength? One of the glass plates is colored red to absorb this light.

39. W When a liquid is introduced into the air space between the lens and the plate in a Newton's-rings apparatus, the diameter of the tenth ring changes from 1.50 to 1.31 cm. Find the index of refraction of the liquid.

40. A lens made of glass (n_g = 1.52) is coated with a thin film of MgF_2 (n_s = 1.38) of thickness t. Visible light is incident normally on the coated lens as in Figure P37.40. (a) For what minimum value of t will the

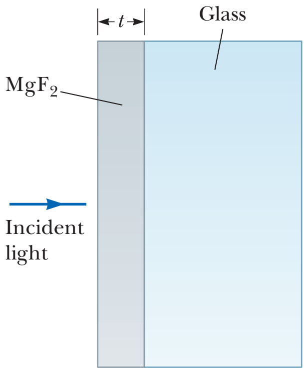


Figure P37.40

reflected light of wavelength 540 nm (in air) be missing? (b) Are there other values of t that will minimize the reflected light at this wavelength? Explain.

41. Two glass plates 10.0 cm long are in contact at one end and separated at the other end by a thread with a diameter $d = 0.050\,0$ mm (Fig. P37.37). Light containing the two wavelengths 400 nm and 600 nm is incident perpendicularly and viewed by reflection. At what distance from the contact point is the next dark fringe?

Section 37.6 The Michelson Interferometer

42. M Mirror M_1 in Figure 37.13 is moved through a displacement ΔL. During this displacement, 250 fringe reversals (formation of successive dark or bright bands) are counted. The light being used has a wavelength of 632.8 nm. Calculate the displacement ΔL.

43. M The Michelson interferometer can be used to measure the index of refraction of a gas by placing an evacuated transparent tube in the light path along one arm of the device. Fringe shifts occur as the gas is slowly added to the tube. Assume 600-nm light is used, the tube is 5.00 cm long, and 160 bright fringes pass on the screen as the pressure of the gas in the tube increases to atmospheric pressure. What is the index of refraction of the gas? *Hint:* The fringe shifts occur because the wavelength of the light changes inside the gas-filled tube.

44. One leg of a Michelson interferometer contains an evacuated cylinder of length L, having glass plates on each end. A gas is slowly leaked into the cylinder until a pressure of 1 atm is reached. If N bright fringes pass on the screen during this process when light of wavelength λ is used, what is the index of refraction of the gas? *Hint:* The fringe shifts occur because the wavelength of the light changes inside the gas-filled tube.

Additional Problems

45. Radio transmitter A operating at 60.0 MHz is 10.0 m from another similar transmitter B that is 180° out of phase with A. How far must an observer move from A toward B along the line connecting the two transmitters to reach the nearest point where the two beams are in phase?

46. A room is 6.0 m long and 3.0 m wide. At the front of the room, along one of the 3.0-m-wide walls, two loudspeakers are set 1.0 m apart, with the center point between them coinciding with the center point of the wall. The speakers emit a sound wave of a single frequency and a maximum in sound intensity is heard at the center of the back wall, 6.0 m from the speakers. What is the highest possible frequency of the sound from the speakers if no other maxima are heard anywhere along the back wall?

47. In an experiment similar to that of Example 37.1, green light with wavelength 560 nm, sent through a pair of slits 30.0 μm apart, produces bright fringes 2.24 cm apart on a screen 1.20 m away. If the apparatus is now submerged in a tank containing a sugar solution with index of refraction 1.38, calculate the fringe separation for this same arrangement.

48. In the What If? section of Example 37.2, it was claimed that overlapping fringes in a two-slit interference pattern for two different wavelengths obey the following relationship even for large values of the angle θ:

$$\frac{m'}{m} = \frac{\lambda}{\lambda'}$$

(a) Prove this assertion. (b) Using the data in Example 37.2, find the nonzero value of y on the screen at which the fringes from the two wavelengths first coincide.

49. An investigator finds a fiber at a crime scene that he wishes to use as evidence against a suspect. He gives the fiber to a technician to test the properties of the fiber. To measure the diameter d of the fiber, the technician places it between two flat glass plates at their ends as in Figure P37.37. When the plates, of length 14.0 cm, are illuminated from above with light of wavelength 650 nm, she observes interference bands separated by 0.580 mm. What is the diameter of the fiber?

50. Raise your hand and hold it flat. Think of the space between your index finger and your middle finger as one slit and think of the space between middle finger and ring finger as a second slit. (a) Consider the interference resulting from sending coherent visible light perpendicularly through this pair of openings. Compute an order-of-magnitude estimate for the angle between adjacent zones of constructive interference. (b) To make the angles in the interference pattern easy to measure with a plastic protractor, you should use an electromagnetic wave with frequency of what order of magnitude? (c) How is this wave classified on the electromagnetic spectrum?

51. Two coherent waves, coming from sources at different locations, move along the x axis. Their wave functions are

$$E_1 = 860 \sin\left[\frac{2\pi x_1}{650} - 924\pi t + \frac{\pi}{6}\right]$$

and

$$E_2 = 860 \sin\left[\frac{2\pi x_2}{650} - 924\pi t + \frac{\pi}{8}\right]$$

where E_1 and E_2 are in volts per meter, x_1 and x_2 are in nanometers, and t is in picoseconds. When the two waves are superposed, determine the relationship between x_1 and x_2 that produces constructive interference.

52. In a Young's interference experiment, the two slits are separated by 0.150 mm and the incident light includes two wavelengths: $\lambda_1 = 540$ nm (green) and $\lambda_2 = 450$ nm (blue). The overlapping interference patterns are observed on a screen 1.40 m from the slits. Calculate the minimum distance from the center of the screen to a point where a bright fringe of the green light coincides with a bright fringe of the blue light.

53. In a Young's double-slit experiment using light of wavelength λ, a thin piece of Plexiglas having index of refraction n covers one of the slits. If the center point on the screen is a dark spot instead of a bright spot, what is the minimum thickness of the Plexiglas?

54. **Review.** A flat piece of glass is held stationary and horizontal above the highly polished, flat top end of a 10.0-cm-long vertical metal rod that has its lower end rigidly fixed. The thin film of air between the rod and glass is observed to be bright by reflected light when it is illuminated by light of wavelength 500 nm. As the temperature is slowly increased by 25.0°C, the film changes from bright to dark and back to bright 200 times. What is the coefficient of linear expansion of the metal?
AMT

55. A certain grade of crude oil has an index of refraction of 1.25. A ship accidentally spills 1.00 m^3 of this oil into the ocean, and the oil spreads into a thin, uniform slick. If the film produces a first-order maximum of light of wavelength 500 nm normally incident on it, how much surface area of the ocean does the oil slick cover? Assume the index of refraction of the ocean water is 1.34.

56. The waves from a radio station can reach a home receiver by two paths. One is a straight-line path from transmitter to home, a distance of 30.0 km. The second is by reflection from the ionosphere (a layer of ionized air molecules high in the atmosphere). Assume this reflection takes place at a point midway between receiver and transmitter, the wavelength broadcast by the radio station is 350 m, and no phase change occurs on reflection. Find the minimum height of the ionospheric layer that could produce destructive interference between the direct and reflected beams.

57. Interference effects are produced at point *P* on a screen as a result of direct rays from a 500-nm source and reflected rays from the mirror as shown in Figure P37.57. Assume the source is 100 m to the left of the screen and 1.00 cm above the mirror. Find the distance *y* to the first dark band above the mirror.

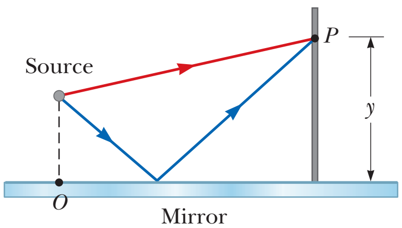


Figure P37.57

58. Measurements are made of the intensity distribution within the central bright fringe in a Young's interference pattern (see Fig. 37.6). At a particular value of *y*, it is found that $I/I_{max} = 0.810$ when 600-nm light is used. What wavelength of light should be used to reduce the relative intensity at the same location to 64.0% of the maximum intensity?

59. Many cells are transparent and colorless. Structures of great interest in biology and medicine can be practically invisible to ordinary microscopy. To indicate the size and shape of cell structures, an *interference microscope* reveals a difference in index of refraction as a shift in interference fringes. The idea is exemplified in the following problem. An air wedge is formed between two glass plates in contact along one edge and slightly separated at the opposite edge as in Figure P37.37. When the plates are illuminated with monochromatic light from above, the reflected light has 85 dark fringes. Calculate the number of dark fringes that appear if water (n = 1.33) replaces the air between the plates.

60. Consider the double-slit arrangement shown in Figure P37.60, where the slit separation is d and the distance from the slit to the screen is L. A sheet of transparent plastic having an index of refraction n and thickness t is placed over the upper slit. As a result, the central maximum of the interference pattern moves upward a distance y'. Find y'.
W

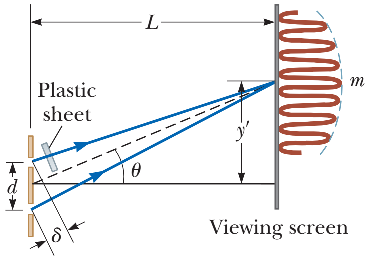


Figure P37.60

61. Figure P37.61 shows a radio-wave transmitter and a receiver separated by a distance d = 50.0 m and both a distance h = 35.0 m above the ground. The receiver can receive signals both directly from the transmitter and indirectly from signals that reflect from the ground. Assume the ground is level between the transmitter and receiver and a 180° phase shift occurs upon reflection. Determine the longest wavelengths that interfere (a) constructively and (b) destructively.

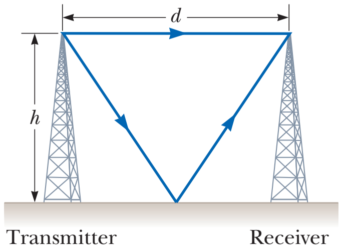


Figure P37.61 Problems 61 and 62.

62. Figure P37.61 shows a radio-wave transmitter and a receiver separated by a distance d and both a distance h above the ground. The receiver can receive signals both directly from the transmitter and indirectly from signals that reflect from the ground. Assume the ground is level between the transmitter and receiver and a 180° phase shift occurs upon reflection. Determine the longest wavelengths that interfere (a) constructively and (b) destructively.

63. In a Newton's-rings experiment, a plano-convex glass (n = 1.52) lens having radius r = 5.00 cm is placed on a flat plate as shown in Figure P37.63 (page 1158). When light of wavelength $\lambda = 650$ nm is incident normally, 55 bright rings are observed, with the last one precisely on the edge of the lens. (a) What is the radius R of curvature of the convex surface of the lens? (b) What is the focal length of the lens?

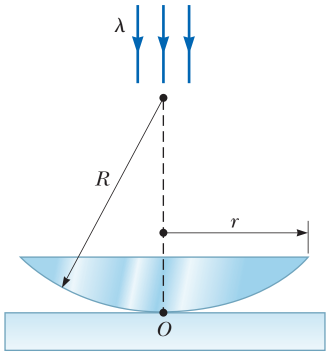


Figure P37.63

64. *Why is the following situation impossible?* A piece of transparent material having an index of refraction $n = 1.50$ is cut into the shape of a wedge as shown in Figure P37.64. Both the top and bottom surfaces of the wedge are in contact with air. Monochromatic light of wavelength $\lambda = 632.8$ nm is normally incident from above, and the wedge is viewed from above. Let $h = 1.00$ mm represent the height of the wedge and $\ell = 0.500$ m its length. A thin-film interference pattern appears in the wedge due to reflection from the top and bottom surfaces. You have been given the task of counting the number of bright fringes that appear in the entire length ℓ of the wedge. You find this task tedious, and your concentration is broken by a noisy distraction after accurately counting 5 000 bright fringes.

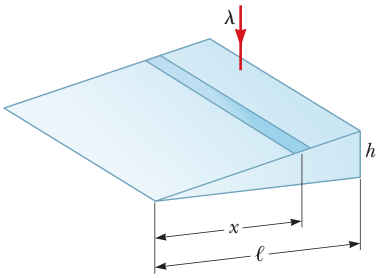


Figure P37.64

65. A plano-concave lens having index of refraction 1.50 is placed on a flat glass plate as shown in Figure P37.65. Its curved surface, with radius of curvature 8.00 m, is on the bottom. The lens is illuminated from above with yellow sodium light of wavelength 589 nm, and a series of concentric bright and dark rings is observed by reflection. The interference pattern has a dark spot at the center that is surrounded by 50 dark rings, the largest of which is at the outer edge of the lens. (a) What is the thickness of the air layer at the center of the interference pattern? (b) Calculate the radius of the outermost dark ring. (c) Find the focal length of the lens.

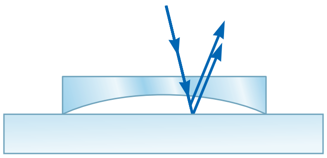


Figure P37.65

66. A plano-convex lens has index of refraction n. The curved side of the lens has radius of curvature R and rests on a flat glass surface of the same index of refraction, with a film of index n_{film} between them, as shown in Figure 37.66. The lens is illuminated from above by light of wavelength λ. Show that the dark Newton's rings have radii given approximately by

$$r \approx \sqrt{\frac{m\lambda R}{n_{\text{film}}}}$$

where $r << R$ and m is an integer.

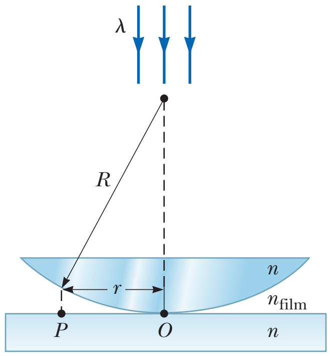


Figure P37.66

67. Interference fringes are produced using Lloyd's mirror and a source S of wavelength $\lambda = 606$ nm as shown in Figure P37.67. Fringes separated by $\Delta y = 1.20$ mm are formed on a screen a distance $L = 2.00$ m from the source. Find the vertical distance h of the source above the reflecting surface.

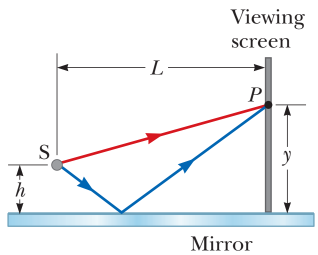


Figure P37.67

68. The quantity nt in Equations 37.17 and 37.18 is called the *optical path length* corresponding to the geometrical distance t and is analogous to the quantity δ in Equation 37.1, the path difference. The optical path length is proportional to n because a larger index of refraction shortens the wavelength, so more cycles of a wave fit into a particular geometrical distance. (a) Assume a mixture of corn syrup and water is prepared in a tank, with its index of refraction n increasing uniformly from 1.33 at $y = 20.0$ cm at the top to 1.90 at $y = 0$. Write the index of refraction $n(y)$ as a function of y.

(b) Compute the optical path length corresponding to the 20.0-cm height of the tank by calculating

$$\int_0^{20\text{ cm}} n(y)\,dy$$

(c) Suppose a narrow beam of light is directed into the mixture at a nonzero angle with respect to the normal to the surface of the mixture. Qualitatively describe its path.

69. M Astronomers observe a 60.0-MHz radio source both directly and by reflection from the sea as shown in Figure P37.17. If the receiving dish is 20.0 m above sea level, what is the angle of the radio source above the horizon at first maximum?

70. Figure CQ37.2 shows an unbroken soap film in a circular frame. The film thickness increases from top to bottom, slowly at first and then rapidly. As a simpler model, consider a soap film (n = 1.33) contained within a rectangular wire frame. The frame is held vertically so that the film drains downward and forms a wedge with flat faces. The thickness of the film at the top is essentially zero. The film is viewed in reflected white light with near-normal incidence, and the first violet (λ = 420 nm) interference band is observed 3.00 cm from the top edge of the film. (a) Locate the first red (λ = 680 nm) interference band. (b) Determine the film thickness at the positions of the violet and red bands. (c) What is the wedge angle of the film?

Challenge Problems

71. Our discussion of the techniques for determining constructive and destructive interference by reflection from a thin film in air has been confined to rays striking the film at nearly normal incidence. **What If?** Assume a ray is incident at an angle of 30.0° (relative to the normal) on a film with index of refraction 1.38 surrounded by vacuum. Calculate the minimum thickness for constructive interference of sodium light with a wavelength of 590 nm.

72. The condition for constructive interference by reflection from a thin film in air as developed in Section 37.5 assumes nearly normal incidence. **What If?** Suppose the light is incident on the film at a nonzero angle θ_1 (relative to the normal). The index of refraction of the film is n, and the film is surrounded by vacuum. Find the condition for constructive interference that relates the thickness t of the film, the index of refraction n of the film, the wavelength λ of the light, and the angle of incidence θ_1.

73. Both sides of a uniform film that has index of refraction n and thickness d are in contact with air. For normal incidence of light, an intensity minimum is observed in the reflected light at λ_2 and an intensity maximum is observed at λ_1, where $\lambda_1 > \lambda_2$. (a) Assuming no intensity minima are observed between λ_1 and λ_2, find an expression for the integer m in Equations 37.17 and 37.18 in terms of the wavelengths λ_1 and λ_2. (b) Assuming $n = 1.40$, $\lambda_1 = 500$ nm, and $\lambda_2 = 370$ nm, determine the best estimate for the thickness of the film.

74. Slit 1 of a double slit is wider than slit 2 so that the light from slit 1 has an amplitude 3.00 times that of the light from slit 2. Show that Equation 37.13 is replaced by the equation $I = I_{max}(1 + 3\cos^2 \phi/2)$ for this situation.

75. Monochromatic light of wavelength 620 nm passes through a very narrow slit S and then strikes a screen in which are two parallel slits, S_1 and S_2, as shown in Figure P37.75. Slit S_1 is directly in line with S and at a distance of $L = 1.20$ m away from S, whereas S_2 is displaced a distance d to one side. The light is detected at point P on a second screen, equidistant from S_1 and S_2. When either slit S_1 or S_2 is open, equal light intensities are measured at point P. When both slits are open, the intensity is three times larger. Find the minimum possible value for the slit separation d.

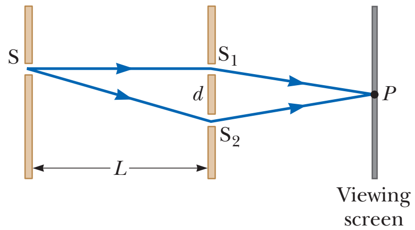


Figure P37.75

76. A plano-convex lens having a radius of curvature of r = 4.00 m is placed on a concave glass surface whose radius of curvature is R = 12.0 m as shown in Figure P37.76. Assuming 500-nm light is incident normal to the flat surface of the lens, determine the radius of the 100th bright ring.

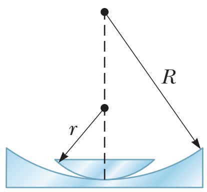


Figure P37.76