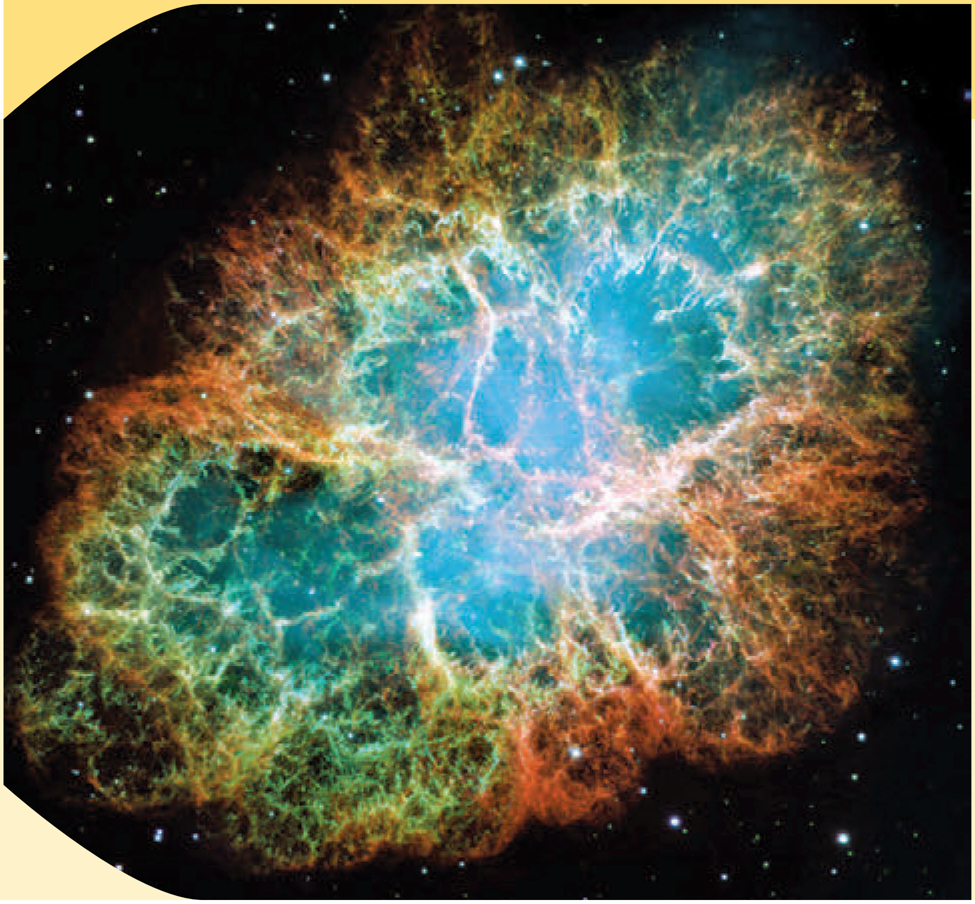
CHAPTER

34

Electromagnetic Waves



This image of the Crab Nebula taken with visible light shows a variety of colors, with each color representing a different wavelength of visible light. *(NASA, ESA, J. Hester, A. Loll (ASU))*

The waves described in Chapters 16, 17, and 18 are mechanical waves. By definition, the propagation of mechanical disturbances—such as sound waves, water waves, and waves on a string—requires the presence of a medium. This chapter is concerned with the properties of electromagnetic waves, which (unlike mechanical waves) can propagate through empty space.

We begin by considering Maxwell's contributions in modifying Ampère's law, which we studied in Chapter 30. We then discuss Maxwell's equations, which form the theoretical basis of all electromagnetic phenomena. These equations predict the existence of electromagnetic waves that propagate through space at the speed of light c according to the traveling wave analysis model. Heinrich Hertz confirmed Maxwell's prediction when he generated and detected electromagnetic waves in 1887. That discovery has led to many practical communication systems, including radio, television, cell phone systems, wireless Internet connectivity, and optoelectronics.

Next, we learn how electromagnetic waves are generated by oscillating electric charges. The waves radiated from the oscillating charges can be detected at great distances. Furthermore, because electromagnetic waves carry energy (T_{ER} in Eq. 8.2) and momentum, they can exert pressure on a surface. The chapter concludes with a description of the various frequency ranges in the electromagnetic spectrum.

34.1 Displacement Current and the General Form of Ampère's Law

In Chapter 30, we discussed using Ampère's law (Eq. 30.13) to analyze the magnetic fields created by currents:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I$$

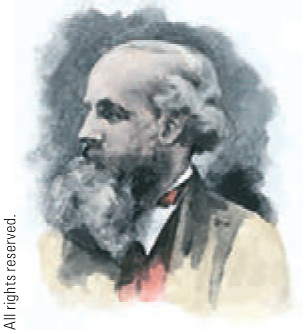
In this equation, the line integral is over any closed path through which conduction current passes, where conduction current is defined by the expression $I = dq/dt$. (In this section, we use the term *conduction current* to refer to the current carried by charge carriers in the wire to distinguish it from a new type of current we shall introduce shortly.) We now show that Ampère's law in this form is valid only if any electric fields present are constant in time. James Clerk Maxwell recognized this limitation and modified Ampère's law to include time-varying electric fields.

Consider a capacitor being charged as illustrated in Figure 34.1. When a conduction current is present, the charge on the positive plate changes, but no conduction current exists in the gap between the plates because there are no charge carriers in the gap. Now consider the two surfaces S_1 and S_2 in Figure 34.1, bounded by the same path P. Ampère's law states that $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$ around this path must equal $\mu_0 I$, where I is the total current through *any* surface bounded by the path P.

When the path P is considered to be the boundary of S_1, $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I$ because the conduction current I passes through S_1. When the path is considered to be the boundary of S_2, however, $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = 0$ because no conduction current passes through S_2. Therefore, we have a contradictory situation that arises from the discontinuity of the current! Maxwell solved this problem by postulating an additional term on the right side of Ampère's law, which includes a factor called the **displacement current** I_d defined as[1]

$$I_d \equiv \epsilon_0 \frac{d\Phi_E}{dt} \tag{34.1}$$

◀ **Displacement current**



James Clerk Maxwell
Scottish Theoretical Physicist (1831–1879)
Maxwell developed the electromagnetic theory of light and the kinetic theory of gases, and explained the nature of Saturn's rings and color vision. Maxwell's successful interpretation of the electromagnetic field resulted in the field equations that bear his name. Formidable mathematical ability combined with great insight enabled him to lead the way in the study of electromagnetism and kinetic theory. He died of cancer before he was 50.

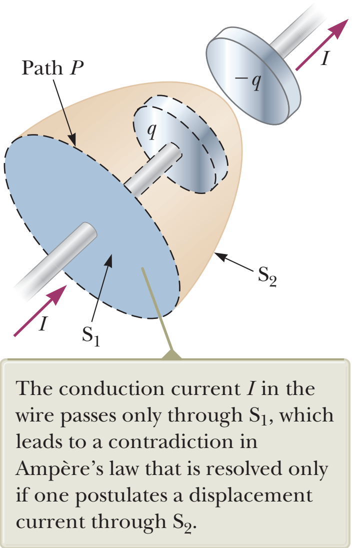


Figure 34.1 Two surfaces S_1 and S_2 near the plate of a capacitor are bounded by the same path P.

[1] *Displacement* in this context does not have the meaning it does in Chapter 2. Despite the inaccurate implications, the word is historically entrenched in the language of physics, so we continue to use it.

where ϵ_0 is the permittivity of free space (see Section 23.3) and $\Phi_E \equiv \int \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$ is the electric flux (see Eq. 24.3) through the surface bounded by the path of integration.

As the capacitor is being charged (or discharged), the changing electric field between the plates may be considered equivalent to a current that acts as a continuation of the conduction current in the wire. When the expression for the displacement current given by Equation 34.1 is added to the conduction current on the right side of Ampère's law, the difficulty represented in Figure 34.1 is resolved. No matter which surface bounded by the path P is chosen, either a conduction current or a displacement current passes through it. With this new term I_d, we can express the general form of Ampère's law (sometimes called the **Ampère–Maxwell law**) as

Ampère–Maxwell law ▶

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0(I + I_d) = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt} \tag{34.2}$$

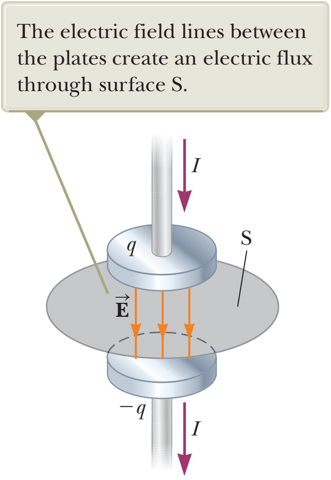


Figure 34.2 When a conduction current exists in the wires, a changing electric field $\vec{\mathbf{E}}$ exists between the plates of the capacitor.

We can understand the meaning of this expression by referring to Figure 34.2. The electric flux through surface S is $\Phi_E = \int \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = EA$, where A is the area of the capacitor plates and E is the magnitude of the uniform electric field between the plates. If q is the charge on the plates at any instant, then $E = q/(\epsilon_0 A)$ (see Section 26.2). Therefore, the electric flux through S is

$$\Phi_E = EA = \frac{q}{\epsilon_0}$$

Hence, the displacement current through S is

$$I_d = \epsilon_0 \frac{d\Phi_E}{dt} = \frac{dq}{dt} \tag{34.3}$$

That is, the displacement current I_d through S is precisely equal to the conduction current I in the wires connected to the capacitor!

By considering surface S, we can identify the displacement current as the source of the magnetic field on the surface boundary. The displacement current has its physical origin in the time-varying electric field. The central point of this formalism is that magnetic fields are produced *both* by conduction currents *and* by time-varying electric fields. This result was a remarkable example of theoretical work by Maxwell, and it contributed to major advances in the understanding of electromagnetism.

Quick Quiz 34.1 In an *RC* circuit, the capacitor begins to discharge. **(i)** During the discharge, in the region of space between the plates of the capacitor, is there (a) conduction current but no displacement current, (b) displacement current but no conduction current, (c) both conduction and displacement current, or (d) no current of any type? **(ii)** In the same region of space, is there (a) an electric field but no magnetic field, (b) a magnetic field but no electric field, (c) both electric and magnetic fields, or (d) no fields of any type?

Example 34.1 Displacement Current in a Capacitor

A sinusoidally varying voltage is applied across a capacitor as shown in Figure 34.3. The capacitance is $C = 8.00\ \mu\text{F}$, the frequency of the applied voltage is $f = 3.00$ kHz, and the voltage amplitude is $\Delta V_{\max} = 30.0$ V. Find the displacement current in the capacitor.

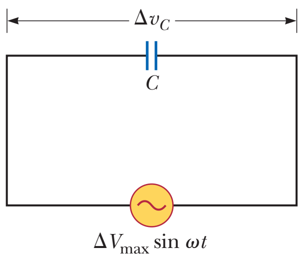


Figure 34.3 (Example 34.1)

SOLUTION

Conceptualize Figure 34.3 represents the circuit diagram for this situation. Figure 34.2 shows a close-up of the capacitor and the electric field between the plates.

Categorize We determine results using equations discussed in this section, so we categorize this example as a substitution problem.

34.1 continued

Evaluate the angular frequency of the source from Equation 15.12:

$$\omega = 2\pi f = 2\pi(3.00 \times 10^3 \text{ Hz}) = 1.88 \times 10^4 \text{ s}^{-1}$$

Use Equation 33.20 to express the potential difference in volts across the capacitor as a function of time in seconds:

$$\Delta v_C = \Delta V_{\max} \sin \omega t = 30.0 \sin (1.88 \times 10^4\, t)$$

Use Equation 34.3 to find the displacement current in amperes as a function of time. Note that the charge on the capacitor is $q = C\,\Delta v_C$:

$$i_d = \frac{dq}{dt} = \frac{d}{dt}(C\,\Delta v_C) = C\frac{d}{dt}(\Delta V_{\max} \sin \omega t)$$
$$= \omega C\,\Delta V_{\max} \cos \omega t$$

Substitute numerical values:

$$i_d = (1.88 \times 10^4 \text{ s}^{-1})(8.00 \times 10^{-6} \text{ C})(30.0 \text{ V}) \cos (1.88 \times 10^4\, t)$$
$$= \boxed{4.51 \cos (1.88 \times 10^4\, t)}$$

34.2 Maxwell's Equations and Hertz's Discoveries

We now present four equations that are regarded as the basis of all electrical and magnetic phenomena. These equations, developed by Maxwell, are as fundamental to electromagnetic phenomena as Newton's laws are to mechanical phenomena. In fact, the theory that Maxwell developed was more far-reaching than even he imagined because it turned out to be in agreement with the special theory of relativity, as Einstein showed in 1905.

Maxwell's equations represent the laws of electricity and magnetism that we have already discussed, but they have additional important consequences. For simplicity, we present **Maxwell's equations** as applied to free space, that is, in the absence of any dielectric or magnetic material. The four equations are

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q}{\epsilon_0} \tag{34.4}$$

◀ Gauss's law

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0 \tag{34.5}$$

◀ Gauss's law in magnetism

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt} \tag{34.6}$$

◀ Faraday's law

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I + \epsilon_0 \mu_0 \frac{d\Phi_E}{dt} \tag{34.7}$$

◀ Ampère–Maxwell law

Equation 34.4 is Gauss's law: the total electric flux through any closed surface equals the net charge inside that surface divided by ϵ_0. This law relates an electric field to the charge distribution that creates it.

Equation 34.5 is Gauss's law in magnetism, and it states that the net magnetic flux through a closed surface is zero. That is, the number of magnetic field lines that enter a closed volume must equal the number that leave that volume, which implies that magnetic field lines cannot begin or end at any point. If they did, it would mean that isolated magnetic monopoles existed at those points. That isolated magnetic monopoles have not been observed in nature can be taken as a confirmation of Equation 34.5.

Equation 34.6 is Faraday's law of induction, which describes the creation of an electric field by a changing magnetic flux. This law states that the emf, which is the line integral of the electric field around any closed path, equals the rate of change of magnetic flux through any surface bounded by that path. One consequence of Faraday's law is the current induced in a conducting loop placed in a time-varying magnetic field.

Equation 34.7 is the Ampère–Maxwell law, discussed in Section 34.1, and it describes the creation of a magnetic field by a changing electric field and by electric current: the line integral of the magnetic field around any closed path is the sum of μ_0 multiplied by the net current through that path and $\epsilon_0\mu_0$ multiplied by the rate of change of electric flux through any surface bounded by that path.

Once the electric and magnetic fields are known at some point in space, the force acting on a particle of charge q can be calculated from the electric and magnetic versions of the particle in a field model:

Lorentz force law ▶

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}} + q\vec{\mathbf{v}} \times \vec{\mathbf{B}} \tag{34.8}$$

This relationship is called the **Lorentz force law.** (We saw this relationship earlier as Eq. 29.6.) Maxwell's equations, together with this force law, completely describe all classical electromagnetic interactions in a vacuum.

Notice the symmetry of Maxwell's equations. Equations 34.4 and 34.5 are symmetric, apart from the absence of the term for magnetic monopoles in Equation 34.5. Furthermore, Equations 34.6 and 34.7 are symmetric in that the line integrals of $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ around a closed path are related to the rate of change of magnetic flux and electric flux, respectively. Maxwell's equations are of fundamental importance not only to electromagnetism, but to all science. Hertz once wrote, "One cannot escape the feeling that these mathematical formulas have an independent existence and an intelligence of their own, that they are wiser than we are, wiser even than their discoverers, that we get more out of them than we put into them."

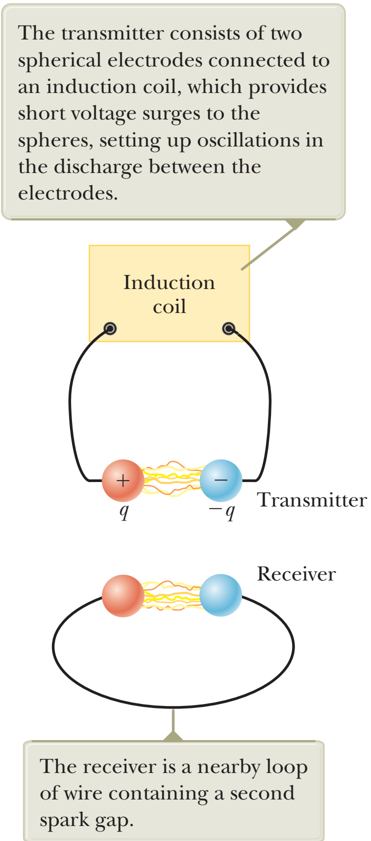


Figure 34.4 Schematic diagram of Hertz's apparatus for generating and detecting electromagnetic waves.

In the next section, we show that Equations 34.6 and 34.7 can be combined to obtain a wave equation for both the electric field and the magnetic field. In empty space, where $q = 0$ and $I = 0$, the solution to these two equations shows that the speed at which electromagnetic waves travel equals the measured speed of light. This result led Maxwell to predict that light waves are a form of electromagnetic radiation.

Hertz performed experiments that verified Maxwell's prediction. The experimental apparatus Hertz used to generate and detect electromagnetic waves is shown schematically in Figure 34.4. An induction coil is connected to a transmitter made up of two spherical electrodes separated by a narrow gap. The coil provides short voltage surges to the electrodes, making one positive and the other negative. A spark is generated between the spheres when the electric field near either electrode surpasses the dielectric strength for air (3×10^6 V/m; see Table 26.1). Free electrons in a strong electric field are accelerated and gain enough energy to ionize any molecules they strike. This ionization provides more electrons, which can accelerate and cause further ionizations. As the air in the gap is ionized, it becomes a much better conductor and the discharge between the electrodes exhibits an oscillatory behavior at a very high frequency. From an electric-circuit viewpoint, this experimental apparatus is equivalent to an LC circuit in which the inductance is that of the coil and the capacitance is due to the spherical electrodes.

Because L and C are small in Hertz's apparatus, the frequency of oscillation is high, on the order of 100 MHz. (Recall from Eq. 32.22 that $\omega = 1/\sqrt{LC}$ for an LC circuit.) Electromagnetic waves are radiated at this frequency as a result of the oscillation of free charges in the transmitter circuit. Hertz was able to detect these waves using a single loop of wire with its own spark gap (the receiver). Such a receiver loop, placed several meters from the transmitter, has its own effective inductance, capacitance, and natural frequency of oscillation. In Hertz's experiment, sparks were induced across the gap of the receiving electrodes when the receiver's frequency was adjusted to match that of the transmitter. In this way, Hertz demonstrated that the oscillating current induced in the receiver was produced by electromagnetic waves radiated by the transmitter. His experiment is analogous to the mechanical phenomenon in which a tuning fork responds to acoustic vibrations from an identical tuning fork that is oscillating nearby.

In addition, Hertz showed in a series of experiments that the radiation generated by his spark-gap device exhibited the wave properties of interference, diffraction, reflection, refraction, and polarization, which are all properties exhibited by light as we shall see in Part 5. Therefore, it became evident that the radio-frequency waves Hertz was generating had properties similar to those of light waves and that they differed only in frequency and wavelength. Perhaps his most convincing experiment was the measurement of the speed of this radiation. Waves of known frequency were reflected from a metal sheet and created a standing-wave interference pattern whose nodal points could be detected. The measured distance between the nodal points enabled determination of the wavelength λ. Using the relationship $v = \lambda f$ (Eq. 16.12) from the traveling wave model, Hertz found that v was close to 3×10^8 m/s, the known speed c of visible light.

Heinrich Rudolf Hertz
German Physicist (1857–1894)
Hertz made his most important discovery of electromagnetic waves in 1887. After finding that the speed of an electromagnetic wave was the same as that of light, Hertz showed that electromagnetic waves, like light waves, could be reflected, refracted, and diffracted. The hertz, equal to one complete vibration or cycle per second, is named after him.

34.3 Plane Electromagnetic Waves

The properties of electromagnetic waves can be deduced from Maxwell's equations. One approach to deriving these properties is to solve the second-order differential equation obtained from Maxwell's third and fourth equations. A rigorous mathematical treatment of that sort is beyond the scope of this text. To circumvent this problem, let's assume the vectors for the electric field and magnetic field in an electromagnetic wave have a specific space–time behavior that is simple but consistent with Maxwell equations.

To understand the prediction of electromagnetic waves more fully, let's focus our attention on an electromagnetic wave that travels in the x direction (the *direction of propagation*). For this wave, the electric field $\vec{\mathbf{E}}$ is in the y direction and the magnetic field $\vec{\mathbf{B}}$ is in the z direction as shown in Figure 34.5. Such waves, in which the electric and magnetic fields are restricted to being parallel to a pair of perpendicular axes, are said to be **linearly polarized waves.** Furthermore, let's assume the field magnitudes E and B depend on x and t only, not on the y or z coordinate.

Let's also imagine that the source of the electromagnetic waves is such that a wave radiated from *any* position in the yz plane (not only from the origin as might be suggested by Fig. 34.5) propagates in the x direction and all such waves are emitted in phase. If we define a **ray** as the line along which the wave travels, all rays for these waves are parallel. This entire collection of waves is often called a **plane wave.** A surface connecting points of equal phase on all waves is a geometric plane called a **wave front,** introduced in Chapter 17. In comparison, a point source of radiation sends waves out radially in all directions. A surface connecting points of equal phase for this situation is a sphere, so this wave is called a **spherical wave.**

Pitfall Prevention 34.1
What Is "a" Wave? What do we mean by a *single* wave? The word *wave* represents both the emission from a *single point* ("wave radiated from *any* position in the yz plane" in the text) and the collection of waves from *all points* on the source ("**plane wave**" in the text). You should be able to use this term in both ways and understand its meaning from the context.

To generate the prediction of plane electromagnetic waves, we start with Faraday's law, Equation 34.6:

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt}$$

To apply this equation to the wave in Figure 34.5, consider a rectangle of width dx and height ℓ lying in the xy plane as shown in Figure 34.6 (page 1036). Let's first evaluate the line integral of $\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$ around this rectangle in the counterclockwise direction at an instant of time when the wave is passing through the rectangle. The contributions from the top and bottom of the rectangle are zero because $\vec{\mathbf{E}}$ is perpendicular to $d\vec{\mathbf{s}}$ for these paths. We can express the electric field on the right side of the rectangle as

$$E(x + dx) \approx E(x) + \left.\frac{dE}{dx}\right|_{t\ \text{constant}} dx = E(x) + \frac{\partial E}{\partial x}\,dx$$

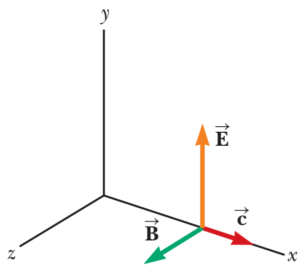


Figure 34.5 Electric and magnetic fields of an electromagnetic wave traveling at velocity $\vec{\mathbf{c}}$ in the positive x direction. The field vectors are shown at one instant of time and at one position in space. These fields depend on x and t.

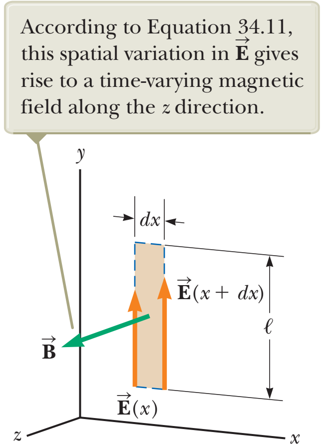


Figure 34.6 At an instant when a plane wave moving in the positive x direction passes through a rectangular path of width dx lying in the xy plane, the electric field in the y direction varies from $\vec{E}(x)$ to $\vec{E}(x + dx)$.

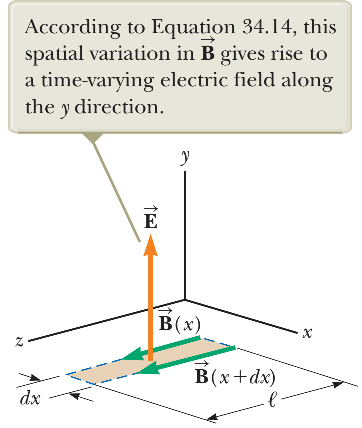


Figure 34.7 At an instant when a plane wave passes through a rectangular path of width dx lying in the xz plane, the magnetic field in the z direction varies from $\vec{B}(x)$ to $\vec{B}(x + dx)$.

where $E(x)$ is the field on the left side of the rectangle at this instant.[2] Therefore, the line integral over this rectangle is approximately

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = [E(x + dx)]\ell - [E(x)]\ell \approx \ell\left(\frac{\partial E}{\partial x}\right) dx \qquad \textbf{(34.9)}$$

Because the magnetic field is in the z direction, the magnetic flux through the rectangle of area $\ell\, dx$ is approximately $\Phi_B = B\ell\, dx$ (assuming dx is very small compared with the wavelength of the wave). Taking the time derivative of the magnetic flux gives

$$\frac{d\Phi_B}{dt} = \ell\, dx \left.\frac{dB}{dt}\right|_{x \text{ constant}} = \ell\, dx \frac{\partial B}{\partial t} \qquad \textbf{(34.10)}$$

Substituting Equations 34.9 and 34.10 into Equation 34.6 gives

$$\ell\left(\frac{\partial E}{\partial x}\right) dx = -\ell\, dx \frac{\partial B}{\partial t}$$

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t} \qquad \textbf{(34.11)}$$

In a similar manner, we can derive a second equation by starting with Maxwell's fourth equation in empty space (Eq. 34.7). In this case, the line integral of $\vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$ is evaluated around a rectangle lying in the xz plane and having width dx and length ℓ as in Figure 34.7. Noting that the magnitude of the magnetic field changes from $B(x)$ to $B(x + dx)$ over the width dx and that the direction for taking the line integral is counterclockwise when viewed from above in Figure 34.7, the line integral over this rectangle is found to be approximately

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = [B(x)]\ell - [B(x + dx)]\ell \approx -\ell\left(\frac{\partial B}{\partial x}\right) dx \qquad \textbf{(34.12)}$$

[2]Because dE/dx in this equation is expressed as the change in E with x at a given instant t, dE/dx is equivalent to the partial derivative $\partial E/\partial x$. Likewise, dB/dt means the change in B with time at a particular position x; therefore, in Equation 34.10, we can replace dB/dt with $\partial B/\partial t$.

The electric flux through the rectangle is $\Phi_E = E\ell\,dx$, which, when differentiated with respect to time, gives

$$\frac{\partial \Phi_E}{\partial t} = \ell\ dx\,\frac{\partial E}{\partial t} \tag{34.13}$$

Substituting Equations 34.12 and 34.13 into Equation 34.7 gives

$$-\ell\left(\frac{\partial B}{\partial x}\right) dx = \mu_0\,\epsilon_0\,\ell\ dx\left(\frac{\partial E}{\partial t}\right)$$

$$\frac{\partial B}{\partial x} = -\mu_0\epsilon_0\,\frac{\partial E}{\partial t} \tag{34.14}$$

Taking the derivative of Equation 34.11 with respect to x and combining the result with Equation 34.14 gives

$$\frac{\partial^2 E}{\partial x^2} = -\frac{\partial}{\partial x}\left(\frac{\partial B}{\partial t}\right) = -\frac{\partial}{\partial t}\left(\frac{\partial B}{\partial x}\right) = -\frac{\partial}{\partial t}\left(-\mu_0\epsilon_0\,\frac{\partial E}{\partial t}\right)$$

$$\boxed{\frac{\partial^2 E}{\partial x^2} = \mu_0\epsilon_0\,\frac{\partial^2 E}{\partial t^2}} \tag{34.15}$$

In the same manner, taking the derivative of Equation 34.14 with respect to x and combining it with Equation 34.11 gives

$$\boxed{\frac{\partial^2 B}{\partial x^2} = \mu_0\epsilon_0\,\frac{\partial^2 B}{\partial t^2}} \tag{34.16}$$

Equations 34.15 and 34.16 both have the form of the linear wave equation[3] with the wave speed v replaced by c, where

$$\boxed{c = \frac{1}{\sqrt{\mu_0\epsilon_0}}} \tag{34.17}$$

◀ **Speed of electromagnetic waves**

Let's evaluate this speed numerically:

$$c = \frac{1}{\sqrt{(4\pi \times 10^{-7}\ \mathrm{T\cdot m/A})(8.854\ 19 \times 10^{-12}\ \mathrm{C^2/N\cdot m^2})}}$$
$$= 2.997\ 92 \times 10^{8}\ \mathrm{m/s}$$

Because this speed is precisely the same as the speed of light in empty space, we are led to believe (correctly) that light is an electromagnetic wave.

The simplest solution to Equations 34.15 and 34.16 is a sinusoidal wave for which the field magnitudes E and B vary with x and t according to the expressions

$$E = E_{\max}\cos\,(kx - \omega t) \tag{34.18}$$

$$B = B_{\max}\cos\,(kx - \omega t) \tag{34.19}$$

◀ **Sinusoidal electric and magnetic fields**

where $E_{\max}$ and $B_{\max}$ are the maximum values of the fields. The angular wave number is $k = 2\pi/\lambda$, where λ is the wavelength. The angular frequency is $\omega = 2\pi f$, where f is the wave frequency. According to the traveling wave model, the ratio ω/k equals the speed of an electromagnetic wave, c:

$$\frac{\omega}{k} = \frac{2\pi f}{2\pi/\lambda} = \lambda f = c$$

[3] The linear wave equation is of the form $(\partial^2 y/\partial x^2) = (1/v^2)(\partial^2 y/\partial t^2)$, where v is the speed of the wave and y is the wave function. The linear wave equation was introduced as Equation 16.27, and we suggest you review Section 16.6.

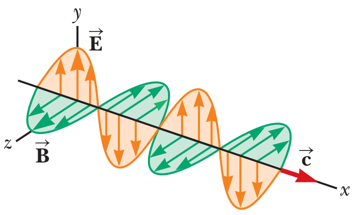


Figure 34.8 A sinusoidal electromagnetic wave moves in the positive x direction with a speed c.

where we have used Equation 16.12, $v = c = \lambda f$, which relates the speed, frequency, and wavelength of a sinusoidal wave. Therefore, for electromagnetic waves, the wavelength and frequency of these waves are related by

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{f} \quad \textbf{(34.20)}$$

Figure 34.8 is a pictorial representation, at one instant, of a sinusoidal, linearly polarized electromagnetic wave moving in the positive x direction.

We can generate other mathematical representations of the traveling wave model for electromagnetic waves. Taking partial derivatives of Equations 34.18 (with respect to x) and 34.19 (with respect to t) gives

$$\frac{\partial E}{\partial x} = -kE_{\text{max}} \sin (kx - \omega t)$$

$$\frac{\partial B}{\partial t} = \omega B_{\text{max}} \sin (kx - \omega t)$$

Substituting these results into Equation 34.11 shows that, at any instant,

$$kE_{\text{max}} = \omega B_{\text{max}}$$

$$\frac{E_{\text{max}}}{B_{\text{max}}} = \frac{\omega}{k} = c$$

Using these results together with Equations 34.18 and 34.19 gives

$$\boxed{\frac{E_{\text{max}}}{B_{\text{max}}} = \frac{E}{B} = c} \quad \textbf{(34.21)}$$

That is, at every instant, the ratio of the magnitude of the electric field to the magnitude of the magnetic field in an electromagnetic wave equals the speed of light.

Finally, note that electromagnetic waves obey the superposition principle as described in the waves in interference analysis model (which we discussed in Section 18.1 with respect to mechanical waves) because the differential equations involving E and B are linear equations. For example, we can add two waves with the same frequency and polarization simply by adding the magnitudes of the two electric fields algebraically.

Pitfall Prevention 34.2

$\vec{E}$ Stronger Than $\vec{B}$? Because the value of c is so large, some students incorrectly interpret Equation 34.21 as meaning that the electric field is much stronger than the magnetic field. Electric and magnetic fields are measured in different units, however, so they cannot be directly compared. In Section 34.4, we find that the electric and magnetic fields contribute equally to the wave's energy.

Quick Quiz 34.2 What is the phase difference between the sinusoidal oscillations of the electric and magnetic fields in Figure 34.8? **(a)** 180° **(b)** 90° **(c)** 0 **(d)** impossible to determine

Quick Quiz 34.3 An electromagnetic wave propagates in the negative y direction. The electric field at a point in space is momentarily oriented in the positive x direction. In which direction is the magnetic field at that point momentarily oriented? **(a)** the negative x direction **(b)** the positive y direction **(c)** the positive z direction **(d)** the negative z direction

Example 34.2 An Electromagnetic Wave AM

A sinusoidal electromagnetic wave of frequency 40.0 MHz travels in free space in the x direction as in Figure 34.9.

(A) Determine the wavelength and period of the wave.

34.2 continued

SOLUTION

Conceptualize Imagine the wave in Figure 34.9 moving to the right along the x axis, with the electric and magnetic fields oscillating in phase.

Categorize We use the mathematical representation of the *traveling wave* model for electromagnetic waves.

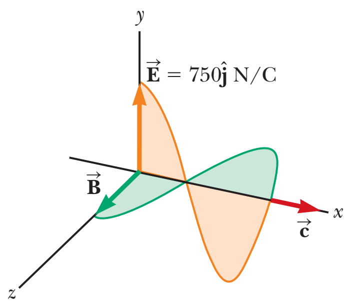


Figure 34.9 (Example 34.2) At some instant, a plane electromagnetic wave moving in the x direction has a maximum electric field of 750 N/C in the positive y direction.

Analyze

Use Equation 34.20 to find the wavelength of the wave:

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{40.0 \times 10^6 \text{ Hz}} = \boxed{7.50 \text{ m}}$$

Find the period T of the wave as the inverse of the frequency:

$$T = \frac{1}{f} = \frac{1}{40.0 \times 10^6 \text{ Hz}} = \boxed{2.50 \times 10^{-8} \text{ s}}$$

(B) At some point and at some instant, the electric field has its maximum value of 750 N/C and is directed along the y axis. Calculate the magnitude and direction of the magnetic field at this position and time.

SOLUTION

Use Equation 34.21 to find the magnitude of the magnetic field:

$$B_{\max} = \frac{E_{\max}}{c} = \frac{750 \text{ N/C}}{3.00 \times 10^8 \text{ m/s}} = \boxed{2.50 \times 10^{-6} \text{ T}}$$

Because $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ must be perpendicular to each other and perpendicular to the direction of wave propagation (x in this case), we conclude that $\vec{\mathbf{B}}$ is in the z direction.

Finalize Notice that the wavelength is several meters. This is relatively long for an electromagnetic wave. As we will see in Section 34.7, this wave belongs to the radio range of frequencies.

34.4 Energy Carried by Electromagnetic Waves

In our discussion of the nonisolated system model for energy in Section 8.1, we identified electromagnetic radiation as one method of energy transfer across the boundary of a system. The amount of energy transferred by electromagnetic waves is symbolized as T_{ER} in Equation 8.2. The rate of transfer of energy by an electromagnetic wave is described by a vector $\vec{\mathbf{S}}$, called the **Poynting vector,** which is defined by the expression

$$\vec{\mathbf{S}} \equiv \frac{1}{\mu_0}\vec{\mathbf{E}} \times \vec{\mathbf{B}} \quad \textbf{(34.22)}$$

◀ Poynting vector

The magnitude of the Poynting vector represents the rate at which energy passes through a unit surface area perpendicular to the direction of wave propagation. Therefore, the magnitude of $\vec{\mathbf{S}}$ represents *power per unit area.* The direction of the vector is along the direction of wave propagation (Fig. 34.10, page 1040). The SI units of $\vec{\mathbf{S}}$ are $\text{J/s} \cdot \text{m}^2 = \text{W/m}^2$.

As an example, let's evaluate the magnitude of $\vec{\mathbf{S}}$ for a plane electromagnetic wave where $|\vec{\mathbf{E}} \times \vec{\mathbf{B}}| = EB$. In this case,

$$S = \frac{EB}{\mu_0} \quad \textbf{(34.23)}$$

Pitfall Prevention 34.3

An Instantaneous Value The Poynting vector given by Equation 34.22 is time dependent. Its magnitude varies in time, reaching a maximum value at the same instant the magnitudes of $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ do. The *average* rate of energy transfer is given by Equation 34.24 on the next page.

Pitfall Prevention 34.4

Irradiance In this discussion, intensity is defined in the same way as in Chapter 17 (as power per unit area). In the optics industry, however, power per unit area is called the *irradiance.* Radiant intensity is defined as the power in watts per solid angle (measured in steradians).

Because $B = E/c$, we can also express this result as

$$S = \frac{E^2}{\mu_0 c} = \frac{cB^2}{\mu_0}$$

These equations for S apply at any instant of time and represent the *instantaneous* rate at which energy is passing through a unit area in terms of the instantaneous values of E and B.

What is of greater interest for a sinusoidal plane electromagnetic wave is the time average of S over one or more cycles, which is called the *wave intensity* I. (We discussed the intensity of sound waves in Chapter 17.) When this average is taken, we obtain an expression involving the time average of $\cos^2(kx - \omega t)$, which equals $\frac{1}{2}$. Hence, the average value of S (in other words, the intensity of the wave) is

Wave intensity ▶

$$I = S_{\text{avg}} = \frac{E_{\max} B_{\max}}{2\mu_0} = \frac{E_{\max}^2}{2\mu_0 c} = \frac{cB_{\max}^2}{2\mu_0} \qquad \textbf{(34.24)}$$

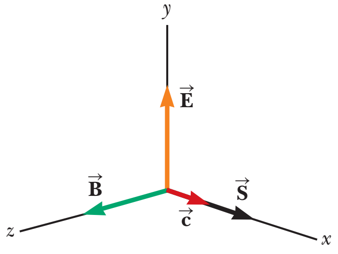


Figure 34.10 The Poynting vector $\vec{\mathbf{S}}$ for a plane electromagnetic wave is along the direction of wave propagation.

Recall that the energy per unit volume associated with an electric field, which is the instantaneous energy density u_E, is given by Equation 26.13:

$$u_E = \tfrac{1}{2}\epsilon_0 E^2$$

Also recall that the instantaneous energy density u_B associated with a magnetic field is given by Equation 32.14:

$$u_B = \frac{B^2}{2\mu_0}$$

Because E and B vary with time for an electromagnetic wave, the energy densities also vary with time. Using the relationships $B = E/c$ and $c = 1/\sqrt{\mu_0\epsilon_0}$, the expression for u_B becomes

$$u_B = \frac{(E/c)^2}{2\mu_0} = \frac{\mu_0\epsilon_0}{2\mu_0}E^2 = \tfrac{1}{2}\epsilon_0 E^2$$

Comparing this result with the expression for u_E, we see that

$$u_B = u_E = \tfrac{1}{2}\epsilon_0 E^2 = \frac{B^2}{2\mu_0}$$

That is, the instantaneous energy density associated with the magnetic field of an electromagnetic wave equals the instantaneous energy density associated with the electric field. Hence, in a given volume, the energy is equally shared by the two fields.

The **total instantaneous energy density** u is equal to the sum of the energy densities associated with the electric and magnetic fields:

Total instantaneous energy density of an electromagnetic wave ▶

$$u = u_E + u_B = \epsilon_0 E^2 = \frac{B^2}{\mu_0}$$

When this total instantaneous energy density is averaged over one or more cycles of an electromagnetic wave, we again obtain a factor of $\frac{1}{2}$. Hence, for any electromagnetic wave, the total average energy per unit volume is

Average energy density of an electromagnetic wave ▶

$$u_{\text{avg}} = \epsilon_0(E^2)_{\text{avg}} = \tfrac{1}{2}\epsilon_0 E_{\max}^2 = \frac{B_{\max}^2}{2\mu_0} \qquad \textbf{(34.25)}$$

Comparing this result with Equation 34.24 for the average value of S, we see that

$$I = S_{\text{avg}} = cu_{\text{avg}} \qquad \textbf{(34.26)}$$

In other words, the intensity of an electromagnetic wave equals the average energy density multiplied by the speed of light.

The Sun delivers about 10^3 W/m^2 of energy to the Earth's surface via electromagnetic radiation. Let's calculate the total power that is incident on the roof of a home. The roof's dimensions are 8.00 m × 20.0 m. We assume the average magnitude of the Poynting vector for solar radiation at the surface of the Earth is $S_{avg} = 1\,000$ W/m^2. This average value represents the power per unit area, or the light intensity. Assuming the radiation is incident normal to the roof, we obtain

$$P_{avg} = S_{avg}A = (1\,000\text{ W/m}^2)(8.00\text{ m} \times 20.0\text{ m}) = 1.60 \times 10^5\text{ W}$$

This power is large compared with the power requirements of a typical home. If this power could be absorbed and made available to electrical devices, it would provide more than enough energy for the average home. Solar energy is not easily harnessed, however, and the prospects for large-scale conversion are not as bright as may appear from this calculation. For example, the efficiency of conversion from solar energy is typically 12–18% for photovoltaic cells, reducing the available power by an order of magnitude. Other considerations reduce the power even further. Depending on location, the radiation is most likely not incident normal to the roof and, even if it is, this situation exists for only a short time near the middle of the day. No energy is available for about half of each day during the nighttime hours, and cloudy days further reduce the available energy. Finally, while energy is arriving at a large rate during the middle of the day, some of it must be stored for later use, requiring batteries or other storage devices. All in all, complete solar operation of homes is not currently cost effective for most homes.

Example 34.3 Fields on the Page

Estimate the maximum magnitudes of the electric and magnetic fields of the light that is incident on this page because of the visible light coming from your desk lamp. Treat the lightbulb as a point source of electromagnetic radiation that is 5% efficient at transforming energy coming in by electrical transmission to energy leaving by visible light.

SOLUTION

Conceptualize The filament in your lightbulb emits electromagnetic radiation. The brighter the light, the larger the magnitudes of the electric and magnetic fields.

Categorize Because the lightbulb is to be treated as a point source, it emits equally in all directions, so the outgoing electromagnetic radiation can be modeled as a spherical wave.

Analyze Recall from Equation 17.13 that the wave intensity I a distance r from a point source is $I = P_{avg}/4\pi r^2$, where P_{avg} is the average power output of the source and $4\pi r^2$ is the area of a sphere of radius r centered on the source.

Set this expression for I equal to the intensity of an electromagnetic wave given by Equation 34.24:

$$I = \frac{P_{avg}}{4\pi r^2} = \frac{E_{max}^2}{2\mu_0 c}$$

Solve for the electric field magnitude:

$$E_{max} = \sqrt{\frac{\mu_0 c P_{avg}}{2\pi r^2}}$$

Let's make some assumptions about numbers to enter in this equation. The visible light output of a 60-W lightbulb operating at 5% efficiency is approximately 3.0 W by visible light. (The remaining energy transfers out of the lightbulb by thermal conduction and invisible radiation.) A reasonable distance from the lightbulb to the page might be 0.30 m.

Substitute these values:

$$E_{max} = \sqrt{\frac{(4\pi \times 10^{-7}\text{ T}\cdot\text{m/A})(3.00 \times 10^8\text{ m/s})(3.0\text{ W})}{2\pi(0.30\text{ m})^2}}$$

$$= \boxed{45\text{ V/m}}$$

continued

34.3 continued

Use Equation 34.21 to find the magnetic field magnitude:

$$B_{\max} = \frac{E_{\max}}{c} = \frac{45 \text{ V/m}}{3.00 \times 10^8 \text{ m/s}} = \boxed{1.5 \times 10^{-7} \text{ T}}$$

Finalize This value of the magnetic field magnitude is two orders of magnitude smaller than the Earth's magnetic field.

34.5 Momentum and Radiation Pressure

Electromagnetic waves transport linear momentum as well as energy. As this momentum is absorbed by some surface, pressure is exerted on the surface. Therefore, the surface is a nonisolated system for momentum. In this discussion, let's assume the electromagnetic wave strikes the surface at normal incidence and transports a total energy T_{ER} to the surface in a time interval Δt. Maxwell showed that if the surface absorbs all the incident energy T_{ER} in this time interval (as does a black body, introduced in Section 20.7), the total momentum $\vec{\mathbf{p}}$ transported to the surface has a magnitude

Momentum transported to a perfectly absorbing surface

$$p = \frac{T_{ER}}{c} \quad \text{(complete absorption)} \qquad \textbf{(34.27)}$$

The pressure P exerted on the surface is defined as force per unit area F/A, which when combined with Newton's second law gives

$$P = \frac{F}{A} = \frac{1}{A}\frac{dp}{dt}$$

Substituting Equation 34.27 into this expression for pressure P gives

$$P = \frac{1}{A}\frac{dp}{dt} = \frac{1}{A}\frac{d}{dt}\left(\frac{T_{ER}}{c}\right) = \frac{1}{c}\frac{(dT_{ER}/dt)}{A}$$

We recognize $(dT_{ER}/dt)/A$ as the rate at which energy is arriving at the surface per unit area, which is the magnitude of the Poynting vector. Therefore, the radiation pressure P exerted on the perfectly absorbing surface is

Radiation pressure exerted on a perfectly absorbing surface

$$P = \frac{S}{c} \quad \text{(complete absorption)} \qquad \textbf{(34.28)}$$

If the surface is a perfect reflector (such as a mirror) and incidence is normal, the momentum transported to the surface in a time interval Δt is twice that given by Equation 34.27. That is, the momentum transferred to the surface by the incoming light is $p = T_{ER}/c$ and that transferred by the reflected light is also $p = T_{ER}/c$. Therefore,

Pitfall Prevention 34.5

So Many p's We have p for momentum and P for pressure, and they are both related to P for power! Be sure to keep all these symbols straight.

$$p = \frac{2T_{ER}}{c} \quad \text{(complete reflection)} \qquad \textbf{(34.29)}$$

The radiation pressure exerted on a perfectly reflecting surface for normal incidence of the wave is

Radiation pressure exerted on a perfectly reflecting surface

$$P = \frac{2S}{c} \quad \text{(complete reflection)} \qquad \textbf{(34.30)}$$

The pressure on a surface having a reflectivity somewhere between these two extremes has a value between S/c and $2S/c$, depending on the properties of the surface.

Although radiation pressures are very small (about 5×10^{-6} N/m^2 for direct sunlight), *solar sailing* is a low-cost means of sending spacecraft to the planets. Large sheets experience radiation pressure from sunlight and are used in much the way canvas sheets are used on earthbound sailboats. In 2010, the Japan Aerospace Exploration Agency (JAXA) launched the first spacecraft to use solar sailing as its primary propulsion, *IKAROS* (Interplanetary Kite-craft Accelerated by Radiation of the Sun). Successful testing of this spacecraft would lead to a larger effort to send a spacecraft to Jupiter by radiation pressure later in the present decade.

Quick Quiz 34.4 To maximize the radiation pressure on the sails of a spacecraft using solar sailing, should the sheets be **(a)** very black to absorb as much sunlight as possible or **(b)** very shiny to reflect as much sunlight as possible?

Conceptual Example 34.4 Sweeping the Solar System

A great amount of dust exists in interplanetary space. Although in theory these dust particles can vary in size from molecular size to a much larger size, very little of the dust in our solar system is smaller than about 0.2 μm. Why?

SOLUTION

The dust particles are subject to two significant forces: the gravitational force that draws them toward the Sun and the radiation-pressure force that pushes them away from the Sun. The gravitational force is proportional to the cube of the radius of a spherical dust particle because it is proportional to the mass and therefore to the volume $4\pi r^3/3$ of the particle. The radiation pressure is proportional to the square of the radius because it depends on the planar cross section of the particle. For large particles, the gravitational force is greater than the force from radiation pressure. For particles having radii less than about 0.2 μm, the radiation-pressure force is greater than the gravitational force. As a result, these particles are swept out of our solar system by sunlight.

Example 34.5 Pressure from a Laser Pointer

When giving presentations, many people use a laser pointer to direct the attention of the audience to information on a screen. If a 3.0-mW pointer creates a spot on a screen that is 2.0 mm in diameter, determine the radiation pressure on a screen that reflects 70% of the light that strikes it. The power 3.0 mW is a time-averaged value.

SOLUTION

Conceptualize Imagine the waves striking the screen and exerting a radiation pressure on it. The pressure should not be very large.

Categorize This problem involves a calculation of radiation pressure using an approach like that leading to Equation 34.28 or Equation 34.30, but it is complicated by the 70% reflection.

Analyze We begin by determining the magnitude of the beam's Poynting vector.

Divide the time-averaged power delivered via the electromagnetic wave by the cross-sectional area of the beam:

$$S_{\text{avg}} = \frac{(Power)_{\text{avg}}}{A} = \frac{(Power)_{\text{avg}}}{\pi r^2} = \frac{3.0 \times 10^{-3}\ \text{W}}{\pi\left(\dfrac{2.0 \times 10^{-3}\ \text{m}}{2}\right)^2} = 955\ \text{W/m}^2$$

Now let's determine the radiation pressure from the laser beam. Equation 34.30 indicates that a completely reflected beam would apply an average pressure of $P_{\text{avg}} = 2S_{\text{avg}}/c$. We can model the actual reflection as follows. Imagine that the surface absorbs the beam, resulting in pressure $P_{\text{avg}} = S_{\text{avg}}/c$. Then the surface emits the beam, resulting in additional pressure $P_{\text{avg}} = S_{\text{avg}}/c$. If the surface emits only a fraction f of the beam (so that f is the amount of the incident beam reflected), the pressure due to the emitted beam is $P_{\text{avg}} = fS_{\text{avg}}/c$.

Use this model to find the total pressure on the surface due to absorption and re-emission (reflection):

$$P_{\text{avg}} = \frac{S_{\text{avg}}}{c} + f\,\frac{S_{\text{avg}}}{c} = (1+f)\frac{S_{\text{avg}}}{c}$$

continued

34.5 continued

Evaluate this pressure for a beam that is 70% reflected: $P_{avg} = (1 + 0.70)\dfrac{955\ \text{W/m}^2}{3.0 \times 10^8\ \text{m/s}} = \boxed{5.4 \times 10^{-6}\ \text{N/m}^2}$

Finalize The pressure has an extremely small value, as expected. (Recall from Section 14.2 that atmospheric pressure is approximately 10^5 N/m^2.) Consider the magnitude of the Poynting vector, S_{avg} = 955 W/m^2. It is about the same as the intensity of sunlight at the Earth's surface. For this reason, it is not safe to shine the beam of a laser pointer into a person's eyes, which may be more dangerous than looking directly at the Sun.

WHAT IF? What if the laser pointer is moved twice as far away from the screen? Does that affect the radiation pressure on the screen?

Answer Because a laser beam is popularly represented as a beam of light with constant cross section, you might think that the intensity of radiation, and therefore the radiation pressure, is independent of distance from the screen. A laser beam, however, does not have a constant cross section at all distances from the source; rather, there is a small but measurable divergence of the beam. If the laser is moved farther away from the screen, the area of illumination on the screen increases, decreasing the intensity. In turn, the radiation pressure is reduced.

In addition, the doubled distance from the screen results in more loss of energy from the beam due to scattering from air molecules and dust particles as the light travels from the laser to the screen. This energy loss further reduces the radiation pressure on the screen.

34.6 Production of Electromagnetic Waves by an Antenna

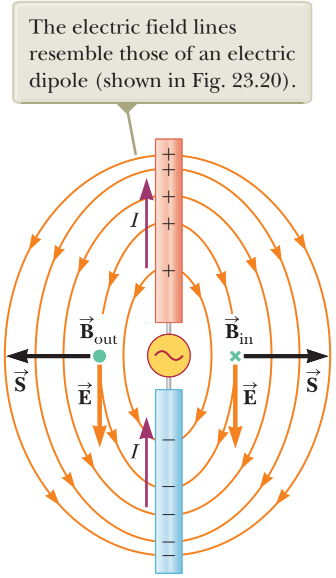


Figure 34.11 A half-wave antenna consists of two metal rods connected to an alternating voltage source. This diagram shows $\vec{E}$ and $\vec{B}$ at an arbitrary instant when the current is upward.

Stationary charges and steady currents cannot produce electromagnetic waves. If the current in a wire changes with time, however, the wire emits electromagnetic waves. The fundamental mechanism responsible for this radiation is the acceleration of a charged particle. **Whenever a charged particle accelerates, energy is transferred away from the particle by electromagnetic radiation.**

Let's consider the production of electromagnetic waves by a *half-wave antenna*. In this arrangement, two conducting rods are connected to a source of alternating voltage (such as an *LC* oscillator) as shown in Figure 34.11. The length of each rod is equal to one-quarter the wavelength of the radiation emitted when the oscillator operates at frequency *f*. The oscillator forces charges to accelerate back and forth between the two rods. Figure 34.11 shows the configuration of the electric and magnetic fields at some instant when the current is upward. The separation of charges in the upper and lower portions of the antenna make the electric field lines resemble those of an electric dipole. (As a result, this type of antenna is sometimes called a *dipole antenna*.) Because these charges are continuously oscillating between the two rods, the antenna can be approximated by an oscillating electric dipole. The current representing the movement of charges between the ends of the antenna produces magnetic field lines forming concentric circles around the antenna that are perpendicular to the electric field lines at all points. The magnetic field is zero at all points along the axis of the antenna. Furthermore, $\vec{E}$ and $\vec{B}$ are 90° out of phase in time; for example, the current is zero when the charges at the outer ends of the rods are at a maximum.

At the two points where the magnetic field is shown in Figure 34.11, the Poynting vector $\vec{S}$ is directed radially outward, indicating that energy is flowing away from the antenna at this instant. At later times, the fields and the Poynting vector reverse direction as the current alternates. Because $\vec{E}$ and $\vec{B}$ are 90° out of phase at points near the dipole, the net energy flow is zero. From this fact, you might conclude (incorrectly) that no energy is radiated by the dipole.

Energy is indeed radiated, however. Because the dipole fields fall off as $1/r^3$ (as shown in Example 23.6 for the electric field of a static dipole), they are negligible at great distances from the antenna. At these great distances, something else causes a type of radiation different from that close to the antenna. The source of this radiation is the continuous induction of an electric field by the time-varying magnetic field and the induction of a magnetic field by the time-varying electric field, predicted by Equations 34.6 and 34.7. The electric and magnetic fields produced in this manner are in phase with each other and vary as $1/r$. The result is an outward flow of energy at all times.

The angular dependence of the radiation intensity produced by a dipole antenna is shown in Figure 34.12. Notice that the intensity and the power radiated are a maximum in a plane that is perpendicular to the antenna and passing through its midpoint. Furthermore, the power radiated is zero along the antenna's axis. A mathematical solution to Maxwell's equations for the dipole antenna shows that the intensity of the radiation varies as $(\sin^2 \theta)/r^2$, where θ is measured from the axis of the antenna.

Electromagnetic waves can also induce currents in a receiving antenna. The response of a dipole receiving antenna at a given position is a maximum when the antenna axis is parallel to the electric field at that point and zero when the axis is perpendicular to the electric field.

Quick Quiz 34.5 If the antenna in Figure 34.11 represents the source of a distant radio station, what would be the best orientation for your portable radio antenna located to the right of the figure? **(a)** up-down along the page **(b)** left-right along the page **(c)** perpendicular to the page

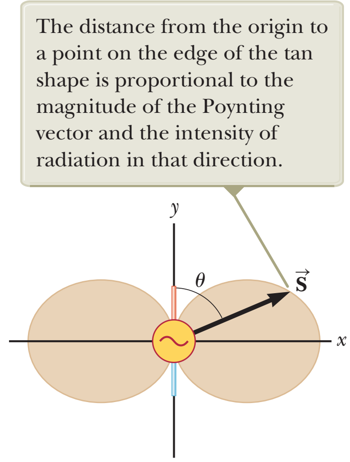


Figure 34.12 Angular dependence of the intensity of radiation produced by an oscillating electric dipole.

34.7 The Spectrum of Electromagnetic Waves

The various types of electromagnetic waves are listed in Figure 34.13 (page 1046), which shows the **electromagnetic spectrum.** Notice the wide ranges of frequencies and wavelengths. No sharp dividing point exists between one type of wave and the next. Remember that all forms of the various types of radiation are produced by the same phenomenon: acceleration of electric charges. The names given to the types of waves are simply a convenient way to describe the region of the spectrum in which they lie.

Radio waves, whose wavelengths range from more than 10^4 m to about 0.1 m, are the result of charges accelerating through conducting wires. They are generated by such electronic devices as *LC* oscillators and are used in radio and television communication systems.

Microwaves have wavelengths ranging from approximately 0.3 m to 10^{-4} m and are also generated by electronic devices. Because of their short wavelengths, they are well suited for radar systems and for studying the atomic and molecular properties of matter. Microwave ovens are an interesting domestic application of these waves. It has been suggested that solar energy could be harnessed by beaming microwaves to the Earth from a solar collector in space.

Infrared waves have wavelengths ranging from approximately 10^{-3} m to the longest wavelength of visible light, 7×10^{-7} m. These waves, produced by molecules and room-temperature objects, are readily absorbed by most materials. The infrared (IR) energy absorbed by a substance appears as internal energy because the energy agitates the object's atoms, increasing their vibrational or translational motion, which results in a temperature increase. Infrared radiation has practical and scientific applications in many areas, including physical therapy, IR photography, and vibrational spectroscopy.

Visible light, the most familiar form of electromagnetic waves, is the part of the electromagnetic spectrum the human eye can detect. Light is produced by the rearrangement of electrons in atoms and molecules. The various wavelengths of visible light, which correspond to different colors, range from red ($\lambda \approx 7 \times 10^{-7}$ m) to violet ($\lambda \approx 4 \times 10^{-7}$ m). The sensitivity of the human eye is a function of wavelength, being a maximum at a wavelength of about 5.5×10^{-7} m. With that in mind, why do you suppose tennis balls often have a yellow-green color? Table 34.1 provides

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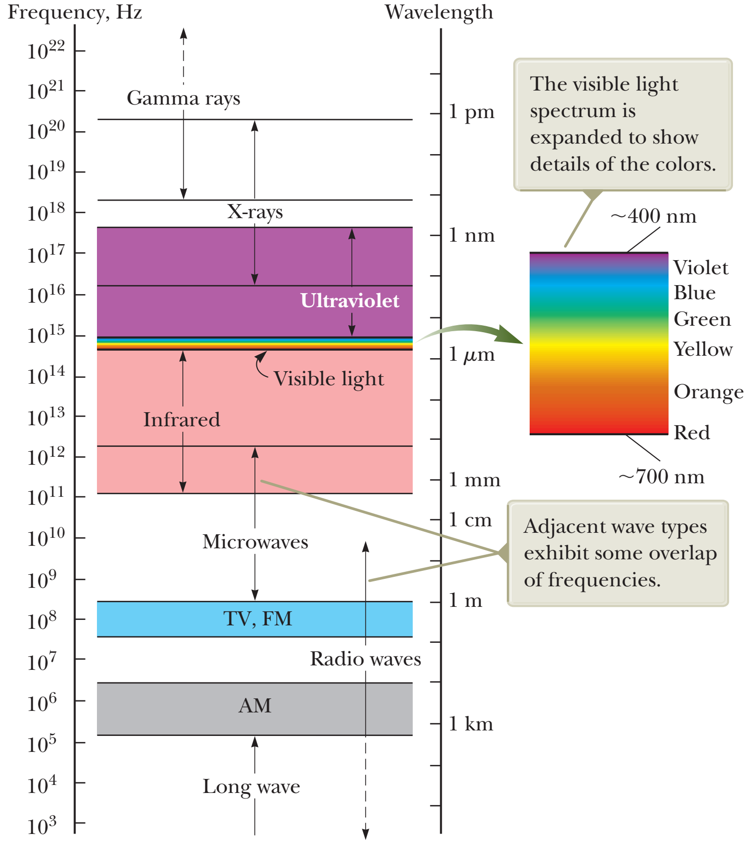
"Heat Rays" Infrared rays are often called "heat rays," but this terminology is a misnomer. Although infrared radiation is used to raise or maintain temperature as in the case of keeping food warm with "heat lamps" at a fast-food restaurant, all wavelengths of electromagnetic radiation carry energy that can cause the temperature of a system to increase. As an example, consider a potato baking in your microwave oven.

Table 34.1 Approximate Correspondence Between Wavelengths of Visible Light and Color

Wavelength Range (nm)	Color Description
400–430	Violet
430–485	Blue
485–560	Green
560–590	Yellow
590–625	Orange
625–700	Red

Note: The wavelength ranges here are approximate. Different people will describe colors differently.

Figure 34.13 The electromagnetic spectrum.



approximate correspondences between the wavelength of visible light and the color assigned to it by humans. Light is the basis of the science of optics and optical instruments, to be discussed in Chapters 35 through 38.

Raymond A. Serway

Wearing sunglasses that do not block ultraviolet (UV) light is worse for your eyes than wearing no sunglasses at all. The lenses of any sunglasses absorb some visible light, thereby causing the wearer's pupils to dilate. If the glasses do not also block UV light, more damage may be done to the lens of the eye because of the dilated pupils. If you wear no sunglasses at all, your pupils are contracted, you squint, and much less UV light enters your eyes. High-quality sunglasses block nearly all the eye-damaging UV light.

Ultraviolet waves cover wavelengths ranging from approximately 4×10^{-7} m to 6×10^{-10} m. The Sun is an important source of ultraviolet (UV) light, which is the main cause of sunburn. Sunscreen lotions are transparent to visible light but absorb most UV light. The higher a sunscreen's solar protection factor, or SPF, the greater the percentage of UV light absorbed. Ultraviolet rays have also been implicated in the formation of cataracts, a clouding of the lens inside the eye.

Most of the UV light from the Sun is absorbed by ozone (O_3) molecules in the Earth's upper atmosphere, in a layer called the stratosphere. This ozone shield converts lethal high-energy UV radiation to IR radiation, which in turn warms the stratosphere.

X-rays have wavelengths in the range from approximately 10^{-8} m to 10^{-12} m. The most common source of x-rays is the stopping of high-energy electrons upon bombarding a metal target. X-rays are used as a diagnostic tool in medicine and as a treatment for certain forms of cancer. Because x-rays can damage or destroy living tissues and organisms, care must be taken to avoid unnecessary exposure or overexposure. X-rays are also used in the study of crystal structure because x-ray wavelengths are comparable to the atomic separation distances in solids (about 0.1 nm).

Gamma rays are electromagnetic waves emitted by radioactive nuclei and during certain nuclear reactions. High-energy gamma rays are a component of cosmic rays that enter the Earth's atmosphere from space. They have wavelengths ranging from approximately 10^{-10} m to less than 10^{-14} m. Gamma rays are highly penetrating and produce serious damage when absorbed by living tissues. Consequently, those working near such dangerous radiation must be protected with heavily absorbing materials such as thick layers of lead.

Quick Quiz 34.6 In many kitchens, a microwave oven is used to cook food. The frequency of the microwaves is on the order of 10^{10} Hz. Are the wavelengths of these microwaves on the order of **(a)** kilometers, **(b)** meters, **(c)** centimeters, or **(d)** micrometers?

Quick Quiz 34.7 A radio wave of frequency on the order of 10^5 Hz is used to carry a sound wave with a frequency on the order of 10^3 Hz. Is the wavelength of this radio wave on the order of **(a)** kilometers, **(b)** meters, **(c)** centimeters, or **(d)** micrometers?

Summary

Definitions

In a region of space in which there is a changing electric field, there is a **displacement current** defined as

$$I_d \equiv \epsilon_0 \frac{d\Phi_E}{dt} \quad \textbf{(34.1)}$$

where ϵ_0 is the permittivity of free space (see Section 23.3) and $\Phi_E = \int \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$ is the electric flux.

The rate at which energy passes through a unit area by electromagnetic radiation is described by the **Poynting vector $\vec{\mathbf{S}}$,** where

$$\vec{\mathbf{S}} \equiv \frac{1}{\mu_0} \vec{\mathbf{E}} \times \vec{\mathbf{B}} \quad \textbf{(34.22)}$$

Concepts and Principles

When used with the **Lorentz force law, $\vec{\mathbf{F}} = q\vec{\mathbf{E}} + q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$, Maxwell's equations** describe all electromagnetic phenomena:

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q}{\epsilon_0} \quad \textbf{(34.4)}$$

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0 \quad \textbf{(34.5)}$$

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt} \quad \textbf{(34.6)}$$

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I + \epsilon_0\mu_0 \frac{d\Phi_E}{dt} \quad \textbf{(34.7)}$$

Electromagnetic waves, which are predicted by Maxwell's equations, have the following properties and are described by the following mathematical representations of the traveling wave model for electromagnetic waves.

- The electric field and the magnetic field each satisfy a wave equation. These two wave equations, which can be obtained from Maxwell's third and fourth equations, are

$$\frac{\partial^2 E}{\partial x^2} = \mu_0\epsilon_0 \frac{\partial^2 E}{\partial t^2} \quad \textbf{(34.15)}$$

$$\frac{\partial^2 B}{\partial x^2} = \mu_0\epsilon_0 \frac{\partial^2 B}{\partial t^2} \quad \textbf{(34.16)}$$

- The waves travel through a vacuum with the speed of light c, where

$$c = \frac{1}{\sqrt{\mu_0\epsilon_0}} \quad \textbf{(34.17)}$$

- Numerically, the speed of electromagnetic waves in a vacuum is 3.00×10^8 m/s.
- The wavelength and frequency of electromagnetic waves are related by

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{f} \quad \textbf{(34.20)}$$

- The electric and magnetic fields are perpendicular to each other and perpendicular to the direction of wave propagation.
- The instantaneous magnitudes of $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ in an electromagnetic wave are related by the expression

$$\frac{E}{B} = c \quad \textbf{(34.21)}$$

- Electromagnetic waves carry energy.
- Electromagnetic waves carry momentum.

continued

Because electromagnetic waves carry momentum, they exert pressure on surfaces. If an electromagnetic wave whose Poynting vector is $\vec{\mathbf{S}}$ is completely absorbed by a surface upon which it is normally incident, the radiation pressure on that surface is

$$P = \frac{S}{c} \quad \text{(complete absorption)} \tag{34.28}$$

If the surface totally reflects a normally incident wave, the pressure is doubled.

The electric and magnetic fields of a sinusoidal plane electromagnetic wave propagating in the positive x direction can be written as

$$E = E_{\max} \cos (kx - \omega t) \tag{34.18}$$

$$B = B_{\max} \cos (kx - \omega t) \tag{34.19}$$

where k is the angular wave number and ω is the angular frequency of the wave. These equations represent special solutions to the wave equations for E and B.

The average value of the Poynting vector for a plane electromagnetic wave has a magnitude

$$S_{\text{avg}} = \frac{E_{\max} B_{\max}}{2\mu_0} = \frac{E_{\max}^2}{2\mu_0 c} = \frac{cB_{\max}^2}{2\mu_0} \tag{34.24}$$

The intensity of a sinusoidal plane electromagnetic wave equals the average value of the Poynting vector taken over one or more cycles.

The electromagnetic spectrum includes waves covering a broad range of wavelengths, from long radio waves at more than 10^4 m to gamma rays at less than 10^{-14} m.

Objective Questions

1. denotes answer available in *Student Solutions Manual/Study Guide*

1. A spherical interplanetary grain of dust of radius 0.2 μm is at a distance r_1 from the Sun. The gravitational force exerted by the Sun on the grain just balances the force due to radiation pressure from the Sun's light. **(i)** Assume the grain is moved to a distance $2r_1$ from the Sun and released. At this location, what is the net force exerted on the grain? (a) toward the Sun (b) away from the Sun (c) zero (d) impossible to determine without knowing the mass of the grain **(ii)** Now assume the grain is moved back to its original location at r_1, compressed so that it crystallizes into a sphere with significantly higher density, and then released. In this situation, what is the net force exerted on the grain? Choose from the same possibilities as in part (i).

2. A small source radiates an electromagnetic wave with a single frequency into vacuum, equally in all directions. **(i)** As the wave moves, does its frequency (a) increase, (b) decrease, or (c) stay constant? Using the same choices, answer the same question about **(ii)** its wavelength, **(iii)** its speed, **(iv)** its intensity, and **(v)** the amplitude of its electric field.

3. A typical microwave oven operates at a frequency of 2.45 GHz. What is the wavelength associated with the electromagnetic waves in the oven? (a) 8.20 m (b) 12.2 cm (c) 1.20×10^8 m (d) 8.20×10^{-9} m (e) none of those answers

4. A student working with a transmitting apparatus like Heinrich Hertz's wishes to adjust the electrodes to generate electromagnetic waves with a frequency half as large as before. **(i)** How large should she make the effective capacitance of the pair of electrodes? (a) four times larger than before (b) two times larger than before (c) one-half as large as before (d) one-fourth as large as before (e) none of those answers **(ii)** After she makes the required adjustment, what will the wavelength of the transmitted wave be? Choose from the same possibilities as in part (i).

5. Assume you charge a comb by running it through your hair and then hold the comb next to a bar magnet. Do the electric and magnetic fields produced constitute an electromagnetic wave? (a) Yes they do, necessarily. (b) Yes they do because charged particles are moving inside the bar magnet. (c) They can, but only if the electric field of the comb and the magnetic field of the magnet are perpendicular. (d) They can, but only if both the comb and the magnet are moving. (e) They can, if either the comb or the magnet or both are accelerating.

6. Which of the following statements are true regarding electromagnetic waves traveling through a vacuum? More than one statement may be correct. (a) All waves have the same wavelength. (b) All waves have the same frequency. (c) All waves travel at 3.00×10^8 m/s. (d) The electric and magnetic fields associated with the waves are perpendicular to each other and to the direction of wave propagation. (e) The speed of the waves depends on their frequency.

7. A plane electromagnetic wave with a single frequency moves in vacuum in the positive x direction. Its amplitude is uniform over the yz plane. **(i)** As the wave moves, does its frequency (a) increase, (b) decrease, or (c) stay constant? Using the same choices, answer the same question about **(ii)** its wavelength, **(iii)** its speed, **(iv)** its intensity, and **(v)** the amplitude of its magnetic field.

8. Assume the amplitude of the electric field in a plane electromagnetic wave is E_1 and the amplitude of the magnetic field is B_1. The source of the wave is then adjusted so that the amplitude of the electric field doubles to become $2E_1$. **(i)** What happens to the amplitude of the magnetic field in this process? (a) It becomes four times larger. (b) It becomes two times larger. (c) It can stay constant. (d) It becomes one-half as large. (e) It becomes one-fourth as large. **(ii)** What happens to the intensity of the wave? Choose from the same possibilities as in part (i).

9. An electromagnetic wave with a peak magnetic field magnitude of 1.50×10^{-7} T has an associated peak electric field of what magnitude? (a) 0.500×10^{-15} N/C (b) 2.00×10^{-5} N/C (c) 2.20×10^{4} N/C (d) 45.0 N/C (e) 22.0 N/C

10. **(i)** Rank the following kinds of waves according to their wavelength ranges from those with the largest typical or average wavelength to the smallest, noting any cases of equality: (a) gamma rays (b) microwaves (c) radio waves (d) visible light (e) x-rays **(ii)** Rank the kinds of waves according to their frequencies from highest to lowest. **(iii)** Rank the kinds of waves according to their speeds in vacuum from fastest to slowest.

11. Consider an electromagnetic wave traveling in the positive y direction. The magnetic field associated with the wave at some location at some instant points in the negative x direction as shown in Figure OQ34.11. What is the direction of the electric field at this position and at this instant? (a) the positive x direction (b) the positive y direction (c) the positive z direction (d) the negative z direction (e) the negative y direction

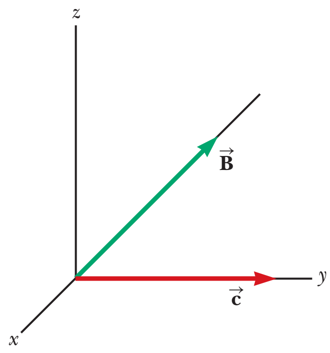


Figure OQ34.11

Conceptual Questions

1. denotes answer available in *Student Solutions Manual/Study Guide*

1. Suppose a creature from another planet has eyes that are sensitive to infrared radiation. Describe what the alien would see if it looked around your library. In particular, what would appear bright and what would appear dim?

2. For a given incident energy of an electromagnetic wave, why is the radiation pressure on a perfectly reflecting surface twice as great as that on a perfectly absorbing surface?

3. Radio stations often advertise "instant news." If that means you can hear the news the instant the radio announcer speaks it, is the claim true? What approximate time interval is required for a message to travel from Maine to California by radio waves? (Assume the waves can be detected at this range.)

4. List at least three differences between sound waves and light waves.

5. If a high-frequency current exists in a solenoid containing a metallic core, the core becomes warm due to induction. Explain why the material rises in temperature in this situation.

6. When light (or other electromagnetic radiation) travels across a given region, (a) what is it that oscillates? (b) What is it that is transported?

7. Why should an infrared photograph of a person look different from a photograph taken with visible light?

8. Do Maxwell's equations allow for the existence of magnetic monopoles? Explain.

9. Despite the advent of digital television, some viewers still use "rabbit ears" atop their sets (Fig. CQ34.9) instead of purchasing cable television service or satellite dishes. Certain orientations of the receiving antenna on a television set give better reception than others. Furthermore, the best orientation varies from station to station. Explain.

Figure CQ34.9 Conceptual Question 9 and Problem 78.

10. What does a radio wave do to the charges in the receiving antenna to provide a signal for your car radio?

11. Describe the physical significance of the Poynting vector.

12. An empty plastic or glass dish being removed from a microwave oven can be cool to the touch, even when food on an adjoining dish is hot. How is this phenomenon possible?

13. What new concept did Maxwell's generalized form of Ampère's law include?

Problems

The problems found in this chapter may be assigned online in Enhanced WebAssign

1. straightforward; **2.** intermediate; **3.** challenging

1. full solution available in the *Student Solutions Manual/Study Guide*

AMT Analysis Model tutorial available in Enhanced WebAssign

GP Guided Problem

M Master It tutorial available in Enhanced WebAssign

W Watch It video solution available in Enhanced WebAssign

Section 34.1 Displacement Current and the General Form of Ampère's Law

1. Consider the situation shown in Figure P34.1. An electric field of 300 V/m is confined to a circular area $d =$ 10.0 cm in diameter and directed outward perpendicular to the plane of the figure. If the field is increasing at a rate of 20.0 V/m · s, what are (a) the direction and (b) the magnitude of the magnetic field at the point P, $r = 15.0$ cm from the center of the circle?

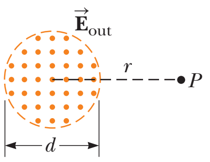


Figure P34.1

2. W A 0.200-A current is charging a capacitor that has circular plates 10.0 cm in radius. If the plate separation is 4.00 mm, (a) what is the time rate of increase of electric field between the plates? (b) What is the magnetic field between the plates 5.00 cm from the center?

3. M A 0.100-A current is charging a capacitor that has square plates 5.00 cm on each side. The plate separation is 4.00 mm. Find (a) the time rate of change of electric flux between the plates and (b) the displacement current between the plates.

Section 34.2 Maxwell's Equations and Hertz's Discoveries

4. AMT W An electron moves through a uniform electric field $\vec{\mathbf{E}} = (2.50\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}})$ V/m and a uniform magnetic field $\vec{\mathbf{B}} = 0.400\hat{\mathbf{k}}$ T. Determine the acceleration of the electron when it has a velocity $\vec{\mathbf{v}} = 10.0\hat{\mathbf{i}}$ m/s.

5. M A proton moves through a region containing a uniform electric field given by $\vec{\mathbf{E}} = 50.0\hat{\mathbf{j}}$ V/m and a uniform magnetic field $\vec{\mathbf{B}} = (0.200\hat{\mathbf{i}} + 0.300\hat{\mathbf{j}} + 0.400\hat{\mathbf{k}})$ T. Determine the acceleration of the proton when it has a velocity $\vec{\mathbf{v}} = 200\hat{\mathbf{i}}$ m/s.

6. A very long, thin rod carries electric charge with the linear density 35.0 nC/m. It lies along the x axis and moves in the x direction at a speed of 1.50×10^7 m/s. (a) Find the electric field the rod creates at the point ($x = 0$, $y = 20.0$ cm, $z = 0$). (b) Find the magnetic field it creates at the same point. (c) Find the force exerted on an electron at this point, moving with a velocity of $(2.40 \times 10^8)\hat{\mathbf{i}}$ m/s.

Section 34.3 Plane Electromagnetic Waves

Note: Assume the medium is vacuum unless specified otherwise.

7. Suppose you are located 180 m from a radio transmitter. (a) How many wavelengths are you from the transmitter if the station calls itself 1150 AM? (The AM band frequencies are in kilohertz.) (b) What if this station is 98.1 FM? (The FM band frequencies are in megahertz.)

8. A diathermy machine, used in physiotherapy, generates electromagnetic radiation that gives the effect of "deep heat" when absorbed in tissue. One assigned frequency for diathermy is 27.33 MHz. What is the wavelength of this radiation?

9. The distance to the North Star, Polaris, is approximately 6.44×10^{18} m. (a) If Polaris were to burn out today, how many years from now would we see it disappear? (b) What time interval is required for sunlight to reach the Earth? (c) What time interval is required for a microwave signal to travel from the Earth to the Moon and back?

10. The red light emitted by a helium–neon laser has a wavelength of 632.8 nm. What is the frequency of the light waves?

11. **Review.** A standing-wave pattern is set up by radio waves between two metal sheets 2.00 m apart, which is the shortest distance between the plates that produces a standing-wave pattern. What is the frequency of the radio waves?

12. W An electromagnetic wave in vacuum has an electric field amplitude of 220 V/m. Calculate the amplitude of the corresponding magnetic field.

13. The speed of an electromagnetic wave traveling in a transparent nonmagnetic substance is $v = 1/\sqrt{\kappa\mu_0\epsilon_0}$, where κ is the dielectric constant of the substance. Determine the speed of light in water, which has a dielectric constant of 1.78 at optical frequencies.

14. A radar pulse returns to the transmitter–receiver after a total travel time of 4.00×10^{-4} s. How far away is the object that reflected the wave?

15. M Figure P34.15 shows a plane electromagnetic sinusoidal wave propagating in the x direction. Suppose the wavelength is 50.0 m and the electric field vibrates in the xy plane with an amplitude of 22.0 V/m. Calculate

(a) the frequency of the wave and (b) the magnetic field $\vec{\mathbf{B}}$ when the electric field has its maximum value in the negative y direction. (c) Write an expression for $\vec{\mathbf{B}}$ with the correct unit vector, with numerical values for B_{max}, k, and ω, and with its magnitude in the form

$$B = B_{max} \cos (kx - \omega t)$$

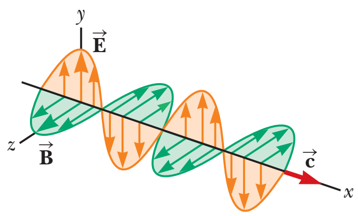


Figure P34.15 Problems 15 and 70.

16. Verify by substitution that the following equations are solutions to Equations 34.15 and 34.16, respectively:

$$E = E_{max} \cos (kx - \omega t)$$

$$B = B_{max} \cos (kx - \omega t)$$

17. AMT **Review.** A microwave oven is powered by a magnetron, an electronic device that generates electromagnetic waves of frequency 2.45 GHz. The microwaves enter the oven and are reflected by the walls. The standing-wave pattern produced in the oven can cook food unevenly, with hot spots in the food at antinodes and cool spots at nodes, so a turntable is often used to rotate the food and distribute the energy. If a microwave oven intended for use with a turntable is instead used with a cooking dish in a fixed position, the antinodes can appear as burn marks on foods such as carrot strips or cheese. The separation distance between the burns is measured to be 6 cm ± 5%. From these data, calculate the speed of the microwaves.

18. *Why is the following situation impossible?* An electromagnetic wave travels through empty space with electric and magnetic fields described by

$$E = 9.00 \times 10^3 \cos [(9.00 \times 10^6)x - (3.00 \times 10^{15})t]$$

$$B = 3.00 \times 10^{-5} \cos [(9.00 \times 10^6)x - (3.00 \times 10^{15})t]$$

where all numerical values and variables are in SI units.

19. M In SI units, the electric field in an electromagnetic wave is described by

$$E_y = 100 \sin (1.00 \times 10^7 x - \omega t)$$

Find (a) the amplitude of the corresponding magnetic field oscillations, (b) the wavelength λ, and (c) the frequency f.

Section 34.4 Energy Carried by Electromagnetic Waves

20. At what distance from the Sun is the intensity of sunlight three times the value at the Earth? (The average Earth–Sun separation is 1.496×10^{11} m.)

21. W If the intensity of sunlight at the Earth's surface under a fairly clear sky is 1 000 W/m^2, how much electromagnetic energy per cubic meter is contained in sunlight?

22. The power of sunlight reaching each square meter of the Earth's surface on a clear day in the tropics is close to 1 000 W. On a winter day in Manitoba, the power concentration of sunlight can be 100 W/m^2. Many human activities are described by a power per unit area on the order of 10^2 W/m^2 or less. (a) Consider, for example, a family of four paying $66 to the electric company every 30 days for 600 kWh of energy carried by electrical transmission to their house, which has floor dimensions of 13.0 m by 9.50 m. Compute the power per unit area used by the family. (b) Consider a car 2.10 m wide and 4.90 m long traveling at 55.0 mi/h using gasoline having "heat of combustion" 44.0 MJ/kg with fuel economy 25.0 mi/gal. One gallon of gasoline has a mass of 2.54 kg. Find the power per unit area used by the car. (c) Explain why direct use of solar energy is not practical for running a conventional automobile. (d) What are some uses of solar energy that are more practical?

23. M A community plans to build a facility to convert solar radiation to electrical power. The community requires 1.00 MW of power, and the system to be installed has an efficiency of 30.0% (that is, 30.0% of the solar energy incident on the surface is converted to useful energy that can power the community). Assuming sunlight has a constant intensity of 1 000 W/m^2, what must be the effective area of a perfectly absorbing surface used in such an installation?

24. In a region of free space, the electric field at an instant of time is $\vec{\mathbf{E}} = (80.0\hat{\mathbf{i}} + 32.0\hat{\mathbf{j}} - 64.0\hat{\mathbf{k}})$ N/C and the magnetic field is $\vec{\mathbf{B}} = (0.200\hat{\mathbf{i}} + 0.080\,0\hat{\mathbf{j}} + 0.290\hat{\mathbf{k}})\,\mu$T. (a) Show that the two fields are perpendicular to each other. (b) Determine the Poynting vector for these fields.

25. When a high-power laser is used in the Earth's atmosphere, the electric field associated with the laser beam can ionize the air, turning it into a conducting plasma that reflects the laser light. In dry air at 0°C and 1 atm, electric breakdown occurs for fields with amplitudes above about 3.00 MV/m. (a) What laser beam intensity will produce such a field? (b) At this maximum intensity, what power can be delivered in a cylindrical beam of diameter 5.00 mm?

26. **Review.** Model the electromagnetic wave in a microwave oven as a plane traveling wave moving to the left, with an intensity of 25.0 kW/m^2. An oven contains two cubical containers of small mass, each full of water. One has an edge length of 6.00 cm, and the other, 12.0 cm. Energy falls perpendicularly on one face of each container. The water in the smaller container absorbs 70.0% of the energy that falls on it. The water in the larger container absorbs 91.0%. That is, the fraction 0.300 of the incoming microwave energy passes through a 6.00-cm thickness of water, and the fraction (0.300)(0.300) = 0.090 passes through a 12.0-cm thickness. Assume a negligible amount of energy leaves either container by heat. Find the temperature change of the water in each container over a time interval of 480 s.

27. W High-power lasers in factories are used to cut through cloth and metal (Fig. P34.27). One such laser has a beam diameter of 1.00 mm and generates an electric field having an amplitude of 0.700 MV/m at the target. Find (a) the amplitude of the magnetic field produced, (b) the intensity of the laser, and (c) the power delivered by the laser.

Philippe Plailly/SPL/Photo Researchers, Inc.

Figure P34.27

28. Consider a bright star in our night sky. Assume its distance from the Earth is 20.0 light-years (ly) and its power output is 4.00×10^{28} W, about 100 times that of the Sun. (a) Find the intensity of the starlight at the Earth. (b) Find the power of the starlight the Earth intercepts. One light-year is the distance traveled by light through a vacuum in one year.

29. M What is the average magnitude of the Poynting vector 5.00 mi from a radio transmitter broadcasting isotropically (equally in all directions) with an average power of 250 kW?

30. Assuming the antenna of a 10.0-kW radio station radiates spherical electromagnetic waves, (a) compute the maximum value of the magnetic field 5.00 km from the antenna and (b) state how this value compares with the surface magnetic field of the Earth.

31. W **Review.** An AM radio station broadcasts isotropically (equally in all directions) with an average power of 4.00 kW. A receiving antenna 65.0 cm long is at a location 4.00 mi from the transmitter. Compute the amplitude of the emf that is induced by this signal between the ends of the receiving antenna.

32. At what distance from a 100-W electromagnetic wave point source does $E_{max} = 15.0$ V/m?

33. The filament of an incandescent lamp has a 150-Ω resistance and carries a direct current of 1.00 A. The filament is 8.00 cm long and 0.900 mm in radius. (a) Calculate the Poynting vector at the surface of the filament, associated with the static electric field producing the current and the current's static magnetic field. (b) Find the magnitude of the static electric and magnetic fields at the surface of the filament.

34. At one location on the Earth, the rms value of the magnetic field caused by solar radiation is 1.80 μT. From this value, calculate (a) the rms electric field due to solar radiation, (b) the average energy density of the solar component of electromagnetic radiation at this location, and (c) the average magnitude of the Poynting vector for the Sun's radiation.

Section 34.5 Momentum and Radiation Pressure

35. A 25.0-mW laser beam of diameter 2.00 mm is reflected at normal incidence by a perfectly reflecting mirror. Calculate the radiation pressure on the mirror.

36. A radio wave transmits 25.0 W/m^2 of power per unit area. A flat surface of area A is perpendicular to the direction of propagation of the wave. Assuming the surface is a perfect absorber, calculate the radiation pressure on it.

37. M A 15.0-mW helium–neon laser emits a beam of circular cross section with a diameter of 2.00 mm. (a) Find the maximum electric field in the beam. (b) What total energy is contained in a 1.00-m length of the beam? (c) Find the momentum carried by a 1.00-m length of the beam.

38. A helium–neon laser emits a beam of circular cross section with a radius r and a power P. (a) Find the maximum electric field in the beam. (b) What total energy is contained in a length ℓ of the beam? (c) Find the momentum carried by a length ℓ of the beam.

39. AMT A uniform circular disk of mass $m = 24.0$ g and radius $r = 40.0$ cm hangs vertically from a fixed, frictionless, horizontal hinge at a point on its circumference as shown in Figure P34.39a. A beam of electromagnetic radiation with intensity 10.0 MW/m^2 is incident on the disk in a direction perpendicular to its surface. The disk is perfectly absorbing, and the resulting radiation pressure makes the disk rotate. Assuming the radiation is *always* perpendicular to the surface of the disk, find the angle θ through which the disk rotates from the vertical as it reaches its new equilibrium position shown in Figure 34.39b.

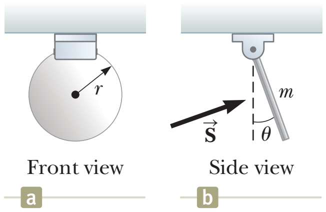


Figure P34.39

40. The intensity of sunlight at the Earth's distance from the Sun is 1 370 W/m^2. Assume the Earth absorbs all the sunlight incident upon it. (a) Find the total force the Sun exerts on the Earth due to radiation pressure. (b) Explain how this force compares with the Sun's gravitational attraction.

41. A plane electromagnetic wave of intensity 6.00 W/m^2, moving in the x direction, strikes a small perfectly reflecting pocket mirror, of area 40.0 cm^2, held in the yz plane. (a) What momentum does the wave transfer to the mirror each second? (b) Find the force the wave exerts on the mirror. (c) Explain the relationship between the answers to parts (a) and (b).

42. Assume the intensity of solar radiation incident on the upper atmosphere of the Earth is 1 370 W/m^2 and use data from Table 13.2 as necessary. Determine (a) the intensity of solar radiation incident on Mars, (b) the total power incident on Mars, and (c) the radiation force that acts on that planet if it absorbs nearly all the light. (d) State how this force compares with the gravitational attraction exerted by the Sun on Mars. (e) Compare the ratio of the gravitational force to the light-pressure force exerted on the Earth and the ratio of these forces exerted on Mars, found in part (d).

43. AMT W A possible means of space flight is to place a perfectly reflecting aluminized sheet into orbit around the Earth and then use the light from the Sun to push this "solar sail." Suppose a sail of area $A = 6.00 \times 10^5$ m^2 and mass $m = 6.00 \times 10^3$ kg is placed in orbit facing the Sun. Ignore all gravitational effects and assume a solar intensity of 1 370 W/m^2. (a) What force is exerted on the sail? (b) What is the sail's acceleration? (c) Assuming the acceleration calculated in part (b) remains constant, find the time interval required for the sail to reach the Moon, 3.84×10^8 m away, starting from rest at the Earth.

Section 34.6 Production of Electromagnetic Waves by an Antenna

44. Extremely low-frequency (ELF) waves that can penetrate the oceans are the only practical means of communicating with distant submarines. (a) Calculate the length of a quarter-wavelength antenna for a transmitter generating ELF waves of frequency 75.0 Hz into air. (b) How practical is this means of communication?

45. A Marconi antenna, used by most AM radio stations, consists of the top half of a Hertz antenna (also known as a half-wave antenna because its length is $\lambda/2$). The lower end of this Marconi (quarter-wave) antenna is connected to Earth ground, and the ground itself serves as the missing lower half. What are the heights of the Marconi antennas for radio stations broadcasting at (a) 560 kHz and (b) 1 600 kHz?

46. A large, flat sheet carries a uniformly distributed electric current with current per unit width J_s. This current creates a magnetic field on both sides of the sheet, parallel to the sheet and perpendicular to the current, with magnitude $B = \frac{1}{2}\mu_0 J_s$. If the current is in the y direction and oscillates in time according to

$$J_{\max}(\cos \omega t)\hat{\mathbf{j}} = J_{\max}[\cos(-\omega t)]\hat{\mathbf{j}}$$

the sheet radiates an electromagnetic wave. Figure P34.46 shows such a wave emitted from one point on the sheet chosen to be the origin. Such electromagnetic waves are emitted from all points on the sheet. The magnetic field of the wave to the right of the sheet is described by the wave function

$$\vec{\mathbf{B}} = \tfrac{1}{2}\mu_0 J_{\max}[\cos(kx - \omega t)]\hat{\mathbf{k}}$$

(a) Find the wave function for the electric field of the wave to the right of the sheet. (b) Find the Poynting vector as a function of x and t. (c) Find the intensity of the wave. (d) **What If?** If the sheet is to emit radiation in each direction (normal to the plane of the sheet) with intensity 570 W/m^2, what maximum value of sinusoidal current density is required?

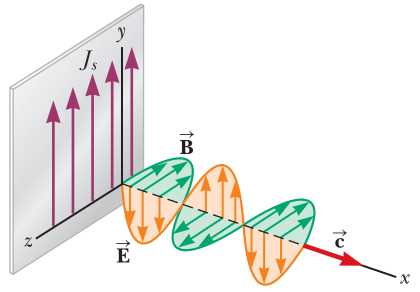


Figure P34.46

47. **Review.** Accelerating charges radiate electromagnetic waves. Calculate the wavelength of radiation produced by a proton in a cyclotron with a magnetic field of 0.350 T.

48. **Review.** Accelerating charges radiate electromagnetic waves. Calculate the wavelength of radiation produced by a proton of mass m_p moving in a circular path perpendicular to a magnetic field of magnitude B.

49. Two vertical radio-transmitting antennas are separated by half the broadcast wavelength and are driven in phase with each other. In what horizontal directions are (a) the strongest and (b) the weakest signals radiated?

Section 34.7 The Spectrum of Electromagnetic Waves

50. W Compute an order-of-magnitude estimate for the frequency of an electromagnetic wave with wavelength equal to (a) your height and (b) the thickness of a sheet of paper. How is each wave classified on the electromagnetic spectrum?

51. What are the wavelengths of electromagnetic waves in free space that have frequencies of (a) 5.00×10^{19} Hz and (b) 4.00×10^9 Hz?

52. An important news announcement is transmitted by radio waves to people sitting next to their radios 100 km from the station and by sound waves to people sitting across the newsroom 3.00 m from the newscaster. Taking the speed of sound in air to be 343 m/s, who receives the news first? Explain.

53. In addition to cable and satellite broadcasts, television stations still use VHF and UHF bands for digitally broadcasting their signals. Twelve VHF television channels (channels 2 through 13) lie in the range of frequencies between 54.0 MHz and 216 MHz. Each channel is assigned a width of 6.00 MHz, with the two ranges 72.0–76.0 MHz and 88.0–174 MHz reserved for non-TV purposes. (Channel 2, for example, lies between 54.0 and 60.0 MHz.) Calculate the broadcast wavelength range for (a) channel 4, (b) channel 6, and (c) channel 8.

Additional Problems

54. Classify waves with frequencies of 2 Hz, 2 kHz, 2 MHz, 2 GHz, 2 THz, 2 PHz, 2 EHz, 2 ZHz, and 2 YHz on the electromagnetic spectrum. Classify waves with wavelengths of 2 km, 2 m, 2 mm, 2 μm, 2 nm, 2 pm, 2 fm, and 2 am.

55. Assume the intensity of solar radiation incident on the cloud tops of the Earth is 1 370 W/m^2. (a) Taking the average Earth–Sun separation to be 1.496×10^{11} m, calculate the total power radiated by the Sun. Determine the maximum values of (b) the electric field and (c) the magnetic field in the sunlight at the Earth's location.

56. In 1965, Arno Penzias and Robert Wilson discovered the cosmic microwave radiation left over from the big bang expansion of the Universe. Suppose the energy density of this background radiation is 4.00×10^{-14} J/m^3. Determine the corresponding electric field amplitude.

57. The eye is most sensitive to light having a frequency of 5.45×10^{14} Hz, which is in the green-yellow region of the visible electromagnetic spectrum. What is the wavelength of this light?

58. Write expressions for the electric and magnetic fields of a sinusoidal plane electromagnetic wave having an electric field amplitude of 300 V/m and a frequency of 3.00 GHz and traveling in the positive x direction.

59. One goal of the Russian space program is to illuminate dark northern cities with sunlight reflected to the Earth from a 200-m diameter mirrored surface in orbit. Several smaller prototypes have already been constructed and put into orbit. (a) Assume that sunlight with intensity 1 370 W/m^2 falls on the mirror nearly perpendicularly and that the atmosphere of the Earth allows 74.6% of the energy of sunlight to pass though it in clear weather. What is the power received by a city when the space mirror is reflecting light to it? (b) The plan is for the reflected sunlight to cover a circle of diameter 8.00 km. What is the intensity of light (the average magnitude of the Poynting vector) received by the city? (c) This intensity is what percentage of the vertical component of sunlight at St. Petersburg in January, when the sun reaches an angle of 7.00° above the horizon at noon?

60. A microwave source produces pulses of 20.0-GHz radiation, with each pulse lasting 1.00 ns. A parabolic reflector with a face area of radius 6.00 cm is used to focus the microwaves into a parallel beam of radiation as shown in Figure P34.60. The average power during each pulse is 25.0 kW. (a) What is the wavelength of these microwaves? (b) What is the total energy contained in each pulse? (c) Compute the average energy density inside each pulse. (d) Determine the amplitude of the electric and magnetic fields in these microwaves. (e) Assuming that this pulsed beam strikes an absorbing surface, compute the force exerted on the surface during the 1.00-ns duration of each pulse.

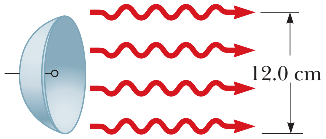


Figure P34.60

61. The intensity of solar radiation at the top of the Earth's atmosphere is 1 370 W/m^2. Assuming 60% of the incoming solar energy reaches the Earth's surface and you absorb 50% of the incident energy, make an order-of-magnitude estimate of the amount of solar energy you absorb if you sunbathe for 60 minutes.

62. Two handheld radio transceivers with dipole antennas are separated by a large, fixed distance. If the transmitting antenna is vertical, what fraction of the maximum received power will appear in the receiving antenna when it is inclined from the vertical (a) by 15.0°? (b) By 45.0°? (c) By 90.0°?

63. AMT Consider a small, spherical particle of radius r located in space a distance $R = 3.75 \times 10^{11}$ m from the Sun. Assume the particle has a perfectly absorbing surface and a mass density of $\rho = 1.50$ g/cm^3. Use $S = 214$ W/m^2 as the value of the solar intensity at the location of the particle. Calculate the value of r for which the particle is in equilibrium between the gravitational force and the force exerted by solar radiation.

64. Consider a small, spherical particle of radius r located in space a distance R from the Sun, of mass M_S. Assume the particle has a perfectly absorbing surface and a mass density ρ. The value of the solar intensity at the particle's location is S. Calculate the value of r for which the particle is in equilibrium between the gravitational force and the force exerted by solar radiation. Your answer should be in terms of S, R, ρ, and other constants.

65. M A dish antenna having a diameter of 20.0 m receives (at normal incidence) a radio signal from a distant source as shown in Figure P34.65. The radio signal is a continuous sinusoidal wave with amplitude $E_{max} = 0.200\ \mu$V/m.

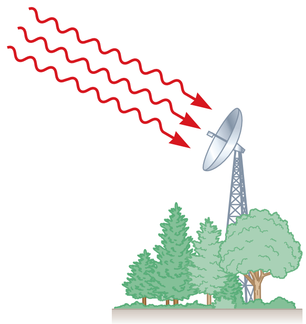


Figure P34.65

Assume the antenna absorbs all the radiation that falls on the dish. (a) What is the amplitude of the magnetic field in this wave? (b) What is the intensity of the radiation received by this antenna? (c) What is the power received by the antenna? (d) What force is exerted by the radio waves on the antenna?

66. The Earth reflects approximately 38.0% of the incident sunlight from its clouds and surface. (a) Given that the intensity of solar radiation at the top of the atmosphere is 1 370 W/m^2, find the radiation pressure on the Earth, in pascals, at the location where the Sun is straight overhead. (b) State how this quantity compares with normal atmospheric pressure at the Earth's surface, which is 101 kPa.

67. **Review.** A 1.00-m-diameter circular mirror focuses the Sun's rays onto a circular absorbing plate 2.00 cm in radius, which holds a can containing 1.00 L of water at 20.0°C. (a) If the solar intensity is 1.00 kW/m^2, what is the intensity on the absorbing plate? At the plate, what are the maximum magnitudes of the fields (b) $\vec{\mathbf{E}}$ and (c) $\vec{\mathbf{B}}$? (d) If 40.0% of the energy is absorbed, what time interval is required to bring the water to its boiling point?

68. (a) A stationary charged particle at the origin creates an electric flux of 487 N · m^2/C through any closed surface surrounding the charge. Find the electric field it creates in the empty space around it as a function of radial distance r away from the particle. (b) A small source at the origin emits an electromagnetic wave with a single frequency into vacuum, equally in all directions, with power 25.0 W. Find the electric field amplitude as a function of radial distance away from the source. (c) At what distance is the amplitude of the electric field in the wave equal to 3.00 MV/m, representing the dielectric strength of air? (d) As the distance from the source doubles, what happens to the field amplitude? (e) State how the behavior shown in part (d) compares with the behavior of the field in part (a).

69. **Review.** (a) A homeowner has a solar water heater installed on the roof of his house (Fig. P34.69). The heater is a flat, closed box with excellent thermal insulation. Its interior is painted black, and its front face is made of insulating glass. Its emissivity for visible light is 0.900, and its emissivity for infrared light is 0.700. Light from the noontime Sun is incident perpendicular to the glass with an intensity of 1 000 W/m^2, and no water enters or leaves the box. Find the steady-state temperature of the box's interior. (b) **What If?** The homeowner builds an identical box with no water tubes. It lies flat on the ground in front of the house. He uses it as a cold frame, where he plants seeds in early spring. Assuming the same noontime Sun is at an elevation angle of 50.0°, find the steady-state temperature of the interior of the box when its ventilation slots are tightly closed.

easyshoot/Shutterstock.com

Figure P34.69

70. GP You may wish to review Sections 16.5 and 17.3 on the transport of energy by string waves and sound. Figure P34.15 is a graphical representation of an electromagnetic wave moving in the x direction. We wish to find an expression for the intensity of this wave by means of a different process from that by which Equation 34.24 was generated. (a) Sketch a graph of the electric field in this wave at the instant $t = 0$, letting your flat paper represent the xy plane. (b) Compute the energy density u_E in the electric field as a function of x at the instant $t = 0$. (c) Compute the energy density in the magnetic field u_B as a function of x at that instant. (d) Find the total energy density u as a function of x, expressed in terms of only the electric field amplitude. (e) The energy in a "shoebox" of length λ and frontal area A is $E_\lambda = \int_0^\lambda uA\,dx$. (The symbol E_λ for energy in a wavelength imitates the notation of Section 16.5.) Perform the integration to compute the amount of this energy in terms of A, λ, E_{max}, and universal constants. (f) We may think of the energy transport by the whole wave as a series of these shoeboxes going past as if carried on a conveyor belt. Each shoebox passes by a point in a time interval defined as the period $T = 1/f$ of the wave. Find the power the wave carries through area A. (g) The intensity of the wave is the power per unit area through which the wave passes. Compute this intensity in terms of E_{max} and universal constants. (h) Explain how your result compares with that given in Equation 34.24.

71. Lasers have been used to suspend spherical glass beads in the Earth's gravitational field. (a) A black bead has a radius of 0.500 mm and a density of 0.200 g/cm^3. Determine the radiation intensity needed to support the bead. (b) What is the minimum power required for this laser?

72. Lasers have been used to suspend spherical glass beads in the Earth's gravitational field. (a) A black bead has radius r and density ρ. Determine the radiation intensity needed to support the bead. (b) What is the minimum power required for this laser?

73. **Review.** A 5.50-kg black cat and her four black kittens, each with mass 0.800 kg, sleep snuggled together on a mat on a cool night, with their bodies forming a hemisphere. Assume the hemisphere has a surface temperature of 31.0°C, an emissivity of 0.970, and a uniform density of 990 kg/m^3. Find (a) the radius of the hemisphere, (b) the area of its curved surface, (c) the radiated power emitted by the cats at their curved surface, and (d) the intensity of radiation at this surface. You may think of the emitted electromagnetic wave as having a single predominant frequency. Find (e) the amplitude of the electric field in the electromagnetic wave just outside the surface of the cozy pile and (f) the amplitude of the magnetic field. (g) **What If?** The next night, the kittens all sleep alone, curling up into separate hemispheres like their mother. Find the total radiated power of the family. (For simplicity, ignore the cats' absorption of radiation from the environment.)

74. The electromagnetic power radiated by a nonrelativistic particle with charge q moving with acceleration a is

$$P = \frac{q^2 a^2}{6\pi\epsilon_0 c^3}$$

where ϵ_0 is the permittivity of free space (also called the permittivity of vacuum) and c is the speed of light in vacuum. (a) Show that the right side of this equation has units of watts. An electron is placed in a constant electric field of magnitude 100 N/C. Determine (b) the acceleration of the electron and (c) the electromagnetic power radiated by this electron. (d) **What If?** If a proton is placed in a cyclotron with a radius of 0.500 m and a magnetic field of magnitude 0.350 T, what electromagnetic power does this proton radiate just before leaving the cyclotron?

75. Review. Gliese 581c is the first Earth-like extrasolar terrestrial planet discovered. Its parent star, Gliese 581, is a red dwarf that radiates electromagnetic waves with power 5.00×10^{24} W, which is only 1.30% of the power of the Sun. Assume the emissivity of the planet is equal for infrared and for visible light and the planet has a uniform surface temperature. Identify (a) the projected area over which the planet absorbs light from Gliese 581 and (b) the radiating area of the planet. (c) If an average temperature of 287 K is necessary for life to exist on Gliese 581c, what should the radius of the planet's orbit be?

Challenge Problems

76. A plane electromagnetic wave varies sinusoidally at 90.0 MHz as it travels through vacuum along the positive x direction. The peak value of the electric field is 2.00 mV/m, and it is directed along the positive y direction. Find (a) the wavelength, (b) the period, and (c) the maximum value of the magnetic field. (d) Write expressions in SI units for the space and time variations of the electric field and of the magnetic field. Include both numerical values and unit vectors to indicate directions. (e) Find the average power per unit area this wave carries through space. (f) Find the average energy density in the radiation (in joules per cubic meter). (g) What radiation pressure would this wave exert upon a perfectly reflecting surface at normal incidence?

77. A linearly polarized microwave of wavelength 1.50 cm is directed along the positive x axis. The electric field vector has a maximum value of 175 V/m and vibrates in the xy plane. Assuming the magnetic field component of the wave can be written in the form $B = B_{\max} \sin (kx - \omega t)$, give values for (a) $B_{\max}$, (b) k, and (c) ω. (d) Determine in which plane the magnetic field vector vibrates. (e) Calculate the average value of the Poynting vector for this wave. (f) If this wave were directed at normal incidence onto a perfectly reflecting sheet, what radiation pressure would it exert? (g) What acceleration would be imparted to a 500-g sheet (perfectly reflecting and at normal incidence) with dimensions of 1.00 m $\times$ 0.750 m?

78. Review. In the absence of cable input or a satellite dish, a television set can use a dipole-receiving antenna for VHF channels and a loop antenna for UHF channels. In Figure CQ34.9, the "rabbit ears" form the VHF antenna and the smaller loop of wire is the UHF antenna. The UHF antenna produces an emf from the changing magnetic flux through the loop. The television station broadcasts a signal with a frequency f, and the signal has an electric field amplitude $E_{\max}$ and a magnetic field amplitude $B_{\max}$ at the location of the receiving antenna. (a) Using Faraday's law, derive an expression for the amplitude of the emf that appears in a single-turn, circular loop antenna with a radius r that is small compared with the wavelength of the wave. (b) If the electric field in the signal points vertically, what orientation of the loop gives the best reception?

79. AMT **Review.** An astronaut, stranded in space 10.0 m from her spacecraft and at rest relative to it, has a mass (including equipment) of 110 kg. Because she has a 100-W flashlight that forms a directed beam, she considers using the beam as a photon rocket to propel herself continuously toward the spacecraft. (a) Calculate the time interval required for her to reach the spacecraft by this method. (b) **What If?** Suppose she throws the 3.00-kg flashlight in the direction away from the spacecraft instead. After being thrown, the flashlight moves at 12.0 m/s relative to the recoiling astronaut. After what time interval will the astronaut reach the spacecraft?