CHAPTER

33

Alternating-Current Circuits

These large transformers are used to increase the voltage at a power plant for distribution of energy by electrical transmission to the power grid. Voltages can be changed relatively easily because power is distributed by alternating current rather than direct current. *(©Lester Lefkowitz/Getty Images)*

In this chapter, we describe alternating-current (AC) circuits. Every time you turn on a television set, a computer, or any of a multitude of other electrical appliances in a home, you are calling on alternating currents to provide the power to operate them. We begin our study by investigating the characteristics of simple series circuits that contain resistors, inductors, and capacitors and that are driven by a sinusoidal voltage. The primary aim of this chapter can be summarized as follows: if an AC source applies an alternating voltage to a series circuit containing resistors, inductors, and capacitors, we want to know the amplitude and time characteristics of the alternating current. We conclude this chapter with two sections concerning transformers, power transmission, and electrical filters.

33.1 AC Sources

An AC circuit consists of circuit elements and a power source that provides an alternating voltage Δv. This time-varying voltage from the source is described by

$$\Delta v = \Delta V_{max} \sin \omega t$$

where ΔV_{max} is the maximum output voltage of the source, or the **voltage amplitude.** There are various possibilities for AC sources, including generators as discussed in Section 31.5 and electrical oscillators. In a home, each electrical outlet serves as an AC source. Because the output voltage of an AC source varies sinusoidally with time, the voltage is positive during one half of the cycle and negative during the other half as in Figure 33.1. Likewise, the current in any circuit driven by an AC source is an alternating current that also varies sinusoidally with time.

From Equation 15.12, the angular frequency of the AC voltage is

$$\omega = 2\pi f = \frac{2\pi}{T}$$

where f is the frequency of the source and T is the period. The source determines the frequency of the current in any circuit connected to it. Commercial electric-power plants in the United States use a frequency of 60.0 Hz, which corresponds to an angular frequency of 377 rad/s.

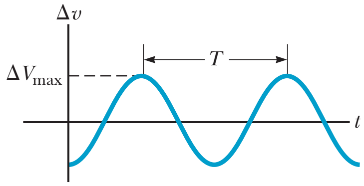


Figure 33.1 The voltage supplied by an AC source is sinusoidal with a period T.

33.2 Resistors in an AC Circuit

Consider a simple AC circuit consisting of a resistor and an AC source  as shown in Figure 33.2. At any instant, the algebraic sum of the voltages around a closed loop in a circuit must be zero (Kirchhoff's loop rule). Therefore, $\Delta v + \Delta v_R = 0$ or, using Equation 27.7 for the voltage across the resistor,

$$\Delta v - i_R R = 0$$

If we rearrange this expression and substitute $\Delta V_{max} \sin \omega t$ for Δv, the instantaneous current in the resistor is

$$i_R = \frac{\Delta v}{R} = \frac{\Delta V_{max}}{R} \sin \omega t = I_{max} \sin \omega t \qquad \textbf{(33.1)}$$

where I_{max} is the maximum current:

$$I_{max} = \frac{\Delta V_{max}}{R} \qquad \textbf{(33.2)}$$

◀ **Maximum current in a resistor**

Equation 33.1 shows that the instantaneous voltage across the resistor is

$$\Delta v_R = i_R R = I_{max} R \sin \omega t \qquad \textbf{(33.3)}$$

◀ **Voltage across a resistor**

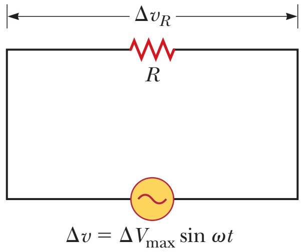


Figure 33.2 A circuit consisting of a resistor of resistance R connected to an AC source, designated by the symbol .

A plot of voltage and current versus time for this circuit is shown in Figure 33.3a on page 1000. At point a, the current has a maximum value in one direction, arbitrarily called the positive direction. Between points a and b, the current is decreasing in magnitude but is still in the positive direction. At point b, the current is momentarily zero; it then begins to increase in the negative direction between points b and c. At point c, the current has reached its maximum value in the negative direction.

The current and voltage are in step with each other because they vary identically with time. Because i_R and Δv_R both vary as $\sin \omega t$ and reach their maximum values at the same time as shown in Figure 33.3a, they are said to be **in phase,** similar to the way two waves can be in phase as discussed in our study of wave motion in Chapter 18. Therefore, for a sinusoidal applied voltage, the current in a resistor is always in phase with the voltage across the resistor. For resistors in AC circuits, there are no new concepts to learn. Resistors behave essentially the same way in both DC and AC circuits. That, however, is not the case for capacitors and inductors.

To simplify our analysis of circuits containing two or more elements, we use a graphical representation called a *phasor diagram.* A **phasor** is a vector whose length is proportional to the maximum value of the variable it represents (ΔV_{max} for voltage and I_{max} for current in this discussion). The phasor rotates counterclockwise at an angular speed equal to the angular frequency associated with the variable. The

Pitfall Prevention 33.1

Time-Varying Values We continue to use lowercase symbols Δv and i to indicate the instantaneous values of time-varying voltages and currents. We will add a subscript to indicate the appropriate circuit element. Capital letters represent fixed values of voltage and current such as ΔV_{max} and I_{max}.

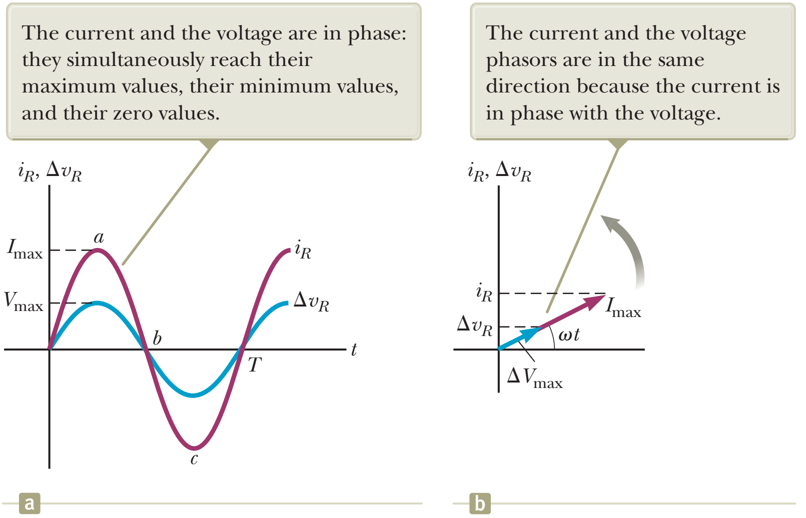


Figure 33.3 (a) Plots of the instantaneous current i_R and instantaneous voltage Δv_R across a resistor as functions of time. At time $t = T$, one cycle of the time-varying voltage and current has been completed. (b) Phasor diagram for the resistive circuit showing that the current is in phase with the voltage.

Pitfall Prevention 33.2

A Phasor Is Like a Graph An alternating voltage can be presented in different representations. One graphical representation is shown in Figure 33.1 in which the voltage is drawn in rectangular coordinates, with voltage on the vertical axis and time on the horizontal axis. Figure 33.3b shows another graphical representation. The phase space in which the phasor is drawn is similar to polar coordinate graph paper. The radial coordinate represents the amplitude of the voltage. The angular coordinate is the phase angle. The vertical-axis coordinate of the tip of the phasor represents the instantaneous value of the voltage. The horizontal coordinate represents nothing at all. As shown in Figure 33.3b, alternating currents can also be represented by phasors.

To help with this discussion of phasors, review Section 15.4, where we represented the simple harmonic motion of a real object by the projection of an imaginary object's uniform circular motion onto a coordinate axis. A phasor is a direct analog to this representation.

projection of the phasor onto the vertical axis represents the instantaneous value of the quantity it represents.

Figure 33.3b shows voltage and current phasors for the circuit of Figure 33.2 at some instant of time. The projections of the phasor arrows onto the vertical axis are determined by a sine function of the angle of the phasor with respect to the horizontal axis. For example, the projection of the current phasor in Figure 33.3b is $I_{max} \sin \omega t$. Notice that this expression is the same as Equation 33.1. Therefore, the projections of phasors represent current values that vary sinusoidally in time. We can do the same with time-varying voltages. The advantage of this approach is that the phase relationships among currents and voltages can be represented as vector additions of phasors using the vector addition techniques discussed in Chapter 3.

In the case of the single-loop resistive circuit of Figure 33.2, the current and voltage phasors are in the same direction in Figure 33.3b because i_R and Δv_R are in phase. The current and voltage in circuits containing capacitors and inductors have different phase relationships.

Quick Quiz 33.1 Consider the voltage phasor in Figure 33.4, shown at three instants of time. **(i)** Choose the part of the figure, (a), (b), or (c), that represents the instant of time at which the instantaneous value of the voltage has the largest magnitude. **(ii)** Choose the part of the figure that represents the instant of time at which the instantaneous value of the voltage has the smallest magnitude.

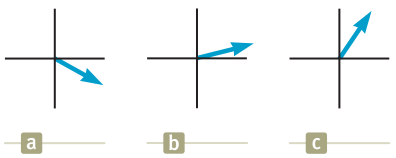


Figure 33.4 (Quick Quiz 33.1) A voltage phasor is shown at three instants of time, (a), (b), and (c).

For the simple resistive circuit in Figure 33.2, notice that the average value of the current over one cycle is zero. That is, the current is maintained in the positive direction for the same amount of time and at the same magnitude as it is maintained in the negative direction. The direction of the current, however, has no effect on the behavior of the resistor. We can understand this concept by realizing that collisions between electrons and the fixed atoms of the resistor result in an

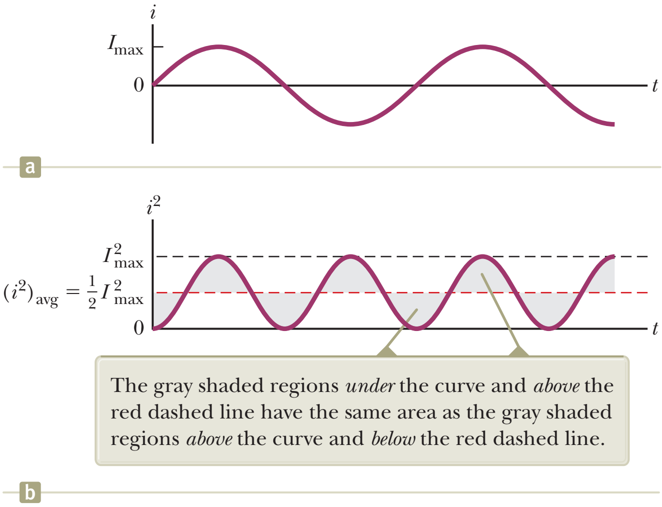


Figure 33.5 (a) Graph of the current in a resistor as a function of time. (b) Graph of the current squared in a resistor as a function of time, showing that the red dashed line is the average of $I^2_{max} \sin^2 \omega t$. In general, the average value of $\sin^2 \omega t$ or $\cos^2 \omega t$ over one cycle is $\frac{1}{2}$.

increase in the resistor's temperature. Although this temperature increase depends on the magnitude of the current, it is independent of the current's direction.

We can make this discussion quantitative by recalling that the rate at which energy is delivered to a resistor is the power $P = i^2R$, where i is the instantaneous current in the resistor. Because this rate is proportional to the square of the current, it makes no difference whether the current is direct or alternating, that is, whether the sign associated with the current is positive or negative. The temperature increase produced by an alternating current having a maximum value I_{max}, however, is not the same as that produced by a direct current equal to I_{max} because the alternating current has this maximum value for only an instant during each cycle (Fig. 33.5a). What is of importance in an AC circuit is an average value of current, referred to as the **rms current.** As we learned in Section 21.1, the notation *rms* stands for *root-mean-square,* which in this case means the square root of the mean (average) value of the square of the current: $I_{rms} = \sqrt{(i^2)_{avg}}$. Because i^2 varies as $\sin^2 \omega t$ and because the average value of i^2 is $\frac{1}{2}I^2_{max}$ (see Fig. 33.5b), the rms current is

$$I_{rms} = \frac{I_{max}}{\sqrt{2}} = 0.707 I_{max} \tag{33.4}$$

◀ **rms current**

This equation states that an alternating current whose maximum value is 2.00 A delivers to a resistor the same power as a direct current that has a value of (0.707)(2.00 A) = 1.41 A. The average power delivered to a resistor that carries an alternating current is

$$P_{avg} = I^2_{rms}R$$

◀ **Average power delivered to a resistor**

Alternating voltage is also best discussed in terms of rms voltage, and the relationship is identical to that for current:

$$\Delta V_{rms} = \frac{\Delta V_{max}}{\sqrt{2}} = 0.707\ \Delta V_{max} \tag{33.5}$$

◀ **rms voltage**

When we speak of measuring a 120-V alternating voltage from an electrical outlet, we are referring to an rms voltage of 120 V. A calculation using Equation 33.5 shows that such an alternating voltage has a maximum value of about 170 V. One reason rms values are often used when discussing alternating currents and voltages is that AC ammeters and voltmeters are designed to read rms values. Furthermore, with rms values, many of the equations we use have the same form as their direct-current counterparts.

Example 33.1 What Is the rms Current?

The voltage output of an AC source is given by the expression $\Delta v = 200 \sin \omega t$, where Δv is in volts. Find the rms current in the circuit when this source is connected to a 100-Ω resistor.

SOLUTION

Conceptualize Figure 33.2 shows the physical situation for this problem.

Categorize We evaluate the current with an equation developed in this section, so we categorize this example as a substitution problem.

Combine Equations 33.2 and 33.4 to find the rms current:

$$I_{\text{rms}} = \frac{I_{\max}}{\sqrt{2}} = \frac{\Delta V_{\max}}{\sqrt{2}\,R}$$

Comparing the expression for voltage output with the general form $\Delta v = \Delta V_{\max} \sin \omega t$ shows that $\Delta V_{\max} = 200$ V. Substitute numerical values:

$$I_{\text{rms}} = \frac{200 \text{ V}}{\sqrt{2}\,(100\ \Omega)} = \boxed{1.41 \text{ A}}$$

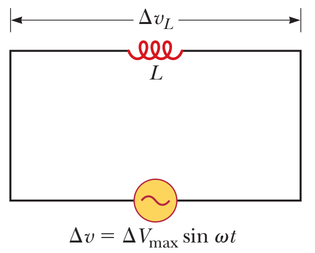


Figure 33.6 A circuit consisting of an inductor of inductance L connected to an AC source.

33.3 Inductors in an AC Circuit

Now consider an AC circuit consisting only of an inductor connected to the terminals of an AC source as shown in Figure 33.6. Because $\Delta v_L = -L(di_L/dt)$ is the self-induced instantaneous voltage across the inductor (see Eq. 32.1), Kirchhoff's loop rule applied to this circuit gives $\Delta v + \Delta v_L = 0$, or

$$\Delta v - L\frac{di_L}{dt} = 0$$

Substituting $\Delta V_{\max} \sin \omega t$ for Δv and rearranging gives

$$\Delta v = L\frac{di_L}{dt} = \Delta V_{\max} \sin \omega t \tag{33.6}$$

Solving this equation for di_L gives

$$di_L = \frac{\Delta V_{\max}}{L} \sin \omega t\, dt$$

Integrating this expression[1] gives the instantaneous current i_L in the inductor as a function of time:

$$i_L = \frac{\Delta V_{\max}}{L}\int \sin \omega t\, dt = -\frac{\Delta V_{\max}}{\omega L}\cos \omega t \tag{33.7}$$

Using the trigonometric identity $\cos \omega t = -\sin(\omega t - \pi/2)$, we can express Equation 33.7 as

Current in an inductor ▶

$$i_L = \frac{\Delta V_{\max}}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right) \tag{33.8}$$

Comparing this result with Equation 33.6 shows that the instantaneous current i_L in the inductor and the instantaneous voltage Δv_L across the inductor are *out* of phase by $\pi/2$ rad = 90°.

A plot of voltage and current versus time is shown in Figure 33.7a. When the current i_L in the inductor is a maximum (point *b* in Fig. 33.7a), it is momentarily

[1]We neglect the constant of integration here because it depends on the initial conditions, which are not important for this situation.

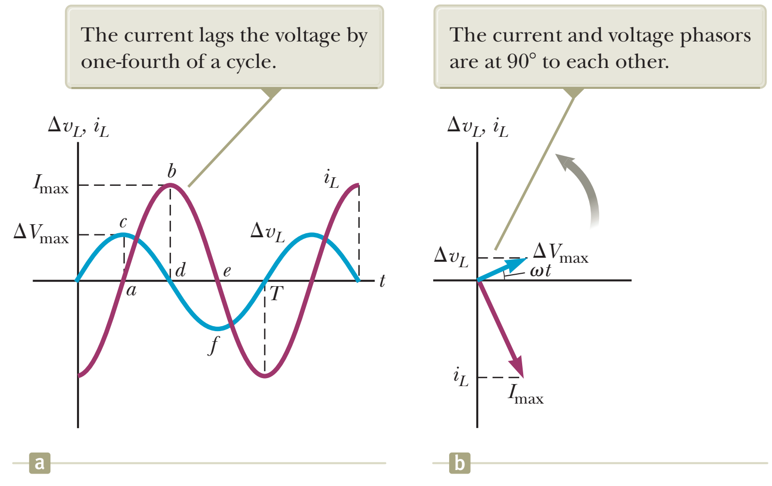


Figure 33.7 (a) Plots of the instantaneous current i_L and instantaneous voltage Δv_L across an inductor as functions of time. (b) Phasor diagram for the inductive circuit.

not changing, so the voltage across the inductor is zero (point *d*). At points such as *a* and *e*, the current is zero and the rate of change of current is at a maximum. Therefore, the voltage across the inductor is also at a maximum (points *c* and *f*). Notice that the voltage reaches its maximum value one-quarter of a period before the current reaches its maximum value. Therefore, for a sinusoidal applied voltage, the current in an inductor always *lags* behind the voltage across the inductor by 90° (one-quarter cycle in time).

As with the relationship between current and voltage for a resistor, we can represent this relationship for an inductor with a phasor diagram as in Figure 33.7b. The phasors are at 90° to each other, representing the 90° phase difference between current and voltage.

Equation 33.7 shows that the current in an inductive circuit reaches its maximum value when $\cos \omega t = \pm 1$:

$$I_{max} = \frac{\Delta V_{max}}{\omega L} \quad (33.9)$$

◀ **Maximum current in an inductor**

This expression is similar to the relationship between current, voltage, and resistance in a DC circuit, $I = \Delta V/R$ (Eq. 27.7). Because I_{max} has units of amperes and ΔV_{max} has units of volts, ωL must have units of ohms. Therefore, ωL has the same units as resistance and is related to current and voltage in the same way as resistance. It must behave in a manner similar to resistance in the sense that it represents opposition to the flow of charge. Because ωL depends on the applied frequency ω, the inductor *reacts* differently, in terms of offering opposition to current, for different frequencies. For this reason, we define ωL as the **inductive reactance** X_L:

$$X_L \equiv \omega L \quad (33.10)$$

◀ **Inductive reactance**

Therefore, we can write Equation 33.9 as

$$I_{max} = \frac{\Delta V_{max}}{X_L} \quad (33.11)$$

The expression for the rms current in an inductor is similar to Equation 33.11, with I_{max} replaced by I_{rms} and ΔV_{max} replaced by ΔV_{rms}.

Equation 33.10 indicates that, for a given applied voltage, the inductive reactance increases as the frequency increases. This conclusion is consistent with Faraday's law: the greater the rate of change of current in the inductor, the larger the back emf. The larger back emf translates to an increase in the reactance and a decrease in the current.

Using Equations 33.6 and 33.11, we find that the instantaneous voltage across the inductor is

Voltage across an inductor ▶

$$\Delta v_L = -L\frac{di_L}{dt} = -\Delta V_{\max} \sin \omega t = -I_{\max} X_L \sin \omega t \tag{33.12}$$

Quick Quiz 33.2 Consider the AC circuit in Figure 33.8. The frequency of the AC source is adjusted while its voltage amplitude is held constant. When does the lightbulb glow the brightest? **(a)** It glows brightest at high frequencies. **(b)** It glows brightest at low frequencies. **(c)** The brightness is the same at all frequencies.

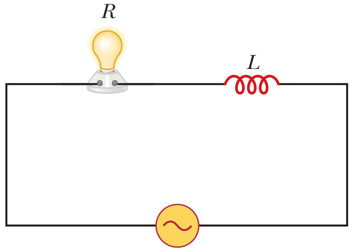


Figure 33.8 (Quick Quiz 33.2) At what frequencies does the lightbulb glow the brightest?

Example 33.2 A Purely Inductive AC Circuit

In a purely inductive AC circuit, $L = 25.0$ mH and the rms voltage is 150 V. Calculate the inductive reactance and rms current in the circuit if the frequency is 60.0 Hz.

SOLUTION

Conceptualize Figure 33.6 shows the physical situation for this problem. Keep in mind that inductive reactance increases with increasing frequency of the applied voltage.

Categorize We determine the reactance and the current from equations developed in this section, so we categorize this example as a substitution problem.

Use Equation 33.10 to find the inductive reactance:

$$X_L = \omega L = 2\pi f L = 2\pi(60.0 \text{ Hz})(25.0 \times 10^{-3} \text{ H})$$
$$= 9.42\ \Omega$$

Use an rms version of Equation 33.11 to find the rms current:

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{X_L} = \frac{150 \text{ V}}{9.42\ \Omega} = 15.9 \text{ A}$$

WHAT IF? If the frequency increases to 6.00 kHz, what happens to the rms current in the circuit?

Answer If the frequency increases, the inductive reactance also increases because the current is changing at a higher rate. The increase in inductive reactance results in a lower current.

Let's calculate the new inductive reactance and the new rms current:

$$X_L = 2\pi(6.00 \times 10^3 \text{ Hz})(25.0 \times 10^{-3} \text{ H}) = 942\ \Omega$$

$$I_{\text{rms}} = \frac{150 \text{ V}}{942\ \Omega} = 0.159 \text{ A}$$

33.4 Capacitors in an AC Circuit

Figure 33.9 shows an AC circuit consisting of a capacitor connected across the terminals of an AC source. Kirchhoff's loop rule applied to this circuit gives $\Delta v + \Delta v_C = 0$, or

$$\Delta v - \frac{q}{C} = 0 \tag{33.13}$$

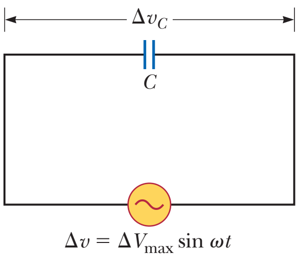


Figure 33.9 A circuit consisting of a capacitor of capacitance C connected to an AC source.

Substituting $\Delta V_{max} \sin \omega t$ for Δv and rearranging gives

$$q = C\,\Delta V_{max} \sin \omega t \tag{33.14}$$

where q is the instantaneous charge on the capacitor. Differentiating Equation 33.14 with respect to time gives the instantaneous current in the circuit:

$$i_C = \frac{dq}{dt} = \omega C\,\Delta V_{max} \cos \omega t \tag{33.15}$$

Using the trigonometric identity

$$\cos \omega t = \sin\left(\omega t + \frac{\pi}{2}\right)$$

we can express Equation 33.15 in the alternative form

$$i_C = \omega C\,\Delta V_{max} \sin\left(\omega t + \frac{\pi}{2}\right) \tag{33.16}$$

◀ **Current in a capacitor**

Comparing this expression with $\Delta v = \Delta V_{max} \sin \omega t$ shows that the current is $\pi/2$ rad $= 90°$ out of phase with the voltage across the capacitor. A plot of current and voltage versus time (Fig. 33.10a) shows that the current reaches its maximum value one-quarter of a cycle sooner than the voltage reaches its maximum value.

Consider a point such as b in Figure 33.10a where the current is zero at this instant. That occurs when the capacitor reaches its maximum charge so that the voltage across the capacitor is a maximum (point d). At points such as a and e, the current is a maximum, which occurs at those instants when the charge on the capacitor reaches zero and the capacitor begins to recharge with the opposite polarity. When the charge is zero, the voltage across the capacitor is zero (points c and f).

As with inductors, we can represent the current and voltage for a capacitor on a phasor diagram. The phasor diagram in Figure 33.10b shows that for a sinusoidally applied voltage, the current always *leads* the voltage across a capacitor by 90°.

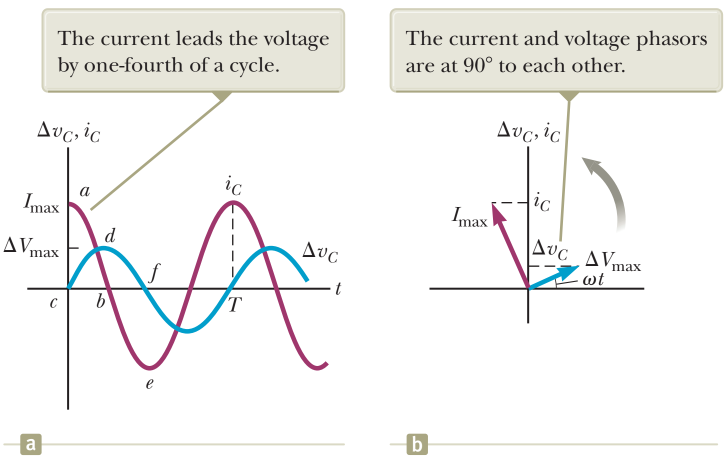


Figure 33.10 (a) Plots of the instantaneous current i_C and instantaneous voltage Δv_C across a capacitor as functions of time. (b) Phasor diagram for the capacitive circuit.

Equation 33.15 shows that the current in the circuit reaches its maximum value when $\cos \omega t = \pm 1$:

$$I_{max} = \omega C \Delta V_{max} = \frac{\Delta V_{max}}{(1/\omega C)} \quad (33.17)$$

As in the case with inductors, this looks like Equation 27.7, so the denominator plays the role of resistance, with units of ohms. We give the combination $1/\omega C$ the symbol X_C, and because this function varies with frequency, we define it as the **capacitive reactance:**

Capacitive reactance ▶

$$X_C \equiv \frac{1}{\omega C} \quad (33.18)$$

We can now write Equation 33.17 as

Maximum current in a capacitor ▶

$$I_{max} = \frac{\Delta V_{max}}{X_C} \quad (33.19)$$

The rms current is given by an expression similar to Equation 33.19, with I_{max} replaced by I_{rms} and ΔV_{max} replaced by ΔV_{rms}.

Using Equation 33.19, we can express the instantaneous voltage across the capacitor as

Voltage across a capacitor ▶

$$\Delta v_C = \Delta V_{max} \sin \omega t = I_{max} X_C \sin \omega t \quad (33.20)$$

Equations 33.18 and 33.19 indicate that as the frequency of the voltage source increases, the capacitive reactance decreases and the maximum current therefore increases. The frequency of the current is determined by the frequency of the voltage source driving the circuit. As the frequency approaches zero, the capacitive reactance approaches infinity and the current therefore approaches zero. This conclusion makes sense because the circuit approaches direct current conditions as ω approaches zero and the capacitor represents an open circuit.

Quick Quiz 33.3 Consider the AC circuit in Figure 33.11. The frequency of the AC source is adjusted while its voltage amplitude is held constant. When does the lightbulb glow the brightest? **(a)** It glows brightest at high frequencies. **(b)** It glows brightest at low frequencies. **(c)** The brightness is the same at all frequencies.

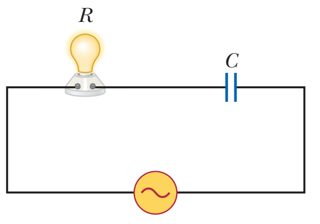


Figure 33.11 (Quick Quiz 33.3)

Quick Quiz 33.4 Consider the AC circuit in Figure 33.12. The frequency of the AC source is adjusted while its voltage amplitude is held constant. When does the lightbulb glow the brightest? **(a)** It glows brightest at high frequencies. **(b)** It glows brightest at low frequencies. **(c)** The brightness is the same at all frequencies.

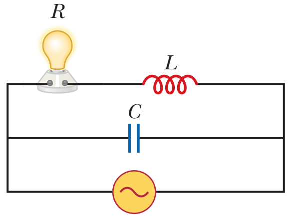


Figure 33.12 (Quick Quiz 33.4)

Example 33.3 A Purely Capacitive AC Circuit

An 8.00-μF capacitor is connected to the terminals of a 60.0-Hz AC source whose rms voltage is 150 V. Find the capacitive reactance and the rms current in the circuit.

SOLUTION

Conceptualize Figure 33.9 shows the physical situation for this problem. Keep in mind that capacitive reactance decreases with increasing frequency of the applied voltage.

Categorize We determine the reactance and the current from equations developed in this section, so we categorize this example as a substitution problem.

Use Equation 33.18 to find the capacitive reactance:

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi(60.0\text{ Hz})(8.00 \times 10^{-6}\text{ F})} = \boxed{332\ \Omega}$$

Use an rms version of Equation 33.19 to find the rms current:

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{X_C} = \frac{150\text{ V}}{332\ \Omega} = \boxed{0.452\text{ A}}$$

WHAT IF? What if the frequency is doubled? What happens to the rms current in the circuit?

Answer If the frequency increases, the capacitive reactance decreases, which is just the opposite from the case of an inductor. The decrease in capacitive reactance results in an increase in the current.

Let's calculate the new capacitive reactance and the new rms current:

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi(120\text{ Hz})(8.00 \times 10^{-6}\text{ F})} = 166\ \Omega$$

$$I_{\text{rms}} = \frac{150\text{ V}}{166\ \Omega} = 0.904\text{ A}$$

33.5 The *RLC* Series Circuit

In the previous sections, we considered individual circuit elements connected to an AC source. Figure 33.13a shows a circuit that contains a combination of circuit elements: a resistor, an inductor, and a capacitor connected in series across an alternating-voltage source. If the applied voltage varies sinusoidally with time, the instantaneous applied voltage is

$$\Delta v = \Delta V_{\text{max}} \sin \omega t$$

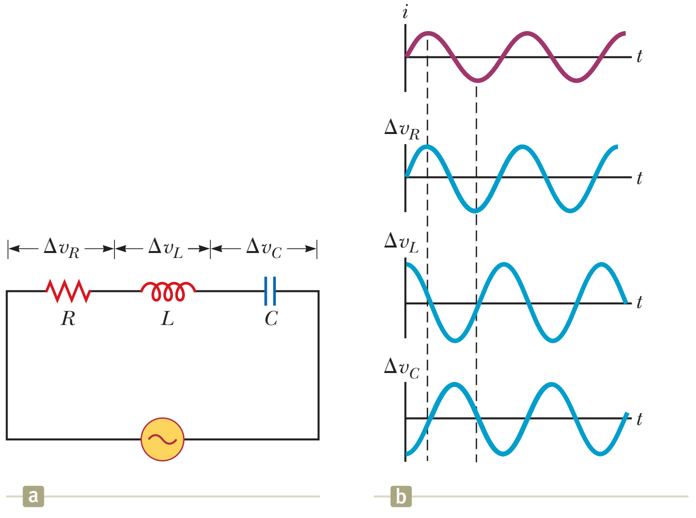


Figure 33.13 (a) A series circuit consisting of a resistor, an inductor, and a capacitor connected to an AC source. (b) Phase relationships between the current and the voltages in the individual circuit elements if they were connected alone to the AC source.

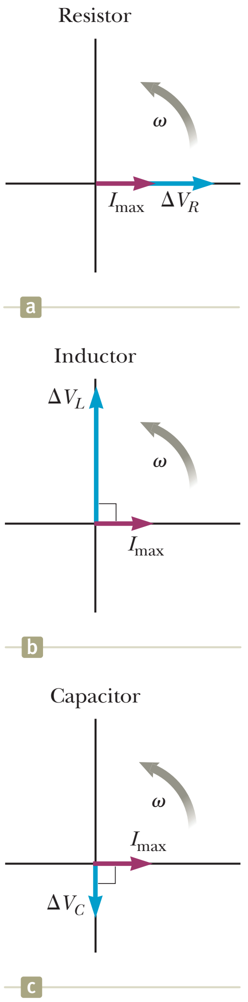


Figure 33.14 Phase relationships between the voltage and current phasors for (a) a resistor, (b) an inductor, and (c) a capacitor connected in series.

Figure 33.13b shows the voltage versus time across each element in the circuit and its phase relationships to the current if it were connected individually to the AC source, as discussed in Sections 33.2–33.4.

When the circuit elements are all connected together to the AC source, as in Figure 33.13a, the current in the circuit is given by

$$i = I_{max} \sin(\omega t - \phi)$$

where ϕ is some **phase angle** between the current and the applied voltage. Based on our discussions of phase in Sections 33.3 and 33.4, we expect that the current will generally not be in phase with the voltage in an *RLC* circuit.

Because the circuit elements in Figure 33.13a are in series, the current everywhere in the circuit must be the same at any instant. That is, the current at all points in a series AC circuit has the same amplitude and phase. Based on the preceding sections, we know that the voltage across each element has a different amplitude and phase. In particular, the voltage across the resistor is in phase with the current, the voltage across the inductor leads the current by 90°, and the voltage across the capacitor lags behind the current by 90°. Using these phase relationships, we can express the instantaneous voltages across the three circuit elements as

$$\Delta v_R = I_{max} R \sin \omega t = \Delta V_R \sin \omega t \tag{33.21}$$

$$\Delta v_L = I_{max} X_L \sin\left(\omega t + \frac{\pi}{2}\right) = \Delta V_L \cos \omega t \tag{33.22}$$

$$\Delta v_C = I_{max} X_C \sin\left(\omega t - \frac{\pi}{2}\right) = -\Delta V_C \cos \omega t \tag{33.23}$$

The sum of these three voltages must equal the instantaneous voltage from the AC source, but it is important to recognize that because the three voltages have different phase relationships with the current, they cannot be added directly. Figure 33.14 represents the phasors at an instant at which the current in all three elements is momentarily zero. The zero current is represented by the current phasor along the horizontal axis in each part of the figure. Next the voltage phasor is drawn at the appropriate phase angle to the current for each element.

Because phasors are rotating vectors, the voltage phasors in Figure 33.14 can be combined using vector addition as in Figure 33.15. In Figure 33.15a, the voltage phasors in Figure 33.14 are combined on the same coordinate axes. Figure 33.15b shows the vector addition of the voltage phasors. The voltage phasors ΔV_L and ΔV_C are in *opposite* directions along the same line, so we can construct the difference phasor $\Delta V_L - \Delta V_C$, which is perpendicular to the phasor ΔV_R. This diagram shows that the vector sum of the voltage amplitudes ΔV_R, ΔV_L, and ΔV_C equals a phasor whose length is the maximum applied voltage ΔV_{max} and which makes an angle ϕ with the current phasor I_{max}. From the right triangle in Figure 33.15b, we see that

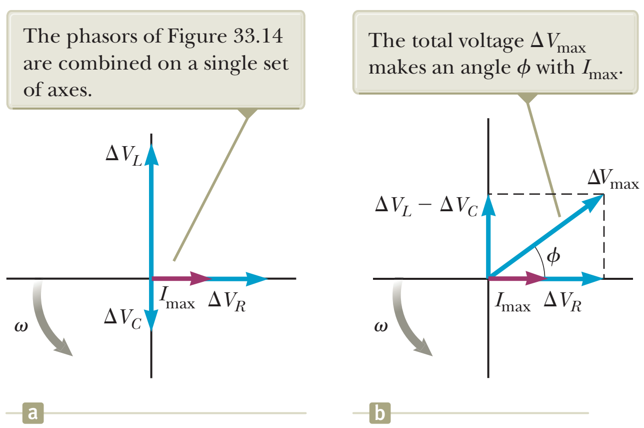


Figure 33.15 (a) Phasor diagram for the series *RLC* circuit shown in Figure 33.13a. (b) The inductance and capacitance phasors are added together and then added vectorially to the resistance phasor.

$$\Delta V_{\max} = \sqrt{\Delta V_R^2 + (\Delta V_L - \Delta V_C)^2} = \sqrt{(I_{\max} R)^2 + (I_{\max} X_L - I_{\max} X_C)^2}$$

$$\Delta V_{\max} = I_{\max} \sqrt{R^2 + (X_L - X_C)^2}$$

Therefore, we can express the maximum current as

$$I_{\max} = \frac{\Delta V_{\max}}{\sqrt{R^2 + (X_L - X_C)^2}} \quad \textbf{(33.24)}$$

◀ **Maximum current in an *RLC* circuit**

Once again, this expression has the same mathematical form as Equation 27.7. The denominator of the fraction plays the role of resistance and is called the **impedance** Z of the circuit:

$$Z \equiv \sqrt{R^2 + (X_L - X_C)^2} \quad \textbf{(33.25)}$$

◀ **Impedance**

where impedance also has units of ohms. Therefore, Equation 33.24 can be written in the form

$$I_{\max} = \frac{\Delta V_{\max}}{Z} \quad \textbf{(33.26)}$$

Equation 33.26 is the AC equivalent of Equation 27.7. Note that the impedance and therefore the current in an AC circuit depend on the resistance, the inductance, the capacitance, and the frequency (because the reactances are frequency dependent).

From the right triangle in the phasor diagram in Figure 33.15b, the phase angle ϕ between the current and the voltage is found as follows:

$$\phi = \tan^{-1}\left(\frac{\Delta V_L - \Delta V_C}{\Delta V_R}\right) = \tan^{-1}\left(\frac{I_{\max} X_L - I_{\max} X_C}{I_{\max} R}\right)$$

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) \quad \textbf{(33.27)}$$

◀ **Phase angle**

When $X_L > X_C$ (which occurs at high frequencies), the phase angle is positive, signifying that the current lags the applied voltage as in Figure 33.15b. We describe this situation by saying that the circuit is *more inductive than capacitive.* When $X_L < X_C$, the phase angle is negative, signifying that the current leads the applied voltage, and the circuit is *more capacitive than inductive.* When $X_L = X_C$, the phase angle is zero and the circuit is *purely resistive.*

Quick Quiz 33.5 Label each part of Figure 33.16, (a), (b), and (c), as representing $X_L > X_C$, $X_L = X_C$, or $X_L < X_C$.

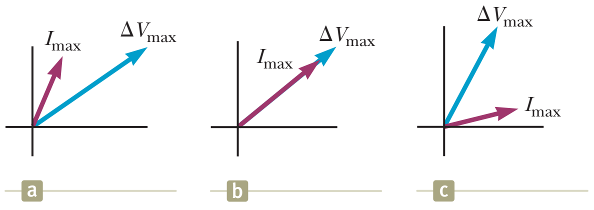


Figure 33.16 (Quick Quiz 33.5) Match the phasor diagrams to the relationships between the reactances.

Example 33.4 Analyzing a Series *RLC* Circuit

A series *RLC* circuit has $R = 425\ \Omega$, $L = 1.25$ H, and $C = 3.50\ \mu$F. It is connected to an AC source with $f = 60.0$ Hz and $\Delta V_{\max} = 150$ V.

(A) Determine the inductive reactance, the capacitive reactance, and the impedance of the circuit.

continued

33.4 continued

SOLUTION

Conceptualize The circuit of interest in this example is shown in Figure 33.13a. The current in the combination of the resistor, inductor, and capacitor oscillates at a particular phase angle with respect to the applied voltage.

Categorize The circuit is a simple series *RLC* circuit, so we can use the approach discussed in this section.

Analyze Find the angular frequency:

$$\omega = 2\pi f = 2\pi(60.0 \text{ Hz}) = 377 \text{ s}^{-1}$$

Use Equation 33.10 to find the inductive reactance:

$$X_L = \omega L = (377 \text{ s}^{-1})(1.25 \text{ H}) = \boxed{471\ \Omega}$$

Use Equation 33.18 to find the capacitive reactance:

$$X_C = \frac{1}{\omega C} = \frac{1}{(377 \text{ s}^{-1})(3.50 \times 10^{-6} \text{ F})} = \boxed{758\ \Omega}$$

Use Equation 33.25 to find the impedance:

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{(425\ \Omega)^2 + (471\ \Omega - 758\ \Omega)^2} = \boxed{513\ \Omega}$$

(B) Find the maximum current in the circuit.

SOLUTION

Use Equation 33.26 to find the maximum current:

$$I_{\text{max}} = \frac{\Delta V_{\text{max}}}{Z} = \frac{150 \text{ V}}{513\ \Omega} = \boxed{0.293 \text{ A}}$$

(C) Find the phase angle between the current and voltage.

SOLUTION

Use Equation 33.27 to calculate the phase angle:

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{471\ \Omega - 758\ \Omega}{425\ \Omega}\right) = \boxed{-34.0°}$$

(D) Find the maximum voltage across each element.

SOLUTION

Use Equations 33.2, 33.11, and 33.19 to calculate the maximum voltages:

$$\Delta V_R = I_{\text{max}} R = (0.293 \text{ A})(425\ \Omega) = \boxed{124 \text{ V}}$$

$$\Delta V_L = I_{\text{max}} X_L = (0.293 \text{ A})(471\ \Omega) = \boxed{138 \text{ V}}$$

$$\Delta V_C = I_{\text{max}} X_C = (0.293 \text{ A})(758\ \Omega) = \boxed{222 \text{ V}}$$

(E) What replacement value of L should an engineer analyzing the circuit choose such that the current leads the applied voltage by 30.0° rather than 34.0°? All other values in the circuit stay the same.

SOLUTION

Solve Equation 33.27 for the inductive reactance:

$$X_L = X_C + R \tan \phi$$

Substitute Equations 33.10 and 33.18 into this expression:

$$\omega L = \frac{1}{\omega C} + R \tan \phi$$

Solve for L:

$$L = \frac{1}{\omega}\left(\frac{1}{\omega C} + R \tan \phi\right)$$

Substitute the given values:

$$L = \frac{1}{(377 \text{ s}^{-1})}\left[\frac{1}{(377 \text{ s}^{-1})(3.50 \times 10^{-6} \text{ F})} + (425\ \Omega)\tan(-30.0°)\right]$$

$$L = \boxed{1.36 \text{ H}}$$

Finalize Because the capacitive reactance is larger than the inductive reactance, the circuit is more capacitive than inductive. In this case, the phase angle ϕ is negative, so the current leads the applied voltage.

33.4 continued

Using Equations 33.21, 33.22, and 33.23, the instantaneous voltages across the three elements are

$$\Delta v_R = (124 \text{ V}) \sin 377t$$

$$\Delta v_L = (138 \text{ V}) \cos 377t$$

$$\Delta v_C = (-222 \text{ V}) \cos 377t$$

WHAT IF? What if you added up the maximum voltages across the three circuit elements? Is that a physically meaningful quantity?

Answer The sum of the maximum voltages across the elements is $\Delta V_R + \Delta V_L + \Delta V_C = 484$ V. This sum is much greater than the maximum voltage of the source, 150 V. The sum of the maximum voltages is a meaningless quantity because when sinusoidally varying quantities are added, *both their amplitudes and their phases* must be taken into account. The maximum voltages across the various elements occur at different times. Therefore, the voltages must be added in a way that takes account of the different phases as shown in Figure 33.15.

33.6 Power in an AC Circuit

Now let's take an energy approach to analyzing AC circuits and consider the transfer of energy from the AC source to the circuit. The power delivered by a battery to an external DC circuit is equal to the product of the current and the terminal voltage of the battery. Likewise, the instantaneous power delivered by an AC source to a circuit is the product of the current and the applied voltage. For the *RLC* circuit shown in Figure 33.13a, we can express the instantaneous power P as

$$P = i\,\Delta v = I_{\max} \sin(\omega t - \phi)\,\Delta V_{\max} \sin \omega t$$

$$P = I_{\max}\,\Delta V_{\max} \sin \omega t \sin(\omega t - \phi) \quad \textbf{(33.28)}$$

This result is a complicated function of time and is therefore not very useful from a practical viewpoint. What is generally of interest is the average power over one or more cycles. Such an average can be computed by first using the trigonometric identity $\sin(\omega t - \phi) = \sin \omega t \cos \phi - \cos \omega t \sin \phi$. Substituting this identity into Equation 33.28 gives

$$P = I_{\max}\,\Delta V_{\max} \sin^2 \omega t \cos \phi - I_{\max}\,\Delta V_{\max} \sin \omega t \cos \omega t \sin \phi \quad \textbf{(33.29)}$$

Let's now take the time average of P over one or more cycles, noting that $I_{\max}$, $\Delta V_{\max}$, ϕ, and ω are all constants. The time average of the first term on the right of the equal sign in Equation 33.29 involves the average value of $\sin^2 \omega t$, which is $\frac{1}{2}$. The time average of the second term on the right of the equal sign is identically zero because $\sin \omega t \cos \omega t = \frac{1}{2} \sin 2\omega t$, and the average value of $\sin 2\omega t$ is zero. Therefore, we can express the **average power** P_{avg} as

$$P_{\text{avg}} = \tfrac{1}{2} I_{\max}\,\Delta V_{\max} \cos \phi \quad \textbf{(33.30)}$$

It is convenient to express the average power in terms of the rms current and rms voltage defined by Equations 33.4 and 33.5:

$$P_{\text{avg}} = I_{\text{rms}}\,\Delta V_{\text{rms}} \cos \phi \quad \textbf{(33.31)}$$

◀ **Average power delivered to an *RLC* circuit**

where the quantity $\cos \phi$ is called the **power factor.** Figure 33.15b shows that the maximum voltage across the resistor is given by $\Delta V_R = \Delta V_{\max} \cos \phi = I_{\max} R$. Therefore, $\cos \phi = I_{\max} R / \Delta V_{\max} = R/Z$, and we can express P_{avg} as

$$P_{\text{avg}} = I_{\text{rms}}\,\Delta V_{\text{rms}} \cos \phi = I_{\text{rms}}\,\Delta V_{\text{rms}} \left(\frac{R}{Z}\right) = I_{\text{rms}} \left(\frac{\Delta V_{\text{rms}}}{Z}\right) R$$

Recognizing that $\Delta V_{rms}/Z = I_{rms}$ gives

$$P_{avg} = I_{rms}^2 R \qquad (33.32)$$

The average power delivered by the source is converted to internal energy in the resistor, just as in the case of a DC circuit. When the load is purely resistive, $\phi = 0$, $\cos\phi = 1$, and, from Equation 33.31, we see that

$$P_{avg} = I_{rms}\,\Delta V_{rms}$$

Note that no power losses are associated with pure capacitors and pure inductors in an AC circuit. To see why that is true, let's first analyze the power in an AC circuit containing only a source and a capacitor. When the current begins to increase in one direction in an AC circuit, charge begins to accumulate on the capacitor and a voltage appears across it. When this voltage reaches its maximum value, the energy stored in the capacitor as electric potential energy is $\frac{1}{2}C(\Delta V_{max})^2$. This energy storage, however, is only momentary. The capacitor is charged and discharged twice during each cycle: charge is delivered to the capacitor during two quarters of the cycle and is returned to the voltage source during the remaining two quarters. Therefore, the average power supplied by the source is zero. In other words, no power losses occur in a capacitor in an AC circuit.

Now consider the case of an inductor. When the current in an inductor reaches its maximum value, the energy stored in the inductor is a maximum and is given by $\frac{1}{2}LI_{max}^2$. When the current begins to decrease in the circuit, this stored energy in the inductor returns to the source as the inductor attempts to maintain the current in the circuit.

Equation 33.31 shows that the power delivered by an AC source to any circuit depends on the phase, a result that has many interesting applications. For example, a factory that uses large motors in machines, generators, or transformers has a large inductive load (because of all the windings). To deliver greater power to such devices in the factory without using excessively high voltages, technicians introduce capacitance in the circuits to shift the phase.

Quick Quiz 33.6 An AC source drives an *RLC* circuit with a fixed voltage amplitude. If the driving frequency is ω_1, the circuit is more capacitive than inductive and the phase angle is $-10°$. If the driving frequency is ω_2, the circuit is more inductive than capacitive and the phase angle is $+10°$. At what frequency is the largest amount of power delivered to the circuit? **(a)** It is largest at ω_1. **(b)** It is largest at ω_2. **(c)** The same amount of power is delivered at both frequencies.

Example 33.5 Average Power in an *RLC* Series Circuit

Calculate the average power delivered to the series *RLC* circuit described in Example 33.4.

SOLUTION

Conceptualize Consider the circuit in Figure 33.13a and imagine energy being delivered to the circuit by the AC source. Review Example 33.4 for other details about this circuit.

Categorize We find the result by using equations developed in this section, so we categorize this example as a substitution problem.

Use Equation 33.5 and the maximum voltage from Example 33.4 to find the rms voltage from the source:

$$\Delta V_{rms} = \frac{\Delta V_{max}}{\sqrt{2}} = \frac{150\text{ V}}{\sqrt{2}} = 106\text{ V}$$

Similarly, find the rms current in the circuit:

$$I_{rms} = \frac{I_{max}}{\sqrt{2}} = \frac{0.293\text{ A}}{\sqrt{2}} = 0.207\text{ A}$$

33.5 continued

Use Equation 33.31 to find the power delivered by the source:

$$P_{avg} = I_{rms} V_{rms} \cos \phi = (0.207 \text{ A})(106 \text{ V}) \cos (-34.0°)$$

$$= \boxed{18.2 \text{ W}}$$

33.7 Resonance in a Series *RLC* Circuit

We investigated resonance in mechanical oscillating systems in Chapter 15. As shown in Chapter 32, a series *RLC* circuit is an electrical oscillating system. Such a circuit is said to be **in resonance** when the driving frequency is such that the rms current has its maximum value. In general, the rms current can be written

$$I_{rms} = \frac{\Delta V_{rms}}{Z} \quad \textbf{(33.33)}$$

where Z is the impedance. Substituting the expression for Z from Equation 33.25 into Equation 33.33 gives

$$I_{rms} = \frac{\Delta V_{rms}}{\sqrt{R^2 + (X_L - X_C)^2}} \quad \textbf{(33.34)}$$

Because the impedance depends on the frequency of the source, the current in the *RLC* circuit also depends on the frequency. The angular frequency ω_0 at which $X_L - X_C = 0$ is called the **resonance frequency** of the circuit. To find ω_0, we set $X_L = X_C$, which gives $\omega_0 L = 1/\omega_0 C$, or

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad \textbf{(33.35)}$$

◀ Resonance frequency

This frequency also corresponds to the natural frequency of oscillation of an *LC* circuit (see Section 32.5). Therefore, the rms current in a series *RLC* circuit has its maximum value when the frequency of the applied voltage matches the natural oscillator frequency, which depends only on L and C. Furthermore, at the resonance frequency, the current is in phase with the applied voltage.

Quick Quiz 33.7 What is the impedance of a series *RLC* circuit at resonance? **(a)** larger than R **(b)** less than R **(c)** equal to R **(d)** impossible to determine

A plot of rms current versus angular frequency for a series *RLC* circuit is shown in Figure 33.17a on page 1014. The data assume a constant $\Delta V_{rms} = 5.0$ mV, $L = 5.0\ \mu$H, and $C = 2.0$ nF. The three curves correspond to three values of R. In each case, the rms current has its maximum value at the resonance frequency ω_0. Furthermore, the curves become narrower and taller as the resistance decreases.

Equation 33.34 shows that when $R = 0$, the current becomes infinite at resonance. Real circuits, however, always have some resistance, which limits the value of the current to some finite value.

We can also calculate the average power as a function of frequency for a series *RLC* circuit. Using Equations 33.32, 33.33, and 33.25 gives

$$P_{avg} = I_{rms}^2 R = \frac{(\Delta V_{rms})^2}{Z^2} R = \frac{(\Delta V_{rms})^2 R}{R^2 + (X_L - X_C)^2} \quad \textbf{(33.36)}$$

Because $X_L = \omega L$, $X_C = 1/\omega C$, and $\omega_0{}^2 = 1/LC$, the term $(X_L - X_C)^2$ can be expressed as

$$(X_L - X_C)^2 = \left(\omega L - \frac{1}{\omega C}\right)^2 = \frac{L^2}{\omega^2}(\omega^2 - \omega_0{}^2)^2$$

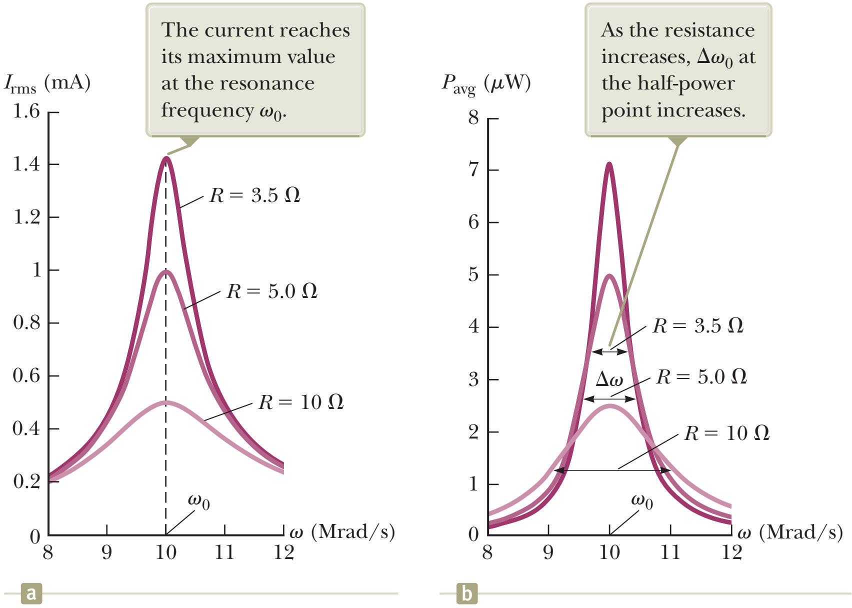


Figure 33.17 (a) The rms current versus frequency for a series *RLC* circuit for three values of *R*. (b) Average power delivered to the circuit versus frequency for the series *RLC* circuit for three values of *R*.

Using this result in Equation 33.36 gives

Average power as a function of frequency in an *RLC* circuit

$$P_{avg} = \frac{(\Delta V_{rms})^2 R\omega^2}{R^2\omega^2 + L^2(\omega^2 - \omega_0^2)^2} \qquad (33.37)$$

Equation 33.37 shows that at resonance, when $\omega = \omega_0$, the average power is a maximum and has the value $(\Delta V_{rms})^2/R$. Figure 33.17b is a plot of average power versus frequency for three values of R in a series *RLC* circuit. As the resistance is made smaller, the curve becomes sharper in the vicinity of the resonance frequency. This curve sharpness is usually described by a dimensionless parameter known as the **quality factor,**[2] denoted by Q:

Quality factor

$$Q = \frac{\omega_0}{\Delta\omega}$$

where $\Delta\omega$ is the width of the curve measured between the two values of ω for which P_{avg} has one-half its maximum value, called the *half-power points* (see Fig. 33.17b.) It is left as a problem (Problem 76) to show that the width at the half-power points has the value $\Delta\omega = R/L$ so that

$$Q = \frac{\omega_0 L}{R} \qquad (33.38)$$

A radio's receiving circuit is an important application of a resonant circuit. The radio is tuned to a particular station (which transmits an electromagnetic wave or signal of a specific frequency) by varying a capacitor, which changes the receiving circuit's resonance frequency. When the circuit is driven by the electromagnetic oscillations a radio signal produces in an antenna, the tuner circuit responds with a large amplitude of electrical oscillation only for the station frequency that matches the resonance frequency. Therefore, only the signal from one radio station is passed on to the amplifier and loudspeakers even though signals from all stations are driving the circuit at the same time. Because many signals are often present over a range of frequencies, it is important to design a high-Q circuit to eliminate unwanted signals. In this manner, stations whose frequencies are near but not equal to the resonance frequency have a response at the receiver that is negligibly small relative to the signal that matches the resonance frequency.

[2]The quality factor is also defined as the ratio $2\pi E/\Delta E$, where E is the energy stored in the oscillating system and ΔE is the energy decrease per cycle of oscillation due to the resistance.

Example 33.6 A Resonating Series *RLC* Circuit

Consider a series *RLC* circuit for which $R = 150\ \Omega$, $L = 20.0$ mH, $\Delta V_{rms} = 20.0$ V, and $\omega = 5\ 000\ s^{-1}$. Determine the value of the capacitance for which the current is a maximum.

SOLUTION

Conceptualize Consider the circuit in Figure 33.13a and imagine varying the frequency of the AC source. The current in the circuit has its maximum value at the resonance frequency ω_0.

Categorize We find the result by using equations developed in this section, so we categorize this example as a substitution problem.

Use Equation 33.35 to solve for the required capacitance in terms of the resonance frequency:

$$\omega_0 = \frac{1}{\sqrt{LC}} \rightarrow C = \frac{1}{\omega_0^2 L}$$

Substitute numerical values:

$$C = \frac{1}{(5.00 \times 10^3\ s^{-1})^2(20.0 \times 10^{-3}\ H)} = 2.00\ \mu F$$

33.8 The Transformer and Power Transmission

As discussed in Section 27.6, it is economical to use a high voltage and a low current to minimize the I^2R loss in transmission lines when electric power is transmitted over great distances. Consequently, 350-kV lines are common, and in many areas, even higher-voltage (765-kV) lines are used. At the receiving end of such lines, the consumer requires power at a low voltage (for safety and for efficiency in design). In practice, the voltage is decreased to approximately 20 000 V at a distribution substation, then to 4 000 V for delivery to residential areas, and finally to 120 V and 240 V at the customer's site. Therefore, a device is needed that can change the alternating voltage and current without causing appreciable changes in the power delivered. The AC transformer is that device.

In its simplest form, the **AC transformer** consists of two coils of wire wound around a core of iron as illustrated in Figure 33.18. (Compare this arrangement to Faraday's experiment in Figure 31.2.) The coil on the left, which is connected to the input alternating-voltage source and has N_1 turns, is called the *primary winding* (or the *primary*). The coil on the right, consisting of N_2 turns and connected to a load resistor R_L, is called the *secondary winding* (or the *secondary*). The purposes of the iron core are to increase the magnetic flux through the coil and to provide a medium in which nearly all the magnetic field lines through one coil pass through the other coil. Eddy-current losses are reduced by using a laminated core. Transformation of energy to internal energy in the finite resistance of the coil wires is usually quite small. Typical transformers have power efficiencies from 90% to

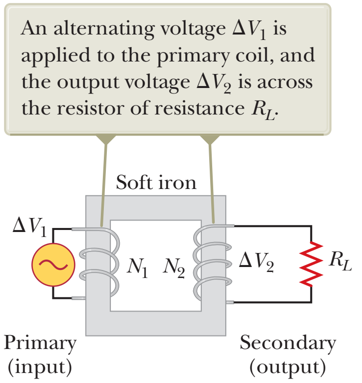


Figure 33.18 An ideal transformer consists of two coils wound on the same iron core.

99%. In the discussion that follows, let's assume we are working with an *ideal transformer,* one in which the energy losses in the windings and core are zero.

Faraday's law states that the voltage Δv_1 across the primary is

$$\Delta v_1 = -N_1 \frac{d\Phi_B}{dt} \quad \textbf{(33.39)}$$

where Φ_B is the magnetic flux through each turn. If we assume all magnetic field lines remain within the iron core, the flux through each turn of the primary equals the flux through each turn of the secondary. Hence, the voltage across the secondary is

$$\Delta v_2 = -N_2 \frac{d\Phi_B}{dt} \quad \textbf{(33.40)}$$

Solving Equation 33.39 for $d\Phi_B/dt$ and substituting the result into Equation 33.40 gives

$$\Delta v_2 = \frac{N_2}{N_1}\Delta v_1 \quad \textbf{(33.41)}$$

When $N_2 > N_1$, the output voltage Δv_2 exceeds the input voltage Δv_1. This configuration is referred to as a *step-up transformer.* When $N_2 < N_1$, the output voltage is less than the input voltage, and we have a *step-down transformer.* A circuit diagram for a transformer connected to a load resistance is shown in Figure 33.19.

When a current I_1 exists in the primary circuit, a current I_2 is induced in the secondary. (In this discussion, uppercase I and ΔV refer to rms values.) If the load in the secondary circuit is a pure resistance, the induced current is in phase with the induced voltage. The power supplied to the secondary circuit must be provided by the AC source connected to the primary circuit. In an ideal transformer where there are no losses, the power $I_1\,\Delta V_1$ supplied by the source is equal to the power $I_2\,\Delta V_2$ in the secondary circuit. That is,

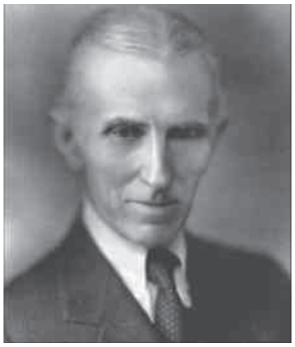
$$I_1\,\Delta V_1 = I_2\,\Delta V_2 \quad \textbf{(33.42)}$$

The value of the load resistance R_L determines the value of the secondary current because $I_2 = \Delta V_2/R_L$. Furthermore, the current in the primary is $I_1 = \Delta V_1/R_{\text{eq}}$, where

$$R_{\text{eq}} = \left(\frac{N_1}{N_2}\right)^2 R_L \quad \textbf{(33.43)}$$

is the equivalent resistance of the load resistance when viewed from the primary side. We see from this analysis that a transformer may be used to match resistances between the primary circuit and the load. In this manner, maximum power transfer can be achieved between a given power source and the load resistance. For example, a transformer connected between the 1-kΩ output of an audio amplifier and an 8-Ω speaker ensures that as much of the audio signal as possible is transferred into the speaker. In stereo terminology, this process is called *impedance matching.*

To operate properly, many common household electronic devices require low voltages. A small transformer that plugs directly into the wall like the one illustrated in Figure 33.20 can provide the proper voltage. The photograph shows the two windings wrapped around a common iron core that is found inside all these



Nikola Tesla
American Physicist (1856–1943)
Tesla was born in Croatia, but he spent most of his professional life as an inventor in the United States. He was a key figure in the development of alternating-current electricity, high-voltage transformers, and the transport of electrical power using AC transmission lines. Tesla's viewpoint was at odds with the ideas of Thomas Edison, who committed himself to the use of direct current in power transmission. Tesla's AC approach won out.

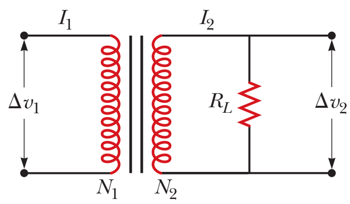


Figure 33.19 Circuit diagram for a transformer.

This transformer is smaller than the one in the opening photograph of this chapter. In addition, it is a step-down transformer. It drops the voltage from 4 000 V to 240 V for delivery to a group of residences.

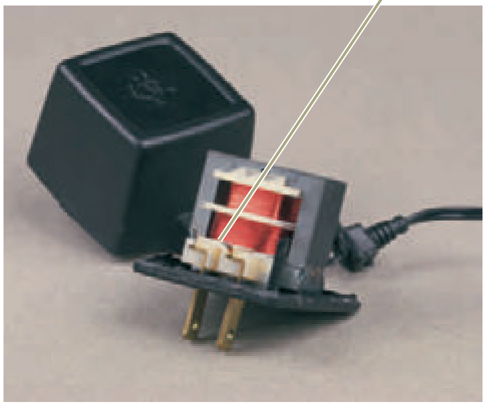


Figure 33.20 Electronic devices are often powered by AC adaptors containing transformers such as this one. These adaptors alter the AC voltage. In many applications, the adaptors also convert alternating current to direct current.

little "black boxes." This particular transformer converts the 120-V AC in the wall socket to 12.5-V AC. (Can you determine the ratio of the numbers of turns in the two coils?) Some black boxes also make use of diodes to convert the alternating current to direct current. (See Section 33.9.)

Example 33.7 The Economics of AC Power

An electricity-generating station needs to deliver energy at a rate of 20 MW to a city 1.0 km away. A common voltage for commercial power generators is 22 kV, but a step-up transformer is used to boost the voltage to 230 kV before transmission.

(A) If the resistance of the wires is 2.0 Ω and the energy costs are about 11¢/kWh, estimate the cost of the energy converted to internal energy in the wires during one day.

SOLUTION

Conceptualize The resistance of the wires is in series with the resistance representing the load (homes and businesses). Therefore, there is a voltage drop in the wires, which means that some of the transmitted energy is converted to internal energy in the wires and never reaches the load.

Categorize This problem involves finding the power delivered to a resistive load in an AC circuit. Let's ignore any capacitive or inductive characteristics of the load and set the power factor equal to 1.

Analyze Calculate I_{rms} in the wires from Equation 33.31:

$$I_{rms} = \frac{P_{avg}}{\Delta V_{rms}} = \frac{20 \times 10^6 \text{ W}}{230 \times 10^3 \text{ V}} = 87 \text{ A}$$

Determine the rate at which energy is delivered to the resistance in the wires from Equation 33.32:

$$P_{wires} = I_{rms}^2 R = (87 \text{ A})^2 (2.0\ \Omega) = 15 \text{ kW}$$

Calculate the energy T_{ET} delivered to the wires over the course of a day:

$$T_{ET} = P_{wires}\,\Delta t = (15 \text{ kW})(24 \text{ h}) = \boxed{363 \text{ kWh}}$$

Find the cost of this energy at a rate of 11¢/kWh:

$$\text{Cost} = (363 \text{ kWh})(\$0.11/\text{kWh}) = \boxed{\$40}$$

(B) Repeat the calculation for the situation in which the power plant delivers the energy at its original voltage of 22 kV.

continued

▸ **33.7** continued

SOLUTION

Calculate I_{rms} in the wires from Equation 33.31:

$$I_{rms} = \frac{P_{avg}}{\Delta V_{rms}} = \frac{20 \times 10^6 \text{ W}}{22 \times 10^3 \text{ V}} = 909 \text{ A}$$

From Equation 33.32, determine the rate at which energy is delivered to the resistance in the wires:

$$P_{wires} = I_{rms}^2 R = (909 \text{ A})^2(2.0\ \Omega) = 1.7 \times 10^3 \text{ kW}$$

Calculate the energy delivered to the wires over the course of a day:

$$T_{ET} = P_{wires}\,\Delta t = (1.7 \times 10^3 \text{ kW})(24 \text{ h}) = 4.0 \times 10^4 \text{ kWh}$$

Find the cost of this energy at a rate of 11¢/kWh:

$$\text{Cost} = (4.0 \times 10^4 \text{ kWh})(\$0.11/\text{kWh}) = \$4.4 \times 10^3$$

Finalize Notice the tremendous savings that are possible through the use of transformers and high-voltage transmission lines. Such savings in combination with the efficiency of using alternating current to operate motors led to the universal adoption of alternating current instead of direct current for commercial power grids.

33.9 Rectifiers and Filters

Portable electronic devices such as radios and laptop computers are often powered by direct current supplied by batteries. Many devices come with AC–DC converters such as that shown in Figure 33.20. Such a converter contains a transformer that steps the voltage down from 120 V to, typically, 6 V or 9 V and a circuit that converts alternating current to direct current. The AC–DC converting process is called **rectification,** and the converting device is called a **rectifier.**

The most important element in a rectifier circuit is a **diode,** a circuit element that conducts current in one direction but not the other. Most diodes used in modern electronics are semiconductor devices. The circuit symbol for a diode is , where the arrow indicates the direction of the current in the diode. A diode has low resistance to current in one direction (the direction of the arrow) and high resistance to current in the opposite direction. To understand how a diode rectifies a current, consider Figure 33.21a, which shows a diode and a resistor connected to the secondary of a transformer. The transformer reduces the voltage from 120-V AC to the lower voltage that is needed for the device having a resistance R (the load

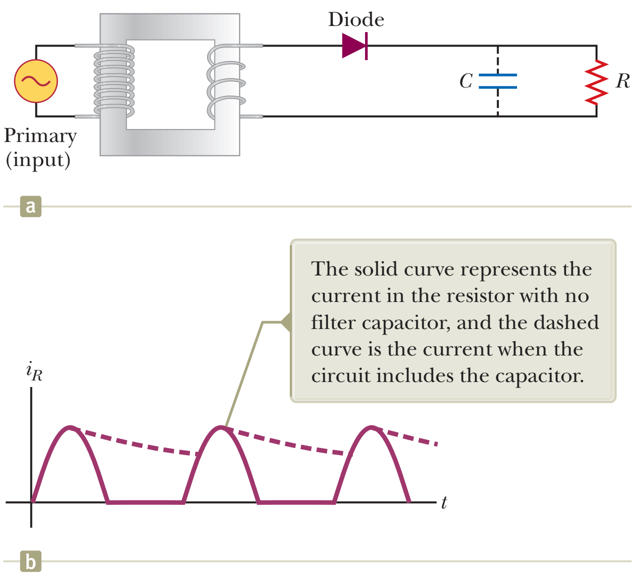


Figure 33.21 (a) A half-wave rectifier with an optional filter capacitor. (b) Current versus time in the resistor.

resistance). Because the diode conducts current in only one direction, the alternating current in the load resistor is reduced to the form shown by the solid curve in Figure 33.21b. The diode conducts current only when the side of the symbol containing the arrowhead has a positive potential relative to the other side. In this situation, the diode acts as a *half-wave rectifier* because current is present in the circuit only during half of each cycle.

When a capacitor is added to the circuit as shown by the dashed lines and the capacitor symbol in Figure 33.21a, the circuit is a simple DC power supply. The time variation of the current in the load resistor (the dashed curve in Fig. 33.21b) is close to being zero, as determined by the *RC* time constant of the circuit. As the current in the circuit begins to rise at $t = 0$ in Figure 33.21b, the capacitor charges up. When the current begins to fall, however, the capacitor discharges through the resistor, so the current in the resistor does not fall as quickly as the current from the transformer.

The *RC* circuit in Figure 33.21a is one example of a **filter circuit,** which is used to smooth out or eliminate a time-varying signal. For example, radios are usually powered by a 60-Hz alternating voltage. After rectification, the voltage still contains a small AC component at 60 Hz (sometimes called *ripple*), which must be filtered. By "filtered," we mean that the 60-Hz ripple must be reduced to a value much less than that of the audio signal to be amplified because without filtering, the resulting audio signal includes an annoying hum at 60 Hz.

We can also design filters that respond differently to different frequencies. Consider the simple series *RC* circuit shown in Figure 33.22a. The input voltage is across the series combination of the two elements. The output is the voltage across the resistor. A plot of the ratio of the output voltage to the input voltage as a function of the logarithm of angular frequency (see Fig. 33.22b) shows that at low frequencies, ΔV_{out} is much smaller than ΔV_{in}, whereas at high frequencies, the two voltages are equal. Because the circuit preferentially passes signals of higher frequency while blocking low-frequency signals, the circuit is called an ***RC* high-pass filter.** (See Problem 54 for an analysis of this filter.)

Physically, a high-pass filter works because a capacitor "blocks out" direct current and AC current at low frequencies. At low frequencies, the capacitive reactance is large and much of the applied voltage appears across the capacitor rather than across the output resistor. As the frequency increases, the capacitive reactance drops and more of the applied voltage appears across the resistor.

Now consider the circuit shown in Figure 33.23a on page 1020, where we have interchanged the resistor and capacitor and where the output voltage is taken across the capacitor. At low frequencies, the reactance of the capacitor and the voltage across the capacitor is high. As the frequency increases, the voltage across the capacitor drops. Therefore, this filter is an ***RC* low-pass filter.** The ratio of output voltage to input voltage (see Problem 56), plotted as a function of the logarithm of ω in Figure 33.23b, shows this behavior.

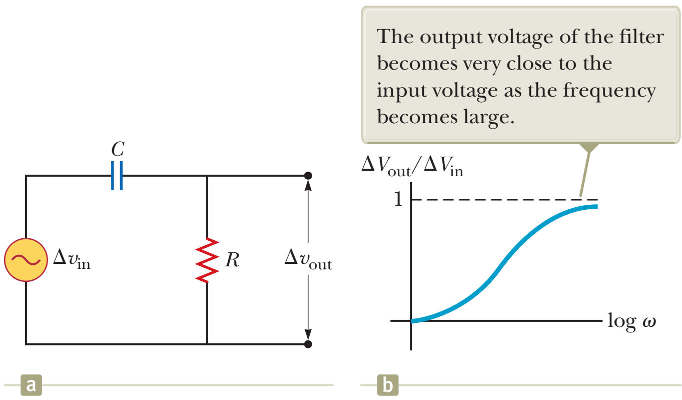


Figure 33.22 (a) A simple *RC* high-pass filter. (b) Ratio of output voltage to input voltage for an *RC* high-pass filter as a function of the angular frequency of the AC source.

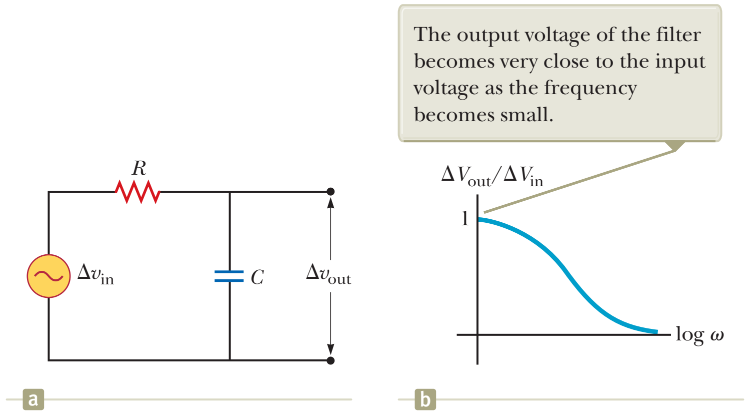


Figure 33.23 (a) A simple *RC* low-pass filter. (b) Ratio of output voltage to input voltage for an *RC* low-pass filter as a function of the angular frequency of the AC source.

You may be familiar with crossover networks, which are an important part of the speaker systems for high-quality audio systems. These networks use low-pass filters to direct low frequencies to a special type of speaker, the "woofer," which is designed to reproduce the low notes accurately. The high frequencies are sent by a high-pass filter to the "tweeter" speaker.

Summary

Definitions

In AC circuits that contain inductors and capacitors, it is useful to define the **inductive reactance** X_L and the **capacitive reactance** X_C as

$$X_L \equiv \omega L \quad (33.10)$$

$$X_C \equiv \frac{1}{\omega C} \quad (33.18)$$

where ω is the angular frequency of the AC source. The SI unit of reactance is the ohm.

The **impedance** Z of an *RLC* series AC circuit is

$$Z \equiv \sqrt{R^2 + (X_L - X_C)^2} \quad (33.25)$$

This expression illustrates that we cannot simply add the resistance and reactances in a circuit. We must account for the applied voltage and current being out of phase, with the **phase angle** ϕ between the current and voltage being

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) \quad (33.27)$$

The sign of ϕ can be positive or negative, depending on whether X_L is greater or less than X_C. The phase angle is zero when $X_L = X_C$.

Concepts and Principles

The **rms current** and **rms voltage** in an AC circuit in which the voltages and current vary sinusoidally are given by

$$I_{rms} = \frac{I_{max}}{\sqrt{2}} = 0.707 I_{max} \quad (33.4)$$

$$\Delta V_{rms} = \frac{\Delta V_{max}}{\sqrt{2}} = 0.707 \Delta V_{max} \quad (33.5)$$

where I_{max} and ΔV_{max} are the maximum values.

If an AC circuit consists of a source and a resistor, the current is in phase with the voltage. That is, the current and voltage reach their maximum values at the same time.

If an AC circuit consists of a source and an inductor, the current lags the voltage by 90°. That is, the voltage reaches its maximum value one-quarter of a period before the current reaches its maximum value.

If an AC circuit consists of a source and a capacitor, the current leads the voltage by 90°. That is, the current reaches its maximum value one-quarter of a period before the voltage reaches its maximum value.

The **average power** delivered by the source in an *RLC* circuit is

$$P_{avg} = I_{rms}\,\Delta V_{rms}\cos\phi \quad (33.31)$$

An equivalent expression for the average power is

$$P_{avg} = I_{rms}^2 R \quad (33.32)$$

The average power delivered by the source results in increasing internal energy in the resistor. No power loss occurs in an ideal inductor or capacitor.

The rms current in a series *RLC* circuit is

$$I_{rms} = \frac{\Delta V_{rms}}{\sqrt{R^2 + (X_L - X_C)^2}} \quad (33.34)$$

A series *RLC* circuit is in resonance when the inductive reactance equals the capacitive reactance. When this condition is met, the rms current given by Equation 33.34 has its maximum value. The **resonance frequency** ω_0 of the circuit is

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad (33.35)$$

The rms current in a series *RLC* circuit has its maximum value when the frequency of the source equals ω_0, that is, when the "driving" frequency matches the resonance frequency.

AC transformers allow for easy changes in alternating voltage according to

$$\Delta v_2 = \frac{N_2}{N_1}\Delta v_1 \quad (33.41)$$

where N_1 and N_2 are the numbers of windings on the primary and secondary coils, respectively, and Δv_1 and Δv_2 are the voltages on these coils.

Objective Questions

1. denotes answer available in *Student Solutions Manual/Study Guide*

1. An inductor and a resistor are connected in series across an AC source as in Figure OQ33.1. Immediately after the switch is closed, which of the following statements is true? (a) The current in the circuit is $\Delta V/R$. (b) The voltage across the inductor is zero. (c) The current in the circuit is zero. (d) The voltage across the resistor is ΔV. (e) The voltage across the inductor is half its maximum value.

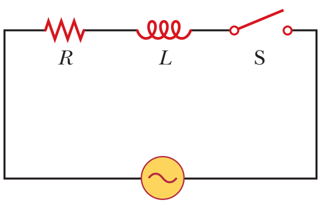


Figure OQ33.1

2. **(i)** When a particular inductor is connected to a source of sinusoidally varying emf with constant amplitude and a frequency of 60.0 Hz, the rms current is 3.00 A. What is the rms current if the source frequency is doubled? (a) 12.0 A (b) 6.00 A (c) 4.24 A (d) 3.00 A (e) 1.50 A **(ii)** Repeat part (i) assuming the load is a capacitor instead of an inductor. **(iii)** Repeat part (i) assuming the load is a resistor instead of an inductor.

3. A capacitor and a resistor are connected in series across an AC source as shown in Figure OQ33.3. After the switch is closed, which of the following statements is true? (a) The voltage across the capacitor lags the current by 90°. (b) The voltage across the resistor is out of phase with the current. (c) The voltage across the capacitor leads the current by 90°. (d) The current decreases as the frequency of the source is increased, but its peak voltage remains the same. (e) None of those statements is correct.

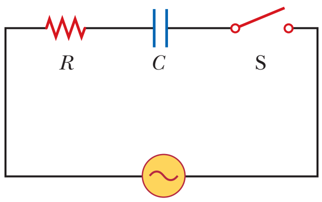


Figure OQ33.3

4. **(i)** What is the time average of the "square-wave" potential shown in Figure OQ33.4? (a) $\sqrt{2}\,\Delta V_{max}$ (b) ΔV_{max} (c) $\Delta V_{max}/\sqrt{2}$ (d) $\Delta V_{max}/2$ (e) $\Delta V_{max}/4$ **(ii)** What is the rms voltage? Choose from the same possibilities as in part (i).

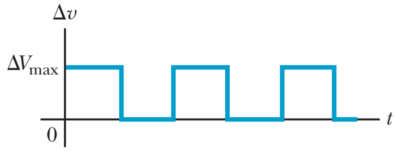


Figure OQ33.4

5. If the voltage across a circuit element has its maximum value when the current in the circuit is zero, which of the following statements *must* be true? (a) The circuit element is a resistor. (b) The circuit element is a capacitor. (c) The circuit element is an inductor. (d) The current and voltage are 90° out of phase. (e) The current and voltage are 180° out of phase.

6. A sinusoidally varying potential difference has amplitude 170 V. **(i)** What is its minimum instantaneous value? (a) 170 V (b) 120 V (c) 0 (d) −120 V (e) −170 V **(ii)** What is its average value? **(iii)** What is its rms value? Choose from the same possibilities as in part (i) in each case.

7. A series *RLC* circuit contains a 20.0-Ω resistor, a 0.750-μF capacitor, and a 120-mH inductor. **(i)** If a sinusoidally varying rms voltage of 120 V at $f = 500$ Hz is applied across this combination of elements, what is the rms current in the circuit? (a) 2.33 A (b) 6.00 A (c) 10.0 A (d) 17.0 A (e) none of those answers **(ii) What If?** What is the rms current in the circuit when operating at its resonance frequency? Choose from the same possibilities as in part (i).

8. A resistor, a capacitor, and an inductor are connected in series across an AC source. Which of the following statements is *false?* (a) The instantaneous voltage across the capacitor lags the current by 90°. (b) The instantaneous voltage across the inductor leads the current by 90°. (c) The instantaneous voltage across the resistor is in phase with the current. (d) The voltages across the resistor, capacitor, and inductor are not in phase. (e) The rms voltage across the combination of the three elements equals the algebraic sum of the rms voltages across each element separately.

9. Under what conditions is the impedance of a series *RLC* circuit equal to the resistance in the circuit? (a) The driving frequency is lower than the resonance frequency. (b) The driving frequency is equal to the resonance frequency. (c) The driving frequency is higher than the resonance frequency. (d) always (e) never

10. What is the phase angle in a series *RLC* circuit at resonance? (a) 180° (b) 90° (c) 0 (d) −90° (e) None of those answers is necessarily correct.

11. A circuit containing an AC source, a capacitor, an inductor, and a resistor has a high-Q resonance at 1 000 Hz. From greatest to least, rank the following contributions to the impedance of the circuit at that frequency and at lower and higher frequencies. Note any cases of equality in your ranking. (a) X_C at 500 Hz (b) X_C at 1 500 Hz (c) X_L at 500 Hz (d) X_L at 1 500 Hz (e) R at 1 000 Hz

12. A 6.00-V battery is connected across the primary coil of a transformer having 50 turns. If the secondary coil of the transformer has 100 turns, what voltage appears across the secondary? (a) 24.0 V (b) 12.0 V (c) 6.00 V (d) 3.00 V (e) none of those answers

13. Do AC ammeters and voltmeters read (a) peak-to-valley, (b) maximum, (c) rms, or (d) average values?

Conceptual Questions

1. denotes answer available in *Student Solutions Manual/Study Guide*

1. (a) Explain how the quality factor is related to the response characteristics of a radio receiver. (b) Which variable most strongly influences the quality factor?

2. (a) Explain how the mnemonic "ELI the ICE man" can be used to recall whether current leads voltage or voltage leads current in *RLC* circuits. Note that E represents emf $\mathcal{E}$. (b) Explain how "CIVIL" works as another mnemonic device, where V represents voltage.

3. Why is the sum of the maximum voltages across each element in a series *RLC* circuit usually greater than the maximum applied voltage? Doesn't that inequality violate Kirchhoff's loop rule?

4. (a) Does the phase angle in an *RLC* series circuit depend on frequency? (b) What is the phase angle for the circuit when the inductive reactance equals the capacitive reactance?

5. Do some research to answer these questions: Who invented the metal detector? Why? What are its limitations?

6. As shown in Figure CQ33.6, a person pulls a vacuum cleaner at speed v across a horizontal floor, exerting on it a force of magnitude F directed upward at an angle θ with the horizontal. (a) At what rate is the person doing work on the cleaner? (b) State as completely as you can the analogy between power in this situation and in an electric circuit.

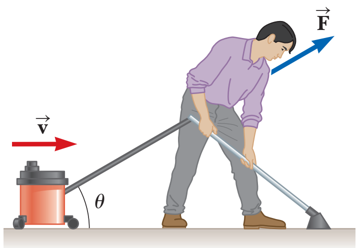


Figure CQ33.6

7. A certain power supply can be modeled as a source of emf in series with both a resistance of 10 Ω and an inductive reactance of 5 Ω. To obtain maximum power delivered to the load, it is found that the load should have a resistance of $R_L = 10$ Ω, an inductive reactance of zero, and a capacitive reactance of 5 Ω. (a) With this load, is the circuit in resonance? (b) With this load, what fraction of the average power put out by the source of emf is delivered to the load? (c) To increase the fraction of the power delivered to the load, how could the load be changed? You may wish to review Example 28.2 and Problem 4 in Chapter 28 on maximum power transfer in DC circuits.

8. Will a transformer operate if a battery is used for the input voltage across the primary? Explain.

9. (a) Why does a capacitor act as a short circuit at high frequencies? (b) Why does a capacitor act as an open circuit at low frequencies?

10. An ice storm breaks a transmission line and interrupts electric power to a town. A homeowner starts a gasoline-powered 120-V generator and clips its output terminals to "hot" and "ground" terminals of the electrical panel for his house. On a power pole down the block is a transformer designed to step down the voltage for household use. It has a ratio of turns N_1/N_2 of 100 to 1. A repairman climbs the pole. What voltage will he encounter on the input side of the transformer? As this question implies, safety precautions must be taken in the use of home generators and during power failures in general.

Problems

ENHANCED WebAssign The problems found in this chapter may be assigned online in Enhanced WebAssign

1. straightforward; **2.** intermediate; **3.** challenging

[1.] full solution available in the *Student Solutions Manual/Study Guide*

AMT Analysis Model tutorial available in Enhanced WebAssign

GP Guided Problem

M Master It tutorial available in Enhanced WebAssign

W Watch It video solution available in Enhanced WebAssign

Section 33.1 AC Sources

Section 33.2 Resistors in an AC Circuit

1. When an AC source is connected across a 12.0-Ω resistor, the rms current in the resistor is 8.00 A. Find (a) the rms voltage across the resistor, (b) the peak voltage of the source, (c) the maximum current in the resistor, and (d) the average power delivered to the resistor.

2. (a) What is the resistance of a lightbulb that uses an average power of 75.0 W when connected to a 60.0-Hz power source having a maximum voltage of 170 V? (b) **What If?** What is the resistance of a 100-W lightbulb?

3. An AC power supply produces a maximum voltage ΔV_{max} = 100 V. This power supply is connected to a resistor R = 24.0 Ω, and the current and resistor voltage are measured with an ideal AC ammeter and voltmeter as shown in Figure P33.3. An ideal ammeter has zero resistance, and an ideal voltmeter has infinite resistance. What is the reading on (a) the ammeter and (b) the voltmeter?

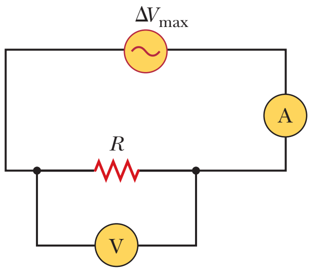


Figure P33.3

4. A certain lightbulb is rated at 60.0 W when operating at an rms voltage of 120 V. (a) What is the peak voltage applied across the bulb? (b) What is the resistance of the bulb? (c) Does a 100-W bulb have greater or less resistance than a 60.0-W bulb? Explain.

[5.] The current in the circuit shown in Figure P33.5 equals 60.0% of the peak current at t = 7.00 ms. What is the lowest source frequency that gives this current?

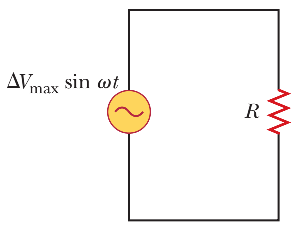


Figure P33.5 Problems 5 and 6.

6. In the AC circuit shown in Figure P33.5, R = 70.0 Ω and the output voltage of the AC source is $\Delta V_{max} \sin \omega t$. (a) If $\Delta V_R = 0.250\ \Delta V_{max}$ for the first time at t = 0.010 0 s, what is the angular frequency of the source? (b) What is the next value of t for which $\Delta V_R = 0.250\ \Delta V_{max}$?

7. An audio amplifier, represented by the AC source and resistor in Figure P33.7, delivers to the speaker alternating voltage at audio frequencies. If the source voltage has an amplitude of 15.0 V, R = 8.20 Ω, and the speaker is equivalent to a resistance of 10.4 Ω, what is the time-averaged power transferred to it?

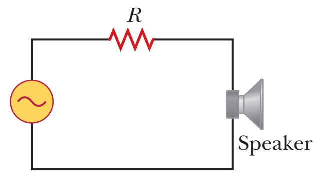


Figure P33.7

8. Figure P33.8 shows three lightbulbs connected to a 120-V AC (rms) household supply voltage. Bulbs 1 and 2 have a power rating of 150 W, and bulb 3 has a 100-W rating. Find (a) the rms current in each bulb and (b) the resistance of each bulb. (c) What is the total resistance of the combination of the three lightbulbs?

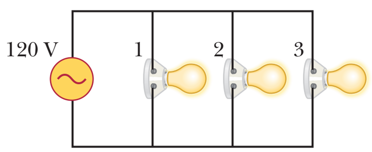


Figure P33.8

Section 33.3 Inductors in an AC Circuit

9. An inductor has a 54.0-Ω reactance when connected to a 60.0–Hz source. The inductor is removed and then connected to a 50.0-Hz source that produces a 100-V rms voltage. What is the maximum current in the inductor?

[10.] M In a purely inductive AC circuit as shown in Figure P33.10 (page 1024), ΔV_{max} = 100 V. (a) The maximum current is 7.50 A at 50.0 Hz. Calculate the inductance L. (b) **What If?** At what angular frequency ω is the maximum current 2.50 A?

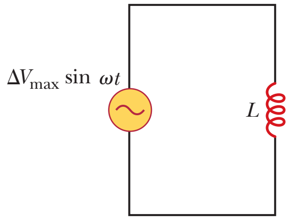


Figure P33.10 Problems 10 and 11.

11. For the circuit shown in Figure P33.10, $\Delta V_{max} = 80.0$ V, $\omega = 65.0\pi$ rad/s, and $L = 70.0$ mH. Calculate the current in the inductor at $t = 15.5$ ms.

12. An inductor is connected to an AC power supply having a maximum output voltage of 4.00 V at a frequency of 300 Hz. What inductance is needed to keep the rms current less than 2.00 mA?

13. An AC source has an output rms voltage of 78.0 V at a frequency of 80.0 Hz. If the source is connected across a 25.0-mH inductor, what are (a) the inductive reactance of the circuit, (b) the rms current in the circuit, and (c) the maximum current in the circuit?

14. A 20.0-mH inductor is connected to a North American electrical outlet ($\Delta V_{rms} = 120$ V, $f = 60.0$ Hz). Assuming the energy stored in the inductor is zero at $t = 0$, determine the energy stored at $t = \frac{1}{180}$ s.

15. Review. Determine the maximum magnetic flux through an inductor connected to a North American electrical outlet ($\Delta V_{rms} = 120$ V, $f = 60.0$ Hz).

16. The output voltage of an AC source is given by $\Delta v = 120 \sin 30.0\pi t$, where Δv is in volts and t is in seconds. The source is connected across a 0.500-H inductor. Find (a) the frequency of the source, (b) the rms voltage across the inductor, (c) the inductive reactance of the circuit, (d) the rms current in the inductor, and (e) the maximum current in the inductor.

Section 33.4 Capacitors in an AC Circuit

17. A 1.00-mF capacitor is connected to a North American electrical outlet ($\Delta V_{rms} = 120$ V, $f = 60.0$ Hz). Assuming the energy stored in the capacitor is zero at $t = 0$, determine the magnitude of the current in the wires at $t = \frac{1}{180}$ s.

18. An AC source with an output rms voltage of 36.0 V at a frequency of 60.0 Hz is connected across a 12.0-μF capacitor. Find (a) the capacitive reactance, (b) the rms current, and (c) the maximum current in the circuit. (d) Does the capacitor have its maximum charge when the current has its maximum value? Explain.

19. (a) For what frequencies does a 22.0-μF capacitor have a reactance below 175 Ω? (b) **What If?** What is the reactance of a 44.0-μF capacitor over this same frequency range?

20. A source delivers an AC voltage of the form $\Delta v = 98.0 \sin 80\pi t$, where Δv is in volts and t is in seconds, to a capacitor. The maximum current in the circuit is 0.500 A. Find (a) the rms voltage of the source, (b) the frequency of the source, and (c) the value of the capacitance.

21. What maximum current is delivered by an AC source with $\Delta V_{max} = 48.0$ V and $f = 90.0$ Hz when connected across a 3.70-μF capacitor?

22. A capacitor C is connected to a power supply that operates at a frequency f and produces an rms voltage ΔV. What is the maximum charge that appears on either capacitor plate?

23. What is the maximum current in a 2.20-μF capacitor when it is connected across (a) a North American electrical outlet having $\Delta V_{rms} = 120$ V and $f = 60.0$ Hz and (b) a European electrical outlet having $\Delta V_{rms} = 240$ V and $f = 50.0$ Hz?

Section 33.5 The *RLC* Series Circuit

24. An AC source with $\Delta V_{max} = 150$ V and $f = 50.0$ Hz is connected between points a and d in Figure P33.24. Calculate the maximum voltages between (a) points a and b, (b) points b and c, (c) points c and d, and (d) points b and d.

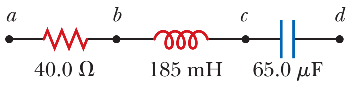


Figure P33.24 Problems 24 and 81.

25. In addition to phasor diagrams showing voltages such as in Figure 33.15, we can draw phasor diagrams with resistance and reactances. The resultant of adding the phasors is the impedance. Draw to scale a phasor diagram showing Z, X_L, X_C, and ϕ for an AC series circuit for which $R = 300\ \Omega$, $C = 11.0\ \mu$F, $L = 0.200$ H, and $f = 500/\pi$ Hz.

26. A sinusoidal voltage $\Delta v = 40.0 \sin 100t$, where Δv is in volts and t is in seconds, is applied to a series *RLC* circuit with $L = 160$ mH, $C = 99.0\ \mu$F, and $R = 68.0\ \Omega$. (a) What is the impedance of the circuit? (b) What is the maximum current? Determine the numerical values for (c) ω and (d) ϕ in the equation $i = I_{max} \sin(\omega t - \phi)$.

27. A series AC circuit contains a resistor, an inductor of 150 mH, a capacitor of 5.00 μF, and a source with $\Delta V_{max} = 240$ V operating at 50.0 Hz. The maximum current in the circuit is 100 mA. Calculate (a) the inductive reactance, (b) the capacitive reactance, (c) the impedance, (d) the resistance in the circuit, and (e) the phase angle between the current and the source voltage.

28. At what frequency does the inductive reactance of a 57.0-μH inductor equal the capacitive reactance of a 57.0-μF capacitor?

29. An *RLC* circuit consists of a 150-Ω resistor, a 21.0-μF capacitor, and a 460-mH inductor connected in series with a 120-V, 60.0-Hz power supply. (a) What is the phase angle between the current and the applied voltage? (b) Which reaches its maximum earlier, the current or the voltage?

30. Draw phasors to scale for the following voltages in SI units: (a) 25.0 sin ωt at $\omega t = 90.0°$, (b) 30.0 sin ωt at $\omega t = 60.0°$, and (c) 18.0 sin ωt at $\omega t = 300°$.

31. M An inductor (L = 400 mH), a capacitor (C = 4.43 μF), and a resistor (R = 500 Ω) are connected in series. A 50.0-Hz AC source produces a peak current of 250 mA in the circuit. (a) Calculate the required peak voltage ΔV_{max}. (b) Determine the phase angle by which the current leads or lags the applied voltage.

32. A 60.0-Ω resistor is connected in series with a 30.0-μF capacitor and a source whose maximum voltage is 120 V, operating at 60.0 Hz. Find (a) the capacitive reactance of the circuit, (b) the impedance of the circuit, and (c) the maximum current in the circuit. (d) Does the voltage lead or lag the current? (e) How will adding an inductor in series with the existing resistor and capacitor affect the current? Explain.

33. **Review.** In an *RLC* series circuit that includes a source of alternating current operating at fixed frequency and voltage, the resistance R is equal to the inductive reactance. If the plate separation of the parallel-plate capacitor is reduced to one-half its original value, the current in the circuit doubles. Find the initial capacitive reactance in terms of R.

Section 33.6 Power in an AC Circuit

34. *Why is the following situation impossible?* A series circuit consists of an ideal AC source (no inductance or capacitance in the source itself) with an rms voltage of ΔV at a frequency f and a magnetic buzzer with a resistance R and an inductance L. By carefully adjusting the inductance L of the circuit, a power factor of exactly 1.00 is attained.

35. W A series *RLC* circuit has a resistance of 45.0 Ω and an impedance of 75.0 Ω. What average power is delivered to this circuit when ΔV_{rms} = 210 V?

36. W An AC voltage of the form $\Delta v = 100 \sin 1\,000t$, where Δv is in volts and t is in seconds, is applied to a series *RLC* circuit. Assume the resistance is 400 Ω, the capacitance is 5.00 μF, and the inductance is 0.500 H. Find the average power delivered to the circuit.

37. A series *RLC* circuit has a resistance of 22.0 Ω and an impedance of 80.0 Ω. If the rms voltage applied to the circuit is 160 V, what average power is delivered to the circuit?

38. An AC voltage of the form $\Delta v = 90.0 \sin 350t$, where Δv is in volts and t is in seconds, is applied to a series *RLC* circuit. If R = 50.0 Ω, C = 25.0 μF, and L = 0.200 H, find (a) the impedance of the circuit, (b) the rms current in the circuit, and (c) the average power delivered to the circuit.

39. W In a certain series *RLC* circuit, I_{rms} = 9.00 A, ΔV_{rms} = 180 V, and the current leads the voltage by 37.0°. (a) What is the total resistance of the circuit? (b) Calculate the reactance of the circuit ($X_L - X_C$).

40. W Suppose you manage a factory that uses many electric motors. The motors create a large inductive load to the electric power line as well as a resistive load. The electric company builds an extra-heavy distribution line to supply you with two components of current: one that is 90° out of phase with the voltage and another that is in phase with the voltage. The electric company charges you an extra fee for "reactive volt-amps" in addition to the amount you pay for the energy you use. You can avoid the extra fee by installing a capacitor between the power line and your factory. The following problem models this solution.

In an *RL* circuit, a 120-V (rms), 60.0-Hz source is in series with a 25.0-mH inductor and a 20.0-Ω resistor. What are (a) the rms current and (b) the power factor? (c) What capacitor must be added in series to make the power factor equal to 1? (d) To what value can the supply voltage be reduced if the power supplied is to be the same as before the capacitor was installed?

41. A diode is a device that allows current to be carried in only one direction (the direction indicated by the arrowhead in its circuit symbol). Find the average power delivered to the diode circuit of Figure P33.41 in terms of ΔV_{rms} and R.

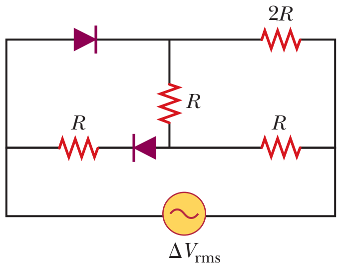


Figure P33.41

Section 33.7 Resonance in a Series *RLC* Circuit

42. W A series *RLC* circuit has components with the following values: L = 20.0 mH, C = 100 nF, R = 20.0 Ω, and ΔV_{max} = 100 V, with $\Delta v = \Delta V_{max} \sin \omega t$. Find (a) the resonant frequency of the circuit, (b) the amplitude of the current at the resonant frequency, (c) the Q of the circuit, and (d) the amplitude of the voltage across the inductor at resonance.

43. AMT M An *RLC* circuit is used in a radio to tune into an FM station broadcasting at f = 99.7 MHz. The resistance in the circuit is R = 12.0 Ω, and the inductance is L = 1.40 μH. What capacitance should be used?

44. The *LC* circuit of a radar transmitter oscillates at 9.00 GHz. (a) What inductance is required for the circuit to resonate at this frequency if its capacitance is 2.00 pF? (b) What is the inductive reactance of the circuit at this frequency?

45. A 10.0-Ω resistor, 10.0-mH inductor, and 100-μF capacitor are connected in series to a 50.0-V (rms) source having variable frequency. If the operating frequency is twice the resonance frequency, find the energy delivered to the circuit during one period.

46. A resistor R, inductor L, and capacitor C are connected in series to an AC source of rms voltage ΔV and variable frequency. If the operating frequency is twice the resonance frequency, find the energy delivered to the circuit during one period.

47. **Review.** A radar transmitter contains an LC circuit oscillating at 1.00×10^{10} Hz. (a) For a one-turn loop having an inductance of 400 pH to resonate at this frequency, what capacitance is required in series with the loop? (b) The capacitor has square, parallel plates separated by 1.00 mm of air. What should the edge length of the plates be? (c) What is the common reactance of the loop and capacitor at resonance?

Section 33.8 The Transformer and Power Transmission

48. W A step-down transformer is used for recharging the batteries of portable electronic devices. The turns ratio N_2/N_1 for a particular transformer used in a DVD player is 1:13. When used with 120-V (rms) household service, the transformer draws an rms current of 20.0 mA from the house outlet. Find (a) the rms output voltage of the transformer and (b) the power delivered to the DVD player.

49. M The primary coil of a transformer has $N_1 = 350$ turns, and the secondary coil has $N_2 = 2\ 000$ turns. If the input voltage across the primary coil is $\Delta v = 170 \cos \omega t$, where Δv is in volts and t is in seconds, what rms voltage is developed across the secondary coil?

50. AMT A transmission line that has a resistance per unit length of $4.50 \times 10^{-4}\ \Omega/\text{m}$ is to be used to transmit 5.00 MW across 400 mi (6.44×10^5 m). The output voltage of the source is 4.50 kV. (a) What is the line loss if a transformer is used to step up the voltage to 500 kV? (b) What fraction of the input power is lost to the line under these circumstances? (c) **What If?** What difficulties would be encountered in attempting to transmit the 5.00 MW at the source voltage of 4.50 kV?

51. In the transformer shown in Figure P33.51, the load resistance R_L is 50.0 Ω. The turns ratio N_1/N_2 is 2.50, and the rms source voltage is $\Delta V_s = 80.0$ V. If a voltmeter across the load resistance measures an rms voltage of 25.0 V, what is the source resistance R_s?

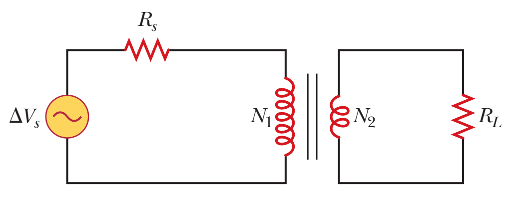


Figure P33.51

52. A person is working near the secondary of a transformer as shown in Figure P33.52. The primary voltage is 120 V at 60.0 Hz. The secondary voltage is 5 000 V. The capacitance C_s, which is the stray capacitance between the hand and the secondary winding, is 20.0 pF. Assuming the person has a body resistance to ground of $R_b = 50.0$ kΩ, determine the rms voltage across the body. *Suggestion:* Model the secondary of the transformer as an AC source.

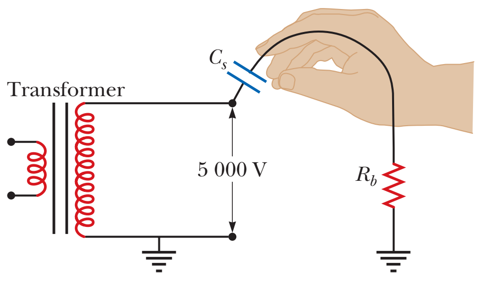


Figure P33.52

Section 33.9 Rectifiers and Filters

53. M The RC high-pass filter shown in Figure P33.53 has a resistance $R = 0.500\ \Omega$ and a capacitance $C = 613\ \mu$F. What is the ratio of the amplitude of the output voltage to that of the input voltage for this filter for a source frequency of 600 Hz?

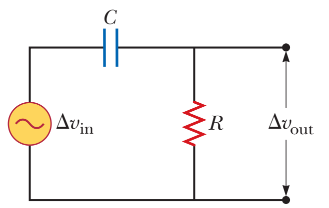


Figure P33.53
Problems 53 and 54.

54. Consider the RC high-pass filter circuit shown in Figure P33.53. (a) Find an expression for the ratio of the amplitude of the output voltage to that of the input voltage in terms of R, C, and the AC source frequency ω. (b) What value does this ratio approach as the frequency decreases toward zero? (c) What value does this ratio approach as the frequency increases without limit?

55. One particular plug-in power supply for a radio looks similar to the one shown in Figure 33.20 and is marked with the following information: Input 120 V AC 8 W Output 9 V DC 300 mA. Assume these values are accurate to two digits. (a) Find the energy efficiency of the device when the radio is operating. (b) At what rate is energy wasted in the device when the radio is operating? (c) Suppose the input power to the transformer is 8.00 W when the radio is switched off and energy costs \$0.110/kWh from the electric company. Find the cost of having six such transformers around the house, each plugged in for 31 days.

56. Consider the filter circuit shown in Figure P33.56. (a) Show that the ratio of the amplitude of the output voltage to that of the input voltage is

$$\frac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \frac{1/\omega C}{\sqrt{R^2 + \left(\dfrac{1}{\omega C}\right)^2}}$$

(b) What value does this ratio approach as the frequency decreases toward zero? (c) What value does this ratio approach as the frequency increases without limit? (d) At what frequency is the ratio equal to one-half?

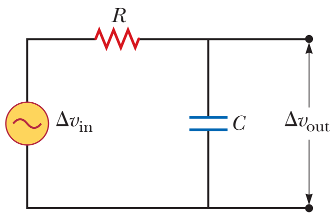


Figure P33.56

Additional Problems

57. W A step-up transformer is designed to have an output voltage of 2 200 V (rms) when the primary is connected across a 110-V (rms) source. (a) If the primary winding has exactly 80 turns, how many turns are required on the secondary? (b) If a load resistor across the secondary draws a current of 1.50 A, what is the current in the primary, assuming ideal conditions? (c) **What If?** If the transformer actually has an efficiency of 95.0%, what is the current in the primary when the secondary current is 1.20 A?

58. *Why is the following situation impossible?* An *RLC* circuit is used in a radio to tune into a North American AM commercial radio station. The values of the circuit components are $R = 15.0\ \Omega$, $L = 2.80\ \mu$H, and $C = 0.910$ pF.

59. **Review.** The voltage phasor diagram for a certain series *RLC* circuit is shown in Figure P33.59. The resistance of the circuit is 75.0 Ω, and the frequency is 60.0 Hz. Find (a) the maximum voltage ΔV_{max}, (b) the phase angle ϕ, (c) the maximum current, (d) the impedance, (e) the capacitance and (f) the inductance of the circuit, and (g) the average power delivered to the circuit.

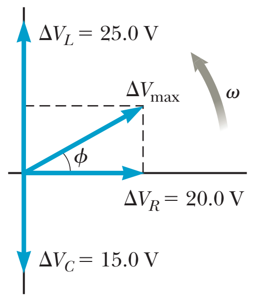


Figure P33.59

60. M Consider a series *RLC* circuit having the parameters $R = 200\ \Omega$, $L = 663$ mH, and $C = 26.5\ \mu$F. The applied voltage has an amplitude of 50.0 V and a frequency of 60.0 Hz. Find (a) the current I_{max} and its phase relative to the applied voltage Δv, (b) the maximum voltage ΔV_R across the resistor and its phase relative to the current, (c) the maximum voltage ΔV_C across the capacitor and its phase relative to the current, and (d) the maximum voltage ΔV_L across the inductor and its phase relative to the current.

61. Energy is to be transmitted over a pair of copper wires in a transmission line at the rate of 20.0 kW with only a 1.00% loss over a distance of 18.0 km at potential difference $\Delta V_{rms} = 1.50 \times 10^3$ V between the wires. Assuming the current density is uniform in the conductors, what is the diameter required for each of the two wires?

62. Energy is to be transmitted over a pair of copper wires in a transmission line at a rate P with only a fractional loss f over a distance ℓ at potential difference ΔV_{rms} between the wires. Assuming the current density is uniform in the conductors, what is the diameter required for each of the two wires?

63. A 400-Ω resistor, an inductor, and a capacitor are in series with an AC source. The reactance of the inductor is 700 Ω, and the circuit impedance is 760 Ω. (a) What are the possible values of the reactance of the capacitor? (b) If you find that the power delivered to the circuit decreases as you raise the frequency, what is the capacitive reactance in the original circuit? (c) Repeat part (a) assuming the resistance is 200 Ω instead of 400 Ω and the circuit impedance continues to be 760 Ω.

64. Show that the rms value for the sawtooth voltage shown in Figure P33.64 is $\Delta V_{max}/\sqrt{3}$.

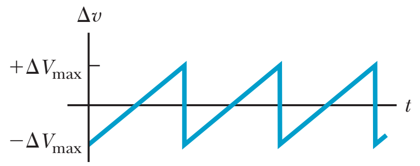


Figure P33.64

65. A transformer may be used to provide maximum power transfer between two AC circuits that have different impedances Z_1 and Z_2. This process is called *impedance matching*. (a) Show that the ratio of turns N_1/N_2 for this transformer is

$$\frac{N_1}{N_2} = \sqrt{\frac{Z_1}{Z_2}}$$

(b) Suppose you want to use a transformer as an impedance-matching device between an audio amplifier that has an output impedance of 8.00 kΩ and a speaker that has an input impedance of 8.00 Ω. What should your N_1/N_2 ratio be?

66. A capacitor, a coil, and two resistors of equal resistance are arranged in an AC circuit as shown in Figure P33.66 (page 1028). An AC source provides an emf of $\Delta V_{rms} = 20.0$ V at a frequency of 60.0 Hz. When the double-throw switch S is open as shown in the figure, the rms current is 183 mA. When the switch is closed in position *a*, the rms current is 298 mA. When the switch is closed in position *b*, the rms current is 137 mA. Determine the values of (a) R, (b) C, and (c) L. (d) Is more than one set of values possible? Explain.

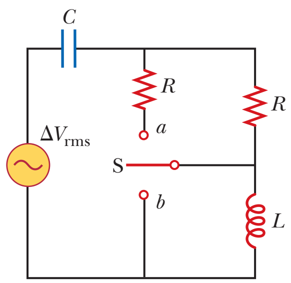


Figure P33.66

67. W Marie Cornu, a physicist at the Polytechnic Institute in Paris, invented phasors in about 1880. This problem helps you see their general utility in representing oscillations. Two mechanical vibrations are represented by the expressions

$$y_1 = 12.0 \sin 4.50t$$

and

$$y_2 = 12.0 \sin (4.50t + 70.0°)$$

where y_1 and y_2 are in centimeters and t is in seconds. Find the amplitude and phase constant of the sum of these functions (a) by using a trigonometric identity (as from Appendix B) and (b) by representing the oscillations as phasors. (c) State the result of comparing the answers to parts (a) and (b). (d) Phasors make it equally easy to add traveling waves. Find the amplitude and phase constant of the sum of the three waves represented by

$$y_1 = 12.0 \sin (15.0x - 4.50t + 70.0°)$$

$$y_2 = 15.5 \sin (15.0x - 4.50t - 80.0°)$$

$$y_3 = 17.0 \sin (15.0x - 4.50t + 160°)$$

where x, y_1, y_2, and y_3 are in centimeters and t is in seconds.

68. A series RLC circuit has resonance angular frequency 2.00×10^3 rad/s. When it is operating at some input frequency, $X_L = 12.0\ \Omega$ and $X_C = 8.00\ \Omega$. (a) Is this input frequency higher than, lower than, or the same as the resonance frequency? Explain how you can tell. (b) Explain whether it is possible to determine the values of both L and C. (c) If it is possible, find L and C. If it is not possible, give a compact expression for the condition that L and C must satisfy.

69. AMT **Review.** One insulated conductor from a household extension cord has a mass per length of 19.0 g/m. A section of this conductor is held under tension between two clamps. A subsection is located in a magnetic field of magnitude 15.3 mT directed perpendicular to the length of the cord. When the cord carries an AC current of 9.00 A at a frequency of 60.0 Hz, it vibrates in resonance in its simplest standing-wave vibration mode. (a) Determine the relationship that must be satisfied between the separation d of the clamps and the tension T in the cord. (b) Determine one possible combination of values for these variables.

70. (a) Sketch a graph of the phase angle for an RLC series circuit as a function of angular frequency from zero to a frequency much higher than the resonance frequency. (b) Identify the value of ϕ at the resonance angular frequency ω_0. (c) Prove that the slope of the graph of ϕ versus ω at the resonance point is $2Q/\omega_0$.

71. In Figure P33.71, find the rms current delivered by the 45.0-V (rms) power supply when (a) the frequency is very large and (b) the frequency is very small.

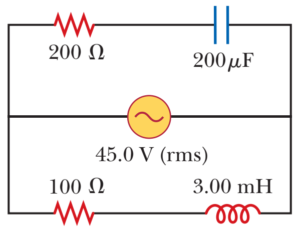


Figure P33.71

72. **Review.** In the circuit shown in Figure P33.72, assume all parameters except C are given. Find (a) the current in the circuit as a function of time and (b) the power delivered to the circuit. (c) Find the current as a function of time after *only* switch 1 is opened. (d) After switch 2 is *also* opened, the current and voltage are in phase. Find the capacitance C. Find (e) the impedance of the circuit when both switches are open, (f) the maximum energy stored in the capacitor during oscillations, and (g) the maximum energy stored in the inductor during oscillations. (h) Now the frequency of the voltage source is doubled. Find the phase difference between the current and the voltage. (i) Find the frequency that makes the inductive reactance one-half the capacitive reactance.

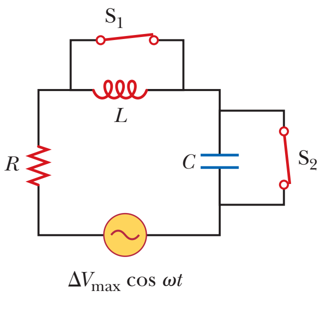


Figure P33.72

73. GP A series RLC circuit contains the following components: $R = 150\ \Omega$, $L = 0.250$ H, $C = 2.00\ \mu$F, and a source with $\Delta V_{max} = 210$ V operating at 50.0 Hz. Our goal is to find the phase angle, the power factor, and the power input for this circuit. (a) Find the inductive reactance in the circuit. (b) Find the capacitive reactance in the circuit. (c) Find the impedance in the circuit. (d) Calculate the maximum current in the circuit. (e) Determine the phase angle between the current and source voltage. (f) Find the power factor for the circuit. (g) Find the power input to the circuit.

74. A series *RLC* circuit is operating at 2.00×10^3 Hz. At this frequency, $X_L = X_C = 1\ 884\ \Omega$. The resistance of the circuit is 40.0 Ω. (a) Prepare a table showing the values of X_L, X_C, and Z for f = 300, 600, 800, 1.00×10^3, 1.50×10^3, 2.00×10^3, 3.00×10^3, 4.00×10^3, 6.00×10^3, and 1.00×10^4 Hz. (b) Plot on the same set of axes X_L, X_C, and Z as a function of ln f.

75. M A series *RLC* circuit consists of an 8.00-Ω resistor, a 5.00-μF capacitor, and a 50.0-mH inductor. A variable-frequency source applies an emf of 400 V (rms) across the combination. Assuming the frequency is equal to one-half the resonance frequency, determine the power delivered to the circuit.

76. A series *RLC* circuit in which $R = 1.00\ \Omega$, $L = 1.00$ mH, and $C = 1.00$ nF is connected to an AC source delivering 1.00 V (rms). (a) Make a precise graph of the power delivered to the circuit as a function of the frequency and (b) verify that the full width of the resonance peak at half-maximum is $R/2\pi L$.

Challenge Problems

77. The resistor in Figure P33.77 represents the midrange speaker in a three-speaker system. Assume its resistance to be constant at 8.00 Ω. The source represents an audio amplifier producing signals of uniform amplitude $\Delta V_{max} = 10.0$ V at all audio frequencies. The inductor and capacitor are to function as a band-pass filter with $\Delta V_{out}/\Delta V_{in} = \frac{1}{2}$ at 200 Hz and at 4.00×10^3 Hz. Determine the required values of (a) L and (b) C. Find (c) the maximum value of the ratio $\Delta V_{out}/\Delta V_{in}$; (d) the frequency f_0 at which the ratio has its maximum value; (e) the phase shift between Δv_{in} and Δv_{out} at 200 Hz, at f_0, and at 4.00×10^3 Hz; and (f) the average power transferred to the speaker at 200 Hz, at f_0, and at 4.00×10^3 Hz. (g) Treating the filter as a resonant circuit, find its quality factor.

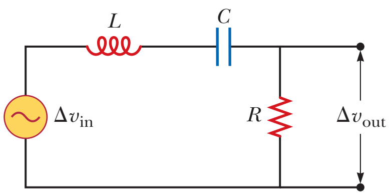


Figure P33.77

78. An 80.0-Ω resistor and a 200-mH inductor are connected in *parallel* across a 100-V (rms), 60.0-Hz source. (a) What is the rms current in the resistor? (b) By what angle does the total current lead or lag behind the voltage?

79. A voltage $\Delta v = 100 \sin \omega t$, where Δv is in volts and t is in seconds, is applied across a series combination of a 2.00-H inductor, a 10.0-μF capacitor, and a 10.0-Ω resistor. (a) Determine the angular frequency ω_0 at which the power delivered to the resistor is a maximum. (b) Calculate the average power delivered at that frequency. (c) Determine the two angular frequencies ω_1 and ω_2 at which the power is one-half the maximum value. *Note:* The Q of the circuit is $\omega_0/(\omega_2 - \omega_1)$.

80. Figure P33.80a shows a parallel *RLC* circuit. The instantaneous voltages (and rms voltages) across each of the three circuit elements are the same, and each is in phase with the current in the resistor. The currents in C and L lead or lag the current in the resistor as shown in the current phasor diagram, Figure P33.80b. (a) Show that the rms current delivered by the source is

$$I_{rms} = \Delta V_{rms}\left[\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2\right]^{1/2}$$

(b) Show that the phase angle ϕ between ΔV_{rms} and I_{rms} is given by

$$\tan \phi = R\left(\frac{1}{X_C} - \frac{1}{X_L}\right)$$

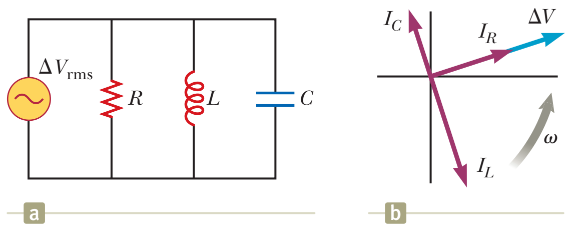


Figure P33.80

81. An AC source with $\Delta V_{rms} = 120$ V is connected between points a and d in Figure P33.24. At what frequency will it deliver a power of 250 W? Explain your answer.