CHAPTER 32

Inductance

A treasure hunter uses a metal detector to search for buried objects at a beach. At the end of the metal detector is a coil of wire that is part of a circuit. When the coil comes near a metal object, the inductance of the coil is affected and the current in the circuit changes. This change triggers a signal in the earphones worn by the treasure hunter. We investigate inductance in this chapter. *(Andy Ryan/Stone/Getty Images)*

In Chapter 31, we saw that an emf and a current are induced in a loop of wire when the magnetic flux through the area enclosed by the loop changes with time. This phenomenon of electromagnetic induction has some practical consequences. In this chapter, we first describe an effect known as *self-induction,* in which a time-varying current in a circuit produces an induced emf opposing the emf that initially set up the time-varying current. Self-induction is the basis of the *inductor,* an electrical circuit element. We discuss the energy stored in the magnetic field of an inductor and the energy density associated with the magnetic field.

Next, we study how an emf is induced in a coil as a result of a changing magnetic flux produced by a second coil, which is the basic principle of *mutual induction.* Finally, we examine the characteristics of circuits that contain inductors, resistors, and capacitors in various combinations.

32.1 Self-Induction and Inductance

In this chapter, we need to distinguish carefully between emfs and currents that are caused by physical sources such as batteries and those that are induced by changing magnetic fields. When we use a term (such as *emf* or *current*) without an adjective, we are describing the parameters associated with a physical source. We use the adjective *induced* to describe those emfs and currents caused by a changing magnetic field.

Consider a circuit consisting of a switch, a resistor, and a source of emf as shown in Figure 32.1. The circuit diagram is represented in perspective to show the orientations of some of the magnetic field lines due to the current in the circuit. When the switch is thrown to its closed position, the current does not immediately jump from zero to its maximum value $\mathcal{E}/R$. Faraday's law of electromagnetic induction (Eq. 31.1) can be used to describe this effect as follows. As the current increases with time, the magnetic field lines surrounding the wires pass through the loop represented by the circuit itself. This magnetic field passing through the loop causes a magnetic flux through the loop. This increasing flux creates an induced emf in the circuit. The direction of the induced emf is such that it would cause an induced current in the loop (if the loop did not already carry a current), which would establish a magnetic field opposing the change in the original magnetic field. Therefore, the direction of the induced emf is opposite the direction of the emf of the battery, which results in a gradual rather than instantaneous increase in the current to its final equilibrium value. Because of the direction of the induced emf, it is also called a *back emf,* similar to that in a motor as discussed in Chapter 31. This effect is called **self-induction** because the changing flux through the circuit and the resultant induced emf arise from the circuit itself. The emf $\mathcal{E}_L$ set up in this case is called a **self-induced emf.**

Brady-Handy Collection, Library of Congress Prints and Photographs Division [LC-BH83-997]

Joseph Henry

American Physicist (1797–1878)

Henry became the first director of the Smithsonian Institution and first president of the Academy of Natural Science. He improved the design of the electromagnet and constructed one of the first motors. He also discovered the phenomenon of self-induction, but he failed to publish his findings. The unit of inductance, the henry, is named in his honor.

To obtain a quantitative description of self-induction, recall from Faraday's law that the induced emf is equal to the negative of the time rate of change of the magnetic flux. The magnetic flux is proportional to the magnetic field, which in turn is proportional to the current in the circuit. Therefore, a self-induced emf is always proportional to the time rate of change of the current. For any loop of wire, we can write this proportionality as

$$\mathcal{E}_L = -L\frac{di}{dt} \quad \textbf{(32.1)}$$

where L is a proportionality constant—called the **inductance** of the loop—that depends on the geometry of the loop and other physical characteristics. If we consider a closely spaced coil of N turns (a toroid or an ideal solenoid) carrying a current i and containing N turns, Faraday's law tells us that $\mathcal{E}_L = -N\,d\Phi_B/dt$. Combining this expression with Equation 32.1 gives

$$L = \frac{N\Phi_B}{i} \quad \textbf{(32.2)}$$

◀ **Inductance of an *N*-turn coil**

where it is assumed the same magnetic flux passes through each turn and L is the inductance of the entire coil.

From Equation 32.1, we can also write the inductance as the ratio

$$L = -\frac{\mathcal{E}_L}{di/dt} \quad \textbf{(32.3)}$$

Recall that resistance is a measure of the opposition to current as given by Equation 27.7, $R = \Delta V/I$; in comparison, Equation 32.3, being of the same mathematical form as Equation 27.7, shows us that inductance is a measure of the opposition to a *change* in current.

The SI unit of inductance is the **henry** (H), which as we can see from Equation 32.3 is 1 volt-second per ampere: 1 H = 1 V · s/A.

As shown in Example 32.1, the inductance of a coil depends on its geometry. This dependence is analogous to the capacitance of a capacitor depending on the geometry of its plates as we found in Equation 26.3 and the resistance of a resistor depending on the length and area of the conducting material in Equation 27.10. Inductance calculations can be quite difficult to perform for complicated geometries, but the examples below involve simple situations for which inductances are easily evaluated.

After the switch is closed, the current produces a magnetic flux through the area enclosed by the loop. As the current increases toward its equilibrium value, this magnetic flux changes in time and induces an emf in the loop.

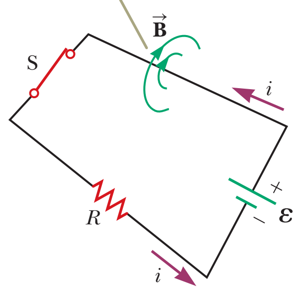


Figure 32.1 Self-induction in a simple circuit.

Quick Quiz 32.1 A coil with zero resistance has its ends labeled a and b. The potential at a is higher than at b. Which of the following could be consistent with this situation? **(a)** The current is constant and is directed from a to b. **(b)** The current is constant and is directed from b to a. **(c)** The current is increasing and is directed from a to b. **(d)** The current is decreasing and is directed from a to b. **(e)** The current is increasing and is directed from b to a. **(f)** The current is decreasing and is directed from b to a.

Example 32.1 Inductance of a Solenoid

Consider a uniformly wound solenoid having N turns and length ℓ. Assume ℓ is much longer than the radius of the windings and the core of the solenoid is air.

(A) Find the inductance of the solenoid.

SOLUTION

Conceptualize The magnetic field lines from each turn of the solenoid pass through all the turns, so an induced emf in each coil opposes changes in the current.

Categorize We categorize this example as a substitution problem. Because the solenoid is long, we can use the results for an ideal solenoid obtained in Chapter 30.

Find the magnetic flux through each turn of area A in the solenoid, using the expression for the magnetic field from Equation 30.17:

$$\Phi_B = BA = \mu_0 niA = \mu_0 \frac{N}{\ell} iA$$

Substitute this expression into Equation 32.2:

$$L = \frac{N\Phi_B}{i} = \mu_0 \frac{N^2}{\ell} A \qquad \textbf{(32.4)}$$

(B) Calculate the inductance of the solenoid if it contains 300 turns, its length is 25.0 cm, and its cross-sectional area is 4.00 cm^2.

SOLUTION

Substitute numerical values into Equation 32.4:

$$L = (4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})\frac{300^2}{25.0 \times 10^{-2}\ \text{m}}(4.00 \times 10^{-4}\ \text{m}^2)$$

$$= 1.81 \times 10^{-4}\ \text{T}\cdot\text{m}^2/\text{A} = 0.181\ \text{mH}$$

(C) Calculate the self-induced emf in the solenoid if the current it carries decreases at the rate of 50.0 A/s.

SOLUTION

Substitute $di/dt = -50.0$ A/s and the answer to part (B) into Equation 32.1:

$$\mathcal{E}_L = -L\frac{di}{dt} = -(1.81 \times 10^{-4}\ \text{H})(-50.0\ \text{A/s})$$

$$= 9.05\ \text{mV}$$

The result for part (A) shows that L depends on geometry and is proportional to the square of the number of turns. Because $N = n\ell$, we can also express the result in the form

$$L = \mu_0 \frac{(n\ell)^2}{\ell} A = \mu_0 n^2 A\ell = \mu_0 n^2 V \qquad \textbf{(32.5)}$$

where $V = A\ell$ is the interior volume of the solenoid.

32.2 *RL* Circuits

If a circuit contains a coil such as a solenoid, the inductance of the coil prevents the current in the circuit from increasing or decreasing instantaneously. A circuit element that has a large inductance is called an **inductor** and has the circuit symbol —ꝏꝏ—. We always assume the inductance of the remainder of a circuit is negligible compared with that of the inductor. Keep in mind, however, that even a circuit without a coil has some inductance that can affect the circuit's behavior.

Because the inductance of an inductor results in a back emf, an inductor in a circuit opposes changes in the current in that circuit. The inductor attempts to keep the current the same as it was before the change occurred. If the battery voltage in the circuit is increased so that the current rises, the inductor opposes this change and the rise is not instantaneous. If the battery voltage is decreased, the inductor causes a slow drop in the current rather than an immediate drop. Therefore, the inductor causes the circuit to be "sluggish" as it reacts to changes in the voltage.

Consider the circuit shown in Figure 32.2, which contains a battery of negligible internal resistance. This circuit is an ***RL* circuit** because the elements connected to the battery are a resistor and an inductor. The curved lines on switch S_2 suggest this switch can never be open; it is always set to either a or b. (If the switch is connected to neither a nor b, any current in the circuit suddenly stops.) Suppose S_2 is set to a and switch S_1 is open for $t < 0$ and then thrown closed at $t = 0$. The current in the circuit begins to increase, and a back emf (Eq. 32.1) that opposes the increasing current is induced in the inductor.

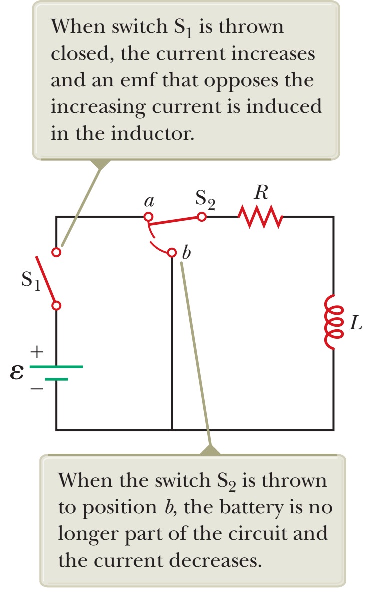


Figure 32.2 An *RL* circuit. When switch S_2 is in position a, the battery is in the circuit.

With this point in mind, let's apply Kirchhoff's loop rule to this circuit, traversing the circuit in the clockwise direction:

$$\mathcal{E} - iR - L\frac{di}{dt} = 0 \qquad \textbf{(32.6)}$$

where iR is the voltage drop across the resistor. (Kirchhoff's rules were developed for circuits with steady currents, but they can also be applied to a circuit in which the current is changing if we imagine them to represent the circuit at one *instant* of time.) Now let's find a solution to this differential equation, which is similar to that for the *RC* circuit (see Section 28.4).

A mathematical solution of Equation 32.6 represents the current in the circuit as a function of time. To find this solution, we change variables for convenience, letting $x = (\mathcal{E}/R) - i$, so $dx = -di$. With these substitutions, Equation 32.6 becomes

$$x + \frac{L}{R}\frac{dx}{dt} = 0$$

Rearranging and integrating this last expression gives

$$\int_{x_0}^{x}\frac{dx}{x} = -\frac{R}{L}\int_0^t dt$$

$$\ln\frac{x}{x_0} = -\frac{R}{L}t$$

where x_0 is the value of x at time $t = 0$. Taking the antilogarithm of this result gives

$$x = x_0 e^{-Rt/L}$$

Because $i = 0$ at $t = 0$, note from the definition of x that $x_0 = \mathcal{E}/R$. Hence, this last expression is equivalent to

$$\frac{\mathcal{E}}{R} - i = \frac{\mathcal{E}}{R}e^{-Rt/L}$$

$$i = \frac{\mathcal{E}}{R}(1 - e^{-Rt/L})$$

This expression shows how the inductor affects the current. The current does not increase instantly to its final equilibrium value when the switch is closed, but instead increases according to an exponential function. If the inductance is removed from the circuit, which corresponds to letting L approach zero, the exponential term

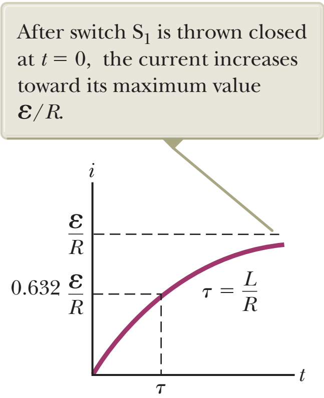


Figure 32.3 Plot of the current versus time for the *RL* circuit shown in Figure 32.2. The time constant τ is the time interval required for i to reach 63.2% of its maximum value.

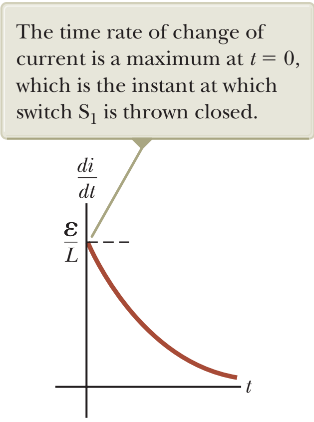


Figure 32.4 Plot of di/dt versus time for the *RL* circuit shown in Figure 32.2. The rate decreases exponentially with time as i increases toward its maximum value.

becomes zero and there is no time dependence of the current in this case; the current increases instantaneously to its final equilibrium value in the absence of the inductance.

We can also write this expression as

$$i = \frac{\mathcal{E}}{R}(1 - e^{-t/\tau}) \tag{32.7}$$

where the constant τ is the **time constant** of the *RL* circuit:

$$\tau = \frac{L}{R} \tag{32.8}$$

Physically, τ is the time interval required for the current in the circuit to reach $(1 - e^{-1}) = 0.632 = 63.2\%$ of its final value $\mathcal{E}/R$. The time constant is a useful parameter for comparing the time responses of various circuits.

Figure 32.3 shows a graph of the current versus time in the *RL* circuit. Notice that the equilibrium value of the current, which occurs as t approaches infinity, is $\mathcal{E}/R$. That can be seen by setting di/dt equal to zero in Equation 32.6 and solving for the current i. (At equilibrium, the change in the current is zero.) Therefore, the current initially increases very rapidly and then gradually approaches the equilibrium value $\mathcal{E}/R$ as t approaches infinity.

Let's also investigate the time rate of change of the current. Taking the first time derivative of Equation 32.7 gives

$$\frac{di}{dt} = \frac{\mathcal{E}}{L}e^{-t/\tau} \tag{32.9}$$

This result shows that the time rate of change of the current is a maximum (equal to $\mathcal{E}/L$) at $t = 0$ and falls off exponentially to zero as t approaches infinity (Fig. 32.4).

Now consider the *RL* circuit in Figure 32.2 again. Suppose switch S_2 has been set at position a long enough (and switch S_1 remains closed) to allow the current to reach its equilibrium value $\mathcal{E}/R$. In this situation, the circuit is described by the outer loop in Figure 32.2. If S_2 is thrown from a to b, the circuit is now described by only the right-hand loop in Figure 32.2. Therefore, the battery has been eliminated from the circuit. Setting $\mathcal{E} = 0$ in Equation 32.6 gives

$$iR + L\frac{di}{dt} = 0$$

It is left as a problem (Problem 22) to show that the solution of this differential equation is

$$i = \frac{\mathcal{E}}{R} e^{-t/\tau} = I_i e^{-t/\tau} \qquad \textbf{(32.10)}$$

where $\mathcal{E}$ is the emf of the battery and $I_i = \mathcal{E}/R$ is the initial current at the instant the switch is thrown to *b*.

If the circuit did not contain an inductor, the current would immediately decrease to zero when the battery is removed. When the inductor is present, it opposes the decrease in the current and causes the current to decrease exponentially. A graph of the current in the circuit versus time (Fig. 32.5) shows that the current is continuously decreasing with time.

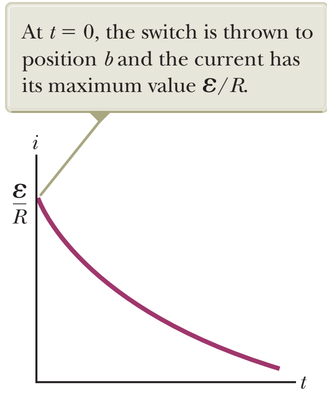


Figure 32.5 Current versus time for the right-hand loop of the circuit shown in Figure 32.2. For $t < 0$, switch S_2 is at position *a*.

Quick Quiz 32.2 Consider the circuit in Figure 32.2 with S_1 open and S_2 at position *a*. Switch S_1 is now thrown closed. **(i)** At the instant it is closed, across which circuit element is the voltage equal to the emf of the battery? (a) the resistor (b) the inductor (c) both the inductor and resistor **(ii)** After a very long time, across which circuit element is the voltage equal to the emf of the battery? Choose from among the same answers.

Example 32.2 Time Constant of an *RL* Circuit

Consider the circuit in Figure 32.2 again. Suppose the circuit elements have the following values: $\mathcal{E} = 12.0$ V, $R = 6.00\ \Omega$, and $L = 30.0$ mH.

(A) Find the time constant of the circuit.

SOLUTION

Conceptualize You should understand the operation and behavior of the circuit in Figure 32.2 from the discussion in this section.

Categorize We evaluate the results using equations developed in this section, so this example is a substitution problem.

Evaluate the time constant from Equation 32.8:

$$\tau = \frac{L}{R} = \frac{30.0 \times 10^{-3}\ \text{H}}{6.00\ \Omega} = \boxed{5.00\ \text{ms}}$$

(B) Switch S_2 is at position *a*, and switch S_1 is thrown closed at $t = 0$. Calculate the current in the circuit at $t = 2.00$ ms.

SOLUTION

Evaluate the current at $t = 2.00$ ms from Equation 32.7:

$$i = \frac{\mathcal{E}}{R}(1 - e^{-t/\tau}) = \frac{12.0\ \text{V}}{6.00\ \Omega}(1 - e^{-2.00\ \text{ms}/5.00\ \text{ms}}) = 2.00\ \text{A}\ (1 - e^{-0.400})$$

$$= \boxed{0.659\ \text{A}}$$

(C) Compare the potential difference across the resistor with that across the inductor.

SOLUTION

At the instant the switch is closed, there is no current and therefore no potential difference across the resistor. At this instant, the battery voltage appears entirely across the inductor in the form of a back emf of 12.0 V as the inductor tries to maintain the zero-current condition. (The top end of the inductor in Fig. 32.2 is at a higher electric potential than the bottom end.) As time passes, the emf across the inductor decreases and the current in the resistor (and hence the voltage across it) increases as shown in Figure 32.6 (page 976). The sum of the two voltages at all times is 12.0 V.

WHAT IF? In Figure 32.6, the voltages across the resistor and inductor are equal at 3.4 ms. What if you wanted to delay the condition in which the voltages are equal to some later instant, such as $t = 10.0$ ms? Which parameter, *L* or *R*, would require the least adjustment, in terms of a percentage change, to achieve that?

continued

Answer Figure 32.6 shows that the voltages are equal when the voltage across the inductor has fallen to half its original value. Therefore, the time interval required for the voltages to become equal is the *half-life* $_{1/2}$ of the decay. We introduced the half-life in the What If? section of Example 28.10 to describe the exponential decay in circuits, where $_{1/2}$ 0.693

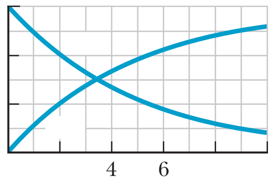


Figure 32.6 (Example 32.2) The time behavior of the voltages across the resistor and inductor in Figure 32.2 given the values provided in this example.

From the desired half-life of 10.0 ms, use the result from Example 28.10 to find the time constant of the circuit:

$$\tau = \frac{}{0.693} \quad \frac{10.0 \text{ ms}}{0.693} \quad 14.4 \text{ ms}$$

Hold fixed and find the value of that gives this time constant:

$$\tau = - \qquad - \quad \frac{30.0 \quad 10 \quad \text{H}}{14.4 \text{ ms}} \quad 2.08$$

Now hold fixed and find the appropriate value of

$$\tau = - \qquad = \tau \qquad 14.4 \text{ ms})(6.00 \qquad 86.4 \quad 10 \quad \text{H}$$

The change in corresponds to a 65% decrease compared with the initial resistance. The change in represents a 188% increase in inductance! Therefore, a much smaller percentage adjustment in can achieve the desired effect than would an adjustment in

Pitfall Prevention 32.1

Capacitors, Resistors, and Inductors Store Energy Differently Different energy-storage mechanisms are at work in capacitors, inductors, and resistors. A charged capacitor stores energy as electrical potential energy. An inductor stores energy as what we could call magnetic potential energy when it carries current. Energy delivered to a resistor is transformed to internal energy.

32.3 Energy in a Magnetic Field

A battery in a circuit containing an inductor must provide more energy than one in a circuit without the inductor. Consider Figure 32.2 with switch S in position When switch S is thrown closed, part of the energy supplied by the battery appears as internal energy in the resistance in the circuit, and the remaining energy is stored in the magnetic field of the inductor. Multiplying each term in Equation 32.6 by and rearranging the expression gives

$$Li\frac{di}{dt} \qquad \textbf{(32.11)}$$

Recognizing as the rate at which energy is supplied by the battery and as the rate at which energy is delivered to the resistor, we see that di) must represent the rate at which energy is being stored in the inductor. If is the energy stored in the inductor at any time, we can write the rate at which energy is stored as

$$\frac{dU}{dt} \qquad Li\frac{di}{dt}$$

To find the total energy stored in the inductor at any instant, let's rewrite this expression as $Li\,di$ and integrate:

$$dU \qquad Li\,di \qquad di$$

Energy stored in an inductor

$$-Li \qquad \textbf{(32.12)}$$

where is constant and has been removed from the integral. Equation 32.12 represents the energy stored in the magnetic field of the inductor when the current is It is similar in form to Equation 26.11 for the energy stored in the electric field of a capacitor, – . In either case, energy is required to establish a field.

We can also determine the energy density of a magnetic field. For simplicity, consider a solenoid whose inductance is given by Equation 32.5:

The magnetic field of a solenoid is given by Equation 30.17:

$$B = \mu_0 n i$$

Substituting the expression for L and $i = B/\mu_0 n$ into Equation 32.12 gives

$$U_B = \tfrac{1}{2}Li^2 = \tfrac{1}{2}\mu_0 n^2 V\left(\frac{B}{\mu_0 n}\right)^2 = \frac{B^2}{2\mu_0}V \quad \textbf{(32.13)}$$

The magnetic energy density, or the energy stored per unit volume in the magnetic field of the inductor, is $u_B = U_B/V$, or

$$u_B = \frac{B^2}{2\mu_0} \quad \textbf{(32.14)}$$

◀ **Magnetic energy density**

Although this expression was derived for the special case of a solenoid, it is valid for any region of space in which a magnetic field exists. Equation 32.14 is similar in form to Equation 26.13 for the energy per unit volume stored in an electric field, $u_E = \frac{1}{2}\epsilon_0 E^2$. In both cases, the energy density is proportional to the square of the field magnitude.

Quick Quiz 32.3 You are performing an experiment that requires the highest-possible magnetic energy density in the interior of a very long current-carrying solenoid. Which of the following adjustments increases the energy density? (More than one choice may be correct.) **(a)** increasing the number of turns per unit length on the solenoid **(b)** increasing the cross-sectional area of the solenoid **(c)** increasing only the length of the solenoid while keeping the number of turns per unit length fixed **(d)** increasing the current in the solenoid

Example 32.3 What Happens to the Energy in the Inductor? AM

Consider once again the *RL* circuit shown in Figure 32.2, with switch S_2 at position *a* and the current having reached its steady-state value. When S_2 is thrown to position *b*, the current in the right-hand loop decays exponentially with time according to the expression $i = I_i e^{-t/\tau}$, where $I_i = \mathcal{E}/R$ is the initial current in the circuit and $\tau = L/R$ is the time constant. Show that all the energy initially stored in the magnetic field of the inductor appears as internal energy in the resistor as the current decays to zero.

SOLUTION

Conceptualize Before S_2 is thrown to *b*, energy is being delivered at a constant rate to the resistor from the battery and energy is stored in the magnetic field of the inductor. After $t = 0$, when S_2 is thrown to *b*, the battery can no longer provide energy and energy is delivered to the resistor only from the inductor.

Categorize We model the right-hand loop of the circuit as an *isolated system* so that energy is transferred between components of the system but does not leave the system.

Analyze We begin by evaluating the energy delivered to the resistor, which appears as internal energy in the resistor.

Begin with Equation 27.22 and recognize that the rate of change of internal energy in the resistor is the power delivered to the resistor:

$$\frac{dE_{\text{int}}}{dt} = P = i^2R$$

Substitute the current given by Equation 32.10 into this equation:

$$\frac{dE_{\text{int}}}{dt} = i^2R = (I_i e^{-Rt/L})^2 R = I_i^2 R e^{-2Rt/L}$$

Solve for dE_{int} and integrate this expression over the limits $t = 0$ to $t \to \infty$:

$$E_{\text{int}} = \int_0^\infty I_i^2 R e^{-2Rt/L}\,dt = I_i^2 R \int_0^\infty e^{-2Rt/L}\,dt$$

The value of the definite integral can be shown to be $L/2R$ (see Problem 36). Use this result to evaluate E_{int}:

$$E_{\text{int}} = I_i^2 R\left(\frac{L}{2R}\right) = \tfrac{1}{2}LI_i^2$$

continued

32.3 continued

Finalize This result is equal to the initial energy stored in the magnetic field of the inductor, given by Equation 32.12, as we set out to prove.

Example 32.4 The Coaxial Cable

Coaxial cables are often used to connect electrical devices, such as your video system, and in receiving signals in television cable systems. Model a long coaxial cable as a thin, cylindrical conducting shell of radius b concentric with a solid cylinder of radius a as in Figure 32.7. The conductors carry the same current I in opposite directions. Calculate the inductance L of a length ℓ of this cable.

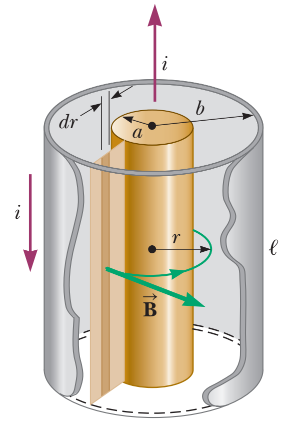


Figure 32.7 (Example 32.4) Section of a long coaxial cable. The inner and outer conductors carry equal currents in opposite directions.

SOLUTION

Conceptualize Consider Figure 32.7. Although we do not have a visible coil in this geometry, imagine a thin, radial slice of the coaxial cable such as the light gold rectangle in Figure 32.7. If the inner and outer conductors are connected at the ends of the cable (above and below the figure), this slice represents one large conducting loop. The current in the loop sets up a magnetic field between the inner and outer conductors that passes through this loop. If the current changes, the magnetic field changes and the induced emf opposes the original change in the current in the conductors.

Categorize We categorize this situation as one in which we must return to the fundamental definition of inductance, Equation 32.2.

Analyze We must find the magnetic flux through the light gold rectangle in Figure 32.7. Ampère's law (see Section 30.3) tells us that the magnetic field in the region between the conductors is due to the inner conductor alone and that its magnitude is $B = \mu_0 i/2\pi r$, where r is measured from the common center of the cylinders. A sample circular field line is shown in Figure 32.7, along with a field vector tangent to the field line. The magnetic field is zero outside the outer shell because the net current passing through the area enclosed by a circular path surrounding the cable is zero; hence, from Ampère's law, $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = 0$.

The magnetic field is perpendicular to the light gold rectangle of length ℓ and width $b - a$, the cross section of interest. Because the magnetic field varies with radial position across this rectangle, we must use calculus to find the total magnetic flux.

Divide the light gold rectangle into strips of width dr such as the darker strip in Figure 32.7. Evaluate the magnetic flux through such a strip:

$$d\Phi_B = B\,dA = B\ell\,dr$$

Substitute for the magnetic field and integrate over the entire light gold rectangle:

$$\Phi_B = \int_a^b \frac{\mu_0 i}{2\pi r}\ell\,dr = \frac{\mu_0 i\ell}{2\pi}\int_a^b \frac{dr}{r} = \frac{\mu_0 i\ell}{2\pi}\ln\left(\frac{b}{a}\right)$$

Use Equation 32.2 to find the inductance of the cable:

$$L = \frac{\Phi_B}{i} = \boxed{\frac{\mu_0\ell}{2\pi}\ln\left(\frac{b}{a}\right)}$$

Finalize The inductance depends only on geometric factors related to the cable. It increases if ℓ increases, if b increases, or if a decreases. This result is consistent with our conceptualization: any of these changes increases the size of the loop represented by our radial slice and through which the magnetic field passes, increasing the inductance.

32.4 Mutual Inductance

Very often, the magnetic flux through the area enclosed by a circuit varies with time because of time-varying currents in nearby circuits. This condition induces an emf through a process known as *mutual induction,* so named because it depends on the interaction of two circuits.

Consider the two closely wound coils of wire shown in cross-sectional view in Figure 32.8. The current i_1 in coil 1, which has N_1 turns, creates a magnetic field. Some of the magnetic field lines pass through coil 2, which has N_2 turns. The magnetic flux caused by the current in coil 1 and passing through coil 2 is represented by Φ_{12}. In analogy to Equation 32.2, we can identify the **mutual inductance** M_{12} of coil 2 with respect to coil 1:

$$M_{12} = \frac{N_2 \Phi_{12}}{i_1} \quad (32.15)$$

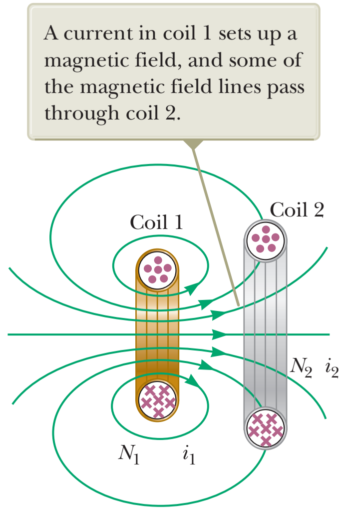


Figure 32.8 A cross-sectional view of two adjacent coils.

Mutual inductance depends on the geometry of both circuits and on their orientation with respect to each other. As the circuit separation distance increases, the mutual inductance decreases because the flux linking the circuits decreases.

If the current i_1 varies with time, we see from Faraday's law and Equation 32.15 that the emf induced by coil 1 in coil 2 is

$$\mathcal{E}_2 = -N_2 \frac{d\Phi_{12}}{dt} = -N_2 \frac{d}{dt}\left(\frac{M_{12} i_1}{N_2}\right) = -M_{12}\frac{di_1}{dt} \quad (32.16)$$

In the preceding discussion, it was assumed the current is in coil 1. Let's also imagine a current i_2 in coil 2. The preceding discussion can be repeated to show that there is a mutual inductance M_{21}. If the current i_2 varies with time, the emf induced by coil 2 in coil 1 is

$$\mathcal{E}_1 = -M_{21}\frac{di_2}{dt} \quad (32.17)$$

In mutual induction, the emf induced in one coil is always proportional to the rate at which the current in the other coil is changing. Although the proportionality constants M_{12} and M_{21} have been treated separately, it can be shown that they are equal. Therefore, with $M_{12} = M_{21} = M$, Equations 32.16 and 32.17 become

$$\mathcal{E}_2 = -M\frac{di_1}{dt} \quad \text{and} \quad \mathcal{E}_1 = -M\frac{di_2}{dt}$$

These two equations are similar in form to Equation 32.1 for the self-induced emf $\mathcal{E} = -L\,(di/dt)$. The unit of mutual inductance is the henry.

Quick Quiz 32.4 In Figure 32.8, coil 1 is moved closer to coil 2, with the orientation of both coils remaining fixed. Because of this movement, the mutual induction of the two coils **(a)** increases, **(b)** decreases, or **(c)** is unaffected.

Example 32.5 "Wireless" Battery Charger

An electric toothbrush has a base designed to hold the toothbrush handle when not in use. As shown in Figure 32.9a, the handle has a cylindrical hole that fits loosely over a matching cylinder on the base. When the handle is placed on the base, a changing current in a solenoid inside the base cylinder induces a current in a coil inside the handle. This induced current charges the battery in the handle.

We can model the base as a solenoid of length ℓ with N_B turns (Fig. 32.9b), carrying a current i, and having a cross-sectional area A. The handle coil contains N_H turns and completely surrounds the base coil. Find the mutual inductance of the system.

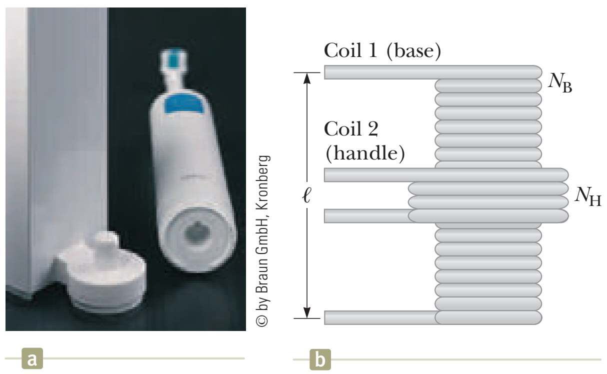


Figure 32.9 (Example 32.5) (a) This electric toothbrush uses the mutual induction of solenoids as part of its battery-charging system. (b) A coil of N_H turns wrapped around the center of a solenoid of N_B turns.

continued

32.5 continued

SOLUTION

Conceptualize Be sure you can identify the two coils in the situation and understand that a changing current in one coil induces a current in the second coil.

Categorize We will determine the result using concepts discussed in this section, so we categorize this example as a substitution problem.

Use Equation 30.17 to express the magnetic field in the interior of the base solenoid:

$$B = \mu_0 \frac{N_B}{\ell} i$$

Find the mutual inductance, noting that the magnetic flux Φ_{BH} through the handle's coil caused by the magnetic field of the base coil is BA:

$$M = \frac{N_H \Phi_{BH}}{i} = \frac{N_H BA}{i} = \mu_0 \frac{N_B N_H}{\ell} A$$

Wireless charging is used in a number of other "cordless" devices. One significant example is the inductive charging used by some manufacturers of electric cars that avoids direct metal-to-metal contact between the car and the charging apparatus.

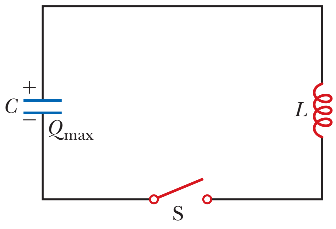


Figure 32.10 A simple LC circuit. The capacitor has an initial charge Q_{max}, and the switch is open for $t < 0$ and then closed at $t = 0$.

32.5 Oscillations in an *LC* Circuit

When a capacitor is connected to an inductor as illustrated in Figure 32.10, the combination is an ***LC* circuit.** If the capacitor is initially charged and the switch is then closed, both the current in the circuit and the charge on the capacitor oscillate between maximum positive and negative values. If the resistance of the circuit is zero, no energy is transformed to internal energy. In the following analysis, the resistance in the circuit is neglected. We also assume an idealized situation in which energy is not radiated away from the circuit. This radiation mechanism is discussed in Chapter 34.

Assume the capacitor has an initial charge Q_{max} (the maximum charge) and the switch is open for $t < 0$ and then closed at $t = 0$. Let's investigate what happens from an energy viewpoint.

When the capacitor is fully charged, the energy U in the circuit is stored in the capacitor's electric field and is equal to $Q_{max}^2/2C$ (Eq. 26.11). At this time, the current in the circuit is zero; therefore, no energy is stored in the inductor. After the switch is closed, the rate at which charges leave or enter the capacitor plates (which is also the rate at which the charge on the capacitor changes) is equal to the current in the circuit. After the switch is closed and the capacitor begins to discharge, the energy stored in its electric field decreases. The capacitor's discharge represents a current in the circuit, and some energy is now stored in the magnetic field of the inductor. Therefore, energy is transferred from the electric field of the capacitor to the magnetic field of the inductor. When the capacitor is fully discharged, it stores no energy. At this time, the current reaches its maximum value and all the energy in the circuit is stored in the inductor. The current continues in the same direction, decreasing in magnitude, with the capacitor eventually becoming fully charged again but with the polarity of its plates now opposite the initial polarity. This process is followed by another discharge until the circuit returns to its original state of maximum charge Q_{max} and the plate polarity shown in Figure 32.10. The energy continues to oscillate between inductor and capacitor.

The oscillations of the LC circuit are an electromagnetic analog to the mechanical oscillations of the particle in simple harmonic motion studied in Chapter 15. Much of what was discussed there is applicable to LC oscillations. For example, we investigated the effect of driving a mechanical oscillator with an external force,

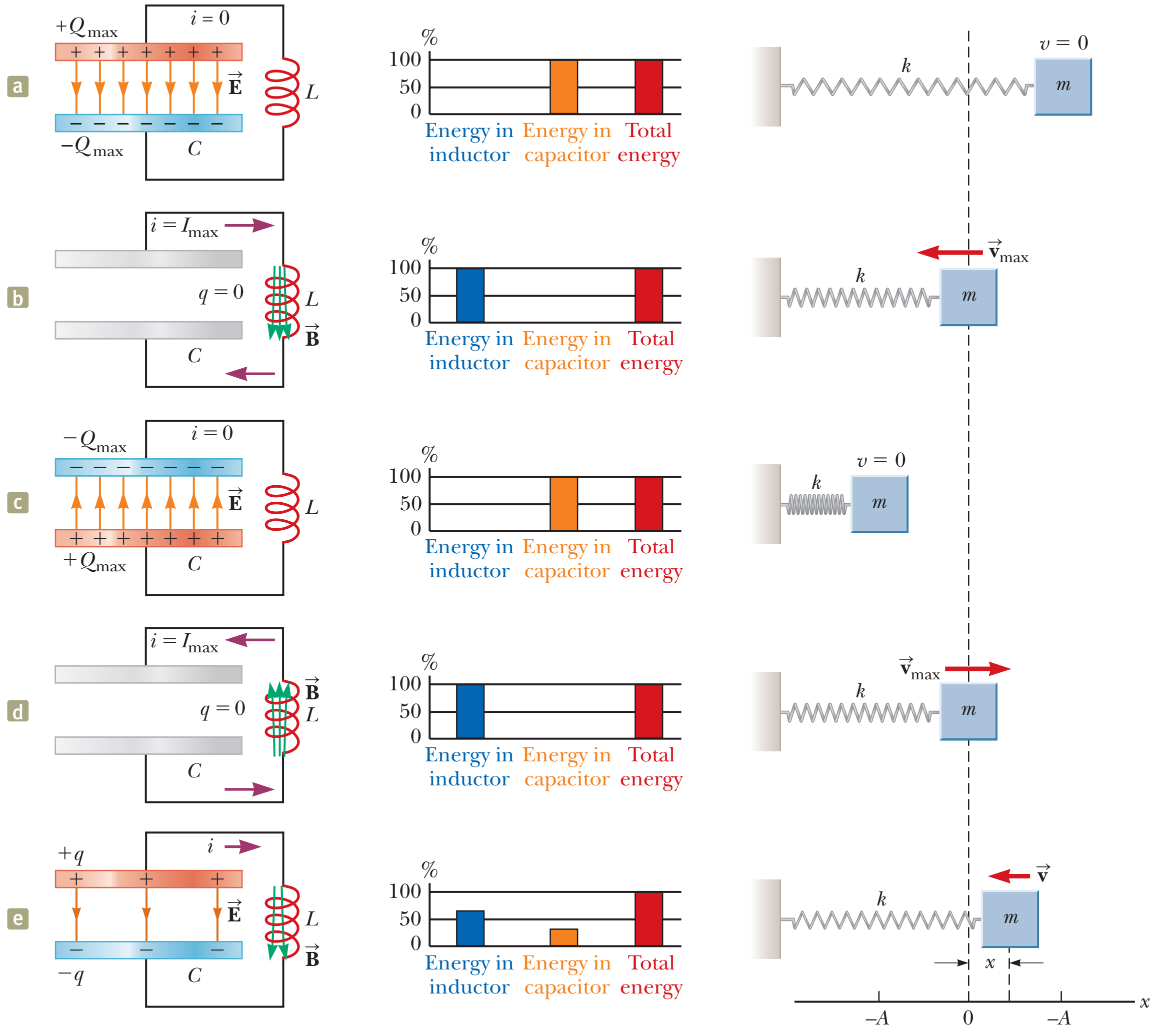


Figure 32.11 Energy transfer in a resistanceless, nonradiating *LC* circuit. The capacitor has a charge Q_{max} at $t = 0$, the instant at which the switch in Figure 32.10 is closed. The mechanical analog of this circuit is the particle in simple harmonic motion, represented by the block–spring system at the right of the figure. (a)–(d) At these special instants, all of the energy in the circuit resides in one of the circuit elements. (e) At an arbitrary instant, the energy is split between the capacitor and the inductor.

which leads to the phenomenon of *resonance.* The same phenomenon is observed in the *LC* circuit. (See Section 33.7.)

A representation of the energy transfer in an *LC* circuit is shown in Figure 32.11. As mentioned, the behavior of the circuit is analogous to that of the particle in simple harmonic motion studied in Chapter 15. For example, consider the block–spring system shown in Figure 15.10. The oscillations of this system are shown at the right of Figure 32.11. The potential energy $\frac{1}{2}kx^2$ stored in the stretched spring is analogous to the potential energy $Q_{max}^2/2C$ stored in the capacitor in Figure 32.11. The kinetic energy $\frac{1}{2}mv^2$ of the moving block is analogous to the magnetic energy $\frac{1}{2}Li^2$ stored in the inductor, which requires the presence of moving charges. In Figure 32.11a, all the energy is stored as electric potential energy in the capacitor at $t = 0$ (because $i = 0$), just as all the energy in a block–spring system is initially stored as potential energy in the spring if it is stretched and released at $t = 0$. In Figure 32.11b, all the energy is stored as magnetic energy $\frac{1}{2}LI_{max}^2$ in the inductor, where I_{max} is the maximum current. Figures 32.11c and 32.11d show subsequent quarter-cycle situations in which the energy is all electric or all magnetic. At intermediate points, part of the energy is electric and part is magnetic.

Consider some arbitrary time t after the switch is closed so that the capacitor has a charge $q < Q_{max}$ and the current is $i < I_{max}$. At this time, both circuit elements store energy, as shown in Figure 32.11e, but the sum of the two energies must equal the total initial energy U stored in the fully charged capacitor at $t = 0$:

Total energy stored in an *LC* circuit ▶

$$U = U_E + U_B = \frac{q^2}{2C} + \tfrac{1}{2}Li^2 = \frac{Q_{max}^2}{2C} \qquad \textbf{(32.18)}$$

Because we have assumed the circuit resistance to be zero and we ignore electromagnetic radiation, no energy is transformed to internal energy and none is transferred out of the system of the circuit. Therefore, with these assumptions, the system of the circuit is isolated: *the total energy of the system must remain constant in time.* We describe the constant energy of the system mathematically by setting $dU/dt = 0$. Therefore, by differentiating Equation 32.18 with respect to time while noting that q and i vary with time gives

$$\frac{dU}{dt} = \frac{d}{dt}\left(\frac{q^2}{2C} + \tfrac{1}{2}Li^2\right) = \frac{q}{C}\frac{dq}{dt} + Li\frac{di}{dt} = 0 \qquad \textbf{(32.19)}$$

We can reduce this result to a differential equation in one variable by remembering that the current in the circuit is equal to the rate at which the charge on the capacitor changes: $i = dq/dt$. It then follows that $di/dt = d^2q/dt^2$. Substitution of these relationships into Equation 32.19 gives

$$\frac{q}{C} + L\frac{d^2q}{dt^2} = 0$$

$$\frac{d^2q}{dt^2} = -\frac{1}{LC}q \qquad \textbf{(32.20)}$$

Let's solve for q by noting that this expression is of the same form as the analogous Equations 15.3 and 15.5 for a particle in simple harmonic motion:

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x = -\omega^2 x$$

where k is the spring constant, m is the mass of the block, and $\omega = \sqrt{k/m}$. The solution of this mechanical equation has the general form (Eq. 15.6):

$$x = A\cos(\omega t + \phi)$$

where A is the amplitude of the simple harmonic motion (the maximum value of x), ω is the angular frequency of this motion, and ϕ is the phase constant; the values of A and ϕ depend on the initial conditions. Because Equation 32.20 is of the same mathematical form as the differential equation of the simple harmonic oscillator, it has the solution

Charge as a function of time for an ideal *LC* circuit ▶

$$q = Q_{max}\cos(\omega t + \phi) \qquad \textbf{(32.21)}$$

where Q_{max} is the maximum charge of the capacitor and the angular frequency ω is

Angular frequency of oscillation in an *LC* circuit ▶

$$\omega = \frac{1}{\sqrt{LC}} \qquad \textbf{(32.22)}$$

Note that the angular frequency of the oscillations depends solely on the inductance and capacitance of the circuit. Equation 32.22 gives the *natural frequency* of oscillation of the LC circuit.

Because q varies sinusoidally with time, the current in the circuit also varies sinusoidally. We can show that by differentiating Equation 32.21 with respect to time:

Current as a function of time for an ideal *LC* current ▶

$$i = \frac{dq}{dt} = -\omega Q_{max}\sin(\omega t + \phi) \qquad \textbf{(32.23)}$$

To determine the value of the phase angle ϕ, let's examine the initial conditions, which in our situation require that at $t = 0$, $i = 0$, and $q = Q_{\max}$. Setting $i = 0$ at $t = 0$ in Equation 32.23 gives

$$0 = -\omega Q_{\max} \sin \phi$$

which shows that $\phi = 0$. This value for ϕ also is consistent with Equation 32.21 and the condition that $q = Q_{\max}$ at $t = 0$. Therefore, in our case, the expressions for q and i are

$$q = Q_{\max} \cos \omega t \tag{32.24}$$

$$i = -\omega Q_{\max} \sin \omega t = -I_{\max} \sin \omega t \tag{32.25}$$

Graphs of q versus t and i versus t are shown in Figure 32.12. The charge on the capacitor oscillates between the extreme values $Q_{\max}$ and $-Q_{\max}$, and the current oscillates between $I_{\max}$ and $-I_{\max}$. Furthermore, the current is 90° out of phase with the charge. That is, when the charge is a maximum, the current is zero, and when the charge is zero, the current has its maximum value.

Let's return to the energy discussion of the LC circuit. Substituting Equations 32.24 and 32.25 in Equation 32.18, we find that the total energy is

$$U = U_E + U_B = \frac{Q_{\max}^2}{2C} \cos^2 \omega t + \tfrac{1}{2} L I_{\max}^2 \sin^2 \omega t \tag{32.26}$$

This expression contains all the features described qualitatively at the beginning of this section. It shows that the energy of the LC circuit continuously oscillates between energy stored in the capacitor's electric field and energy stored in the inductor's magnetic field. When the energy stored in the capacitor has its maximum value $Q_{\max}^2/2C$, the energy stored in the inductor is zero. When the energy stored in the inductor has its maximum value $\frac{1}{2} L I_{\max}^2$, the energy stored in the capacitor is zero.

Plots of the time variations of U_E and U_B are shown in Figure 32.13. The sum $U_E + U_B$ is a constant and is equal to the total energy $Q_{\max}^2/2C$, or $\frac{1}{2} L I_{\max}^2$. Analytical verification is straightforward. The amplitudes of the two graphs in Figure 32.13 must be equal because the maximum energy stored in the capacitor (when $I = 0$) must equal the maximum energy stored in the inductor (when $q = 0$). This equality is expressed mathematically as

$$\frac{Q_{\max}^2}{2C} = \frac{L I_{\max}^2}{2}$$

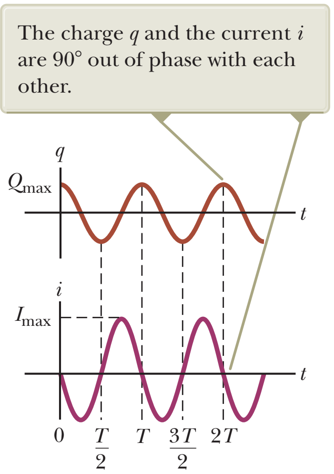


Figure 32.12 Graphs of charge versus time and current versus time for a resistanceless, nonradiating LC circuit.

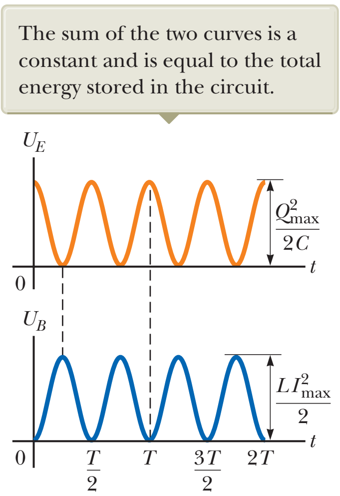


Figure 32.13 Plots of U_E versus t and U_B versus t for a resistanceless, nonradiating LC circuit.

Using this expression in Equation 32.26 for the total energy gives

$$U = \frac{Q_{max}^2}{2C}(\cos^2 \omega t + \sin^2 \omega t) = \frac{Q_{max}^2}{2C} \quad \textbf{(32.27)}$$

because $\cos^2 \omega t + \sin^2 \omega t = 1$.

In our idealized situation, the oscillations in the circuit persist indefinitely; the total energy U of the circuit, however, remains constant only if energy transfers and transformations are neglected. In actual circuits, there is always some resistance and some energy is therefore transformed to internal energy. We mentioned at the beginning of this section that we are also ignoring radiation from the circuit. In reality, radiation is inevitable in this type of circuit, and the total energy in the circuit continuously decreases as a result of this process.

Quick Quiz 32.5 **(i)** At an instant of time during the oscillations of an LC circuit, the current is at its maximum value. At this instant, what happens to the voltage across the capacitor? (a) It is different from that across the inductor. (b) It is zero. (c) It has its maximum value. (d) It is impossible to determine. **(ii)** Now consider an instant when the current is momentarily zero. From the same choices, describe the magnitude of the voltage across the capacitor at this instant.

Example 32.6 Oscillations in an *LC* Circuit

In Figure 32.14, the battery has an emf of 12.0 V, the inductance is 2.81 mH, and the capacitance is 9.00 pF. The switch has been set to position *a* for a long time so that the capacitor is charged. The switch is then thrown to position *b*, removing the battery from the circuit and connecting the capacitor directly across the inductor.

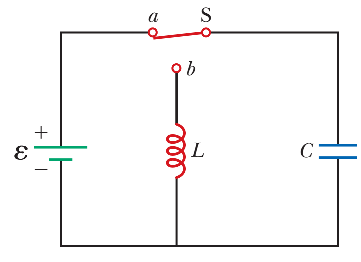


Figure 32.14 (Example 32.6) First the capacitor is fully charged with the switch set to position *a*. Then the switch is thrown to position *b*, and the battery is no longer in the circuit.

(A) Find the frequency of oscillation of the circuit.

SOLUTION

Conceptualize When the switch is thrown to position *b*, the active part of the circuit is the right-hand loop, which is an LC circuit.

Categorize We use equations developed in this section, so we categorize this example as a substitution problem.

Use Equation 32.22 to find the frequency:

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$

Substitute numerical values:

$$f = \frac{1}{2\pi[(2.81 \times 10^{-3}\text{ H})(9.00 \times 10^{-12}\text{ F})]^{1/2}} = \boxed{1.00 \times 10^6 \text{ Hz}}$$

(B) What are the maximum values of charge on the capacitor and current in the circuit?

SOLUTION

Find the initial charge on the capacitor, which equals the maximum charge:

$$Q_{max} = C\,\Delta V = (9.00 \times 10^{-12}\text{ F})(12.0\text{ V}) = \boxed{1.08 \times 10^{-10}\text{ C}}$$

Use Equation 32.25 to find the maximum current from the maximum charge:

$$I_{max} = \omega Q_{max} = 2\pi f Q_{max} = (2\pi \times 10^6\text{ s}^{-1})(1.08 \times 10^{-10}\text{ C})$$

$$= \boxed{6.79 \times 10^{-4}\text{ A}}$$

32.6 The *RLC* Circuit

Let's now turn our attention to a more realistic circuit consisting of a resistor, an inductor, and a capacitor connected in series as shown in Figure 32.15. We assume the resistance of the resistor represents all the resistance in the circuit. Suppose the switch is at position *a* so that the capacitor has an initial charge Q_{max}. The switch is now thrown to position *b*. At this instant, the total energy stored in the capacitor and inductor is $Q_{max}^2/2C$. This total energy, however, is no longer constant as it was in the *LC* circuit because the resistor causes transformation to internal energy. (We continue to ignore electromagnetic radiation from the circuit in this discussion.) Because the rate of energy transformation to internal energy within a resistor is i^2R,

$$\frac{dU}{dt} = -i^2R$$

where the negative sign signifies that the energy U of the circuit is decreasing in time. Substituting $U = U_E + U_B$ gives

$$\frac{q}{C}\frac{dq}{dt} + Li\frac{di}{dt} = -i^2R \tag{32.28}$$

To convert this equation into a form that allows us to compare the electrical oscillations with their mechanical analog, we first use $i = dq/dt$ and move all terms to the left-hand side to obtain

$$Li\frac{d^2q}{dt^2} + i^2R + \frac{q}{C}i = 0$$

Now divide through by i:

$$L\frac{d^2q}{dt^2} + iR + \frac{q}{C} = 0$$

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = 0 \tag{32.29}$$

The *RLC* circuit is analogous to the damped harmonic oscillator discussed in Section 15.6 and illustrated in Figure 15.20. The equation of motion for a damped block–spring system is, from Equation 15.31,

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0 \tag{32.30}$$

Comparing Equations 32.29 and 32.30, we see that q corresponds to the position x of the block at any instant, L to the mass m of the block, R to the damping coefficient b, and C to $1/k$, where k is the force constant of the spring. These and other relationships are listed in Table 32.1 on page 986.

Because the analytical solution of Equation 32.29 is cumbersome, we give only a qualitative description of the circuit behavior. In the simplest case, when $R = 0$, Equation 32.29 reduces to that of a simple *LC* circuit as expected, and the charge and the current oscillate sinusoidally in time. This situation is equivalent to removing all damping in the mechanical oscillator.

When R is small, a situation that is analogous to light damping in the mechanical oscillator, the solution of Equation 32.29 is

$$q = Q_{max}e^{-Rt/2L}\cos\omega_d t \tag{32.31}$$

where ω_d, the angular frequency at which the circuit oscillates, is given by

$$\omega_d = \left[\frac{1}{LC} - \left(\frac{R}{2L}\right)^2\right]^{1/2} \tag{32.32}$$

That is, the value of the charge on the capacitor undergoes a damped harmonic oscillation in analogy with a block–spring system moving in a viscous medium. Equation 32.32 shows that when $R << \sqrt{4L/C}$ (so that the second term in the

The switch is set first to position *a*, and the capacitor is charged. The switch is then thrown to position *b*.

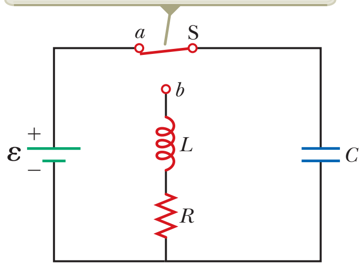


Figure 32.15 A series *RLC* circuit.

Table 32.1 **Analogies Between the *RLC* Circuit and the Particle in Simple Harmonic Motion**

***RLC* Circuit**		**One-Dimensional Particle in Simple Harmonic Motion**
Charge	$q \leftrightarrow x$	Position
Current	$i \leftrightarrow v_x$	Velocity
Potential difference	$\Delta V \leftrightarrow F_x$	Force
Resistance	$R \leftrightarrow b$	Viscous damping coefficient
Capacitance	$C \leftrightarrow 1/k$	(k = spring constant)
Inductance	$L \leftrightarrow m$	Mass
Current = time derivative of charge	$i = \frac{dq}{dt} \leftrightarrow v_x = \frac{dx}{dt}$	Velocity = time derivative of position
Rate of change of current = second time derivative of charge	$\frac{di}{dt} = \frac{d^2q}{dt^2} \leftrightarrow a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2}$	Acceleration = second time derivative of position
Energy in inductor	$U_B = \frac{1}{2}Li^2 \leftrightarrow K = \frac{1}{2}mv^2$	Kinetic energy of moving object
Energy in capacitor	$U_E = \frac{1}{2}\frac{q^2}{C} \leftrightarrow U = \frac{1}{2}kx^2$	Potential energy stored in a spring
Rate of energy loss due to resistance	$i^2R \leftrightarrow bv^2$	Rate of energy loss due to friction
RLC circuit	$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = 0 \leftrightarrow m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$	Damped object on a spring

brackets is much smaller than the first), the frequency ω_d of the damped oscillator is close to that of the undamped oscillator, $1/\sqrt{LC}$. Because $i = dq/dt$, it follows that the current also undergoes damped harmonic oscillation. A plot of the charge versus time for the damped oscillator is shown in Figure 32.16a, and an oscilloscope trace for a real *RLC* circuit is shown in Figure 32.16b. The maximum value of q decreases after each oscillation, just as the amplitude of a damped block–spring system decreases in time.

For larger values of R, the oscillations damp out more rapidly; in fact, there exists a critical resistance value $R_c = \sqrt{4L/C}$ above which no oscillations occur. A system with $R = R_c$ is said to be *critically damped*. When R exceeds R_c, the system is said to be *overdamped*.

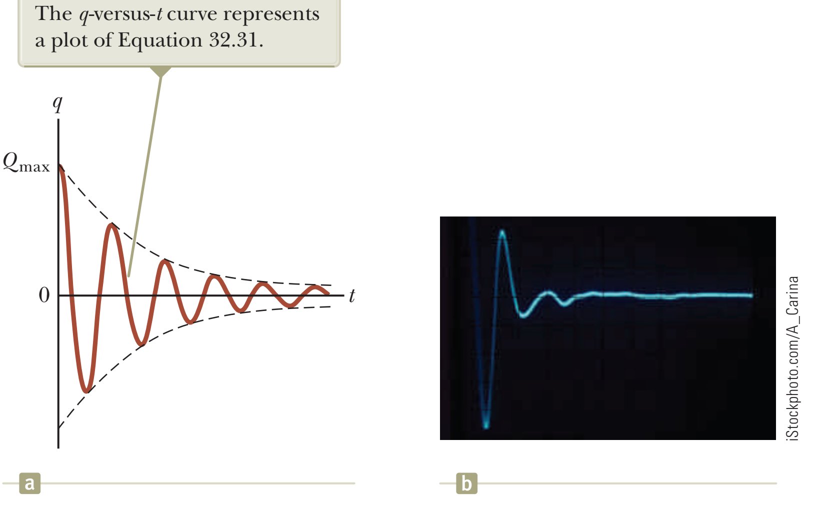


Figure 32.16 (a) Charge versus time for a damped *RLC* circuit. The charge decays in this way when $R < \sqrt{4L/C}$. (b) Oscilloscope pattern showing the decay in the oscillations of an *RLC* circuit.

Summary

Concepts and Principles

When the current in a loop of wire changes with time, an emf is induced in the loop according to Faraday's law. The **self-induced emf** is

$$\mathcal{E}_L = -L\frac{di}{dt} \qquad \textbf{(32.1)}$$

where L is the **inductance** of the loop. Inductance is a measure of how much opposition a loop offers to a change in the current in the loop. Inductance has the SI unit of **henry** (H), where 1 H = 1 V · s/A.

The inductance of any coil is

$$L = \frac{N\Phi_B}{i} \qquad \textbf{(32.2)}$$

where N is the total number of turns and Φ_B is the magnetic flux through the coil. The inductance of a device depends on its geometry. For example, the inductance of an air-core solenoid is

$$L = \mu_0 \frac{N^2}{\ell} A \qquad \textbf{(32.4)}$$

where ℓ is the length of the solenoid and A is the cross-sectional area.

If a resistor and inductor are connected in series to a battery of emf $\mathcal{E}$ at time $t = 0$, the current in the circuit varies in time according to the expression

$$i = \frac{\mathcal{E}}{R}(1 - e^{-t/\tau}) \qquad \textbf{(32.7)}$$

where $\tau = L/R$ is the **time constant** of the RL circuit. If we replace the battery in the circuit by a resistanceless wire, the current decays exponentially with time according to the expression

$$i = \frac{\mathcal{E}}{R}e^{-t/\tau} \qquad \textbf{(32.10)}$$

where $\mathcal{E}/R$ is the initial current in the circuit.

The energy stored in the magnetic field of an inductor carrying a current i is

$$U_B = \tfrac{1}{2}Li^2 \qquad \textbf{(32.12)}$$

This energy is the magnetic counterpart to the energy stored in the electric field of a charged capacitor.

The energy density at a point where the magnetic field is B is

$$u_B = \frac{B^2}{2\mu_0} \qquad \textbf{(32.14)}$$

The **mutual inductance** of a system of two coils is

$$M_{12} = \frac{N_2\Phi_{12}}{i_1} = M_{21} = \frac{N_1\Phi_{21}}{i_2} = M \qquad \textbf{(32.15)}$$

This mutual inductance allows us to relate the induced emf in a coil to the changing source current in a nearby coil using the relationships

$$\mathcal{E}_2 = -M_{12}\frac{di_1}{dt} \quad \text{and} \quad \mathcal{E}_1 = -M_{21}\frac{di_2}{dt} \qquad \textbf{(32.16, 32.17)}$$

In an LC circuit that has zero resistance and does not radiate electromagnetically (an idealization), the values of the charge on the capacitor and the current in the circuit vary sinusoidally in time at an angular frequency given by

$$\omega = \frac{1}{\sqrt{LC}} \qquad \textbf{(32.22)}$$

The energy in an LC circuit continuously transfers between energy stored in the capacitor and energy stored in the inductor.

In an RLC circuit with small resistance, the charge on the capacitor varies with time according to

$$q = Q_{\max}e^{-Rt/2L}\cos\omega_d t \qquad \textbf{(32.31)}$$

where

$$\omega_d = \left[\frac{1}{LC} - \left(\frac{R}{2L}\right)^2\right]^{1/2} \qquad \textbf{(32.32)}$$

Objective Questions

1. denotes answer available in *Student Solutions Manual/Study Guide*

1. The centers of two circular loops are separated by a fixed distance. **(i)** For what relative orientation of the loops is their mutual inductance a maximum? (a) coaxial and lying in parallel planes (b) lying in the same plane (c) lying in perpendicular planes, with the center of one on the axis of the other (d) The orientation makes no difference. **(ii)** For what relative orientation is their mutual inductance a minimum? Choose from the same possibilities as in part (i).

2. A long, fine wire is wound into a coil with inductance 5 mH. The coil is connected across the terminals of a battery, and the current is measured a few seconds after the connection is made. The wire is unwound and wound again into a different coil with $L = 10$ mH. This second coil is connected across the same battery, and the current is measured in the same way. Compared with the current in the first coil, is the current in the second coil (a) four times as large, (b) twice as large, (c) unchanged, (d) half as large, or (e) one-fourth as large?

3. A solenoidal inductor for a printed circuit board is being redesigned. To save weight, the number of turns is reduced by one-half, with the geometric dimensions kept the same. By how much must the current change if the energy stored in the inductor is to remain the same? (a) It must be four times larger. (b) It must be two times larger. (c) It should be left the same. (d) It should be one-half as large. (e) No change in the current can compensate for the reduction in the number of turns.

4. In Figure OQ32.4, the switch is left in position *a* for a long time interval and is then quickly thrown to position *b*. Rank the magnitudes of the voltages across the four circuit elements a short time thereafter from the largest to the smallest.

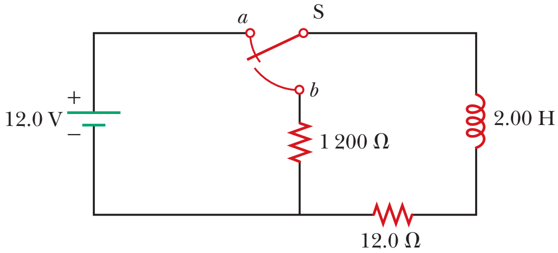


Figure OQ32.4

5. Two solenoids, A and B, are wound using equal lengths of the same kind of wire. The length of the axis of each solenoid is large compared with its diameter. The axial length of A is twice as large as that of B, and A has twice as many turns as B. What is the ratio of the inductance of solenoid A to that of solenoid B? (a) 4 (b) 2 (c) 1 (d) $\frac{1}{2}$ (e) $\frac{1}{4}$

6. If the current in an inductor is doubled, by what factor is the stored energy multiplied? (a) 4 (b) 2 (c) 1 (d) $\frac{1}{2}$ (e) $\frac{1}{4}$

7. Initially, an inductor with no resistance carries a constant current. Then the current is brought to a new constant value twice as large. *After* this change, when the current is constant at its higher value, what has happened to the emf in the inductor? (a) It is larger than before the change by a factor of 4. (b) It is larger by a factor of 2. (c) It has the same nonzero value. (d) It continues to be zero. (e) It has decreased.

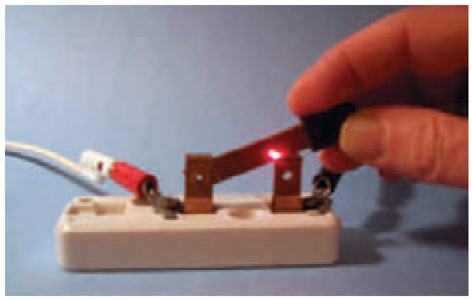
Conceptual Questions

1. denotes answer available in *Student Solutions Manual/Study Guide*

1. Consider this thesis: "Joseph Henry, America's first professional physicist, caused a basic change in the human view of the Universe when he discovered self-induction during a school vacation at the Albany Academy about 1830. Before that time, one could think of the Universe as composed of only one thing: matter. The energy that temporarily maintains the current after a battery is removed from a coil, on the other hand, is not energy that belongs to any chunk of matter. It is energy in the massless magnetic field surrounding the coil. With Henry's discovery, Nature forced us to admit that the Universe consists of fields as well as matter." (a) Argue for or against the statement. (b) In your view, what makes up the Universe?

2. (a) What parameters affect the inductance of a coil? (b) Does the inductance of a coil depend on the current in the coil?

3. A switch controls the current in a circuit that has a large inductance. The electric arc at the switch (Fig. CQ32.3) can melt and oxidize the contact surfaces, resulting in high resistivity of the contacts and eventual destruction of the switch. Is a spark more likely to be produced at the switch when the switch is being closed, when it is being opened, or does it not matter?



Alexandra Héder

Figure CQ32.3

4. Consider the four circuits shown in Figure CQ32.4, each consisting of a battery, a switch, a lightbulb, a

resistor, and either a capacitor or an inductor. Assume the capacitor has a large capacitance and the inductor has a large inductance but no resistance. The lightbulb has high efficiency, glowing whenever it carries electric current. **(i)** Describe what the lightbulb does in each of circuits (a) through (d) after the switch is thrown closed. **(ii)** Describe what the lightbulb does in each of circuits (a) through (d) when, having been closed for a long time interval, the switch is opened.

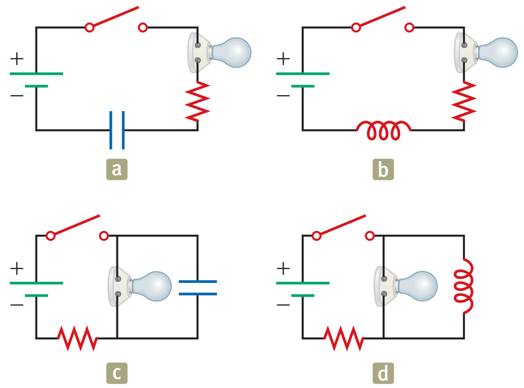


Figure CQ32.4

5. The current in a circuit containing a coil, a resistor, and a battery has reached a constant value. (a) Does the coil have an inductance? (b) Does the coil affect the value of the current?

6. (a) Can an object exert a force on itself? (b) When a coil induces an emf in itself, does it exert a force on itself?

7. The open switch in Figure CQ32.7 is thrown closed at $t = 0$. Before the switch is closed, the capacitor is uncharged and all currents are zero. Determine the currents in L, C, and R, the emf across L, and the potential differences across C and R (a) at the instant after the switch is closed and (b) long after it is closed.

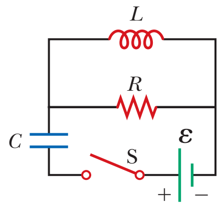


Figure CQ32.7

8. After the switch is closed in the LC circuit shown in Figure CQ32.8, the charge on the capacitor is sometimes zero, but at such instants the current in the circuit is not zero. How is this behavior possible?

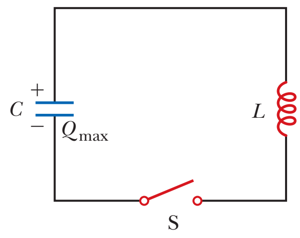


Figure CQ32.8 Conceptual Question 8 and Problems 52, 54, and 55.

9. How can you tell whether an RLC circuit is overdamped or underdamped?

10. Discuss the similarities between the energy stored in the electric field of a charged capacitor and the energy stored in the magnetic field of a current-carrying coil.

Problems

ENHANCED WebAssign The problems found in this chapter may be assigned online in Enhanced WebAssign

1. straightforward; **2.** intermediate; **3.** challenging

1. full solution available in the *Student Solutions Manual/Study Guide*

AMT Analysis Model tutorial available in Enhanced WebAssign

GP Guided Problem

M Master It tutorial available in Enhanced WebAssign

W Watch It video solution available in Enhanced WebAssign

Section 32.1 Self-Induction and Inductance

1. A coil has an inductance of 3.00 mH, and the current in it changes from 0.200 A to 1.50 A in a time interval of 0.200 s. Find the magnitude of the average induced emf in the coil during this time interval.

2. A coiled telephone cord forms a spiral with 70.0 turns, a diameter of 1.30 cm, and an unstretched length of 60.0 cm. Determine the inductance of one conductor in the unstretched cord.

3. A 2.00-H inductor carries a steady current of 0.500 A. When the switch in the circuit is opened, the current is effectively zero after 10.0 ms. What is the average induced emf in the inductor during this time interval?

4. M A solenoid of radius 2.50 cm has 400 turns and a length of 20.0 cm. Find (a) its inductance and (b) the rate at which current must change through it to produce an emf of 75.0 mV.

5. An emf of 24.0 mV is induced in a 500-turn coil when the current is changing at the rate of 10.0 A/s. What is the magnetic flux through each turn of the coil at an instant when the current is 4.00 A?

6. A 40.0-mA current is carried by a uniformly wound air-core solenoid with 450 turns, a 15.0-mm diameter, and 12.0-cm length. Compute (a) the magnetic field inside the solenoid, (b) the magnetic flux through each turn, and (c) the inductance of the solenoid. (d) **What If?** If the current were different, which of these quantities would change?

7. The current in a coil changes from 3.50 A to 2.00 A in the same direction in 0.500 s. If the average emf induced in the coil is 12.0 mV, what is the inductance of the coil?

8. A technician wraps wire around a tube of length 36.0 cm having a diameter of 8.00 cm. When the windings are evenly spread over the full length of the tube, the result is a solenoid containing 580 turns of wire. (a) Find the inductance of this solenoid. (b) If the current in this solenoid increases at the rate of 4.00 A/s, find the self-induced emf in the solenoid.

9. W The current in a 90.0-mH inductor changes with time as $i = 1.00t^2 - 6.00t$, where i is in amperes and t is in seconds. Find the magnitude of the induced emf at (a) $t = 1.00$ s and (b) $t = 4.00$ s. (c) At what time is the emf zero?

10. M An inductor in the form of a solenoid contains 420 turns and is 16.0 cm in length. A uniform rate of decrease of current through the inductor of 0.421 A/s induces an emf of 175 μV. What is the radius of the solenoid?

11. A self-induced emf in a solenoid of inductance L changes in time as $\mathcal{E} = \mathcal{E}_0 e^{-kt}$. Assuming the charge is finite, find the total charge that passes a point in the wire of the solenoid.

12. A toroid has a major radius R and a minor radius r and is tightly wound with N turns of wire on a hollow cardboard torus. Figure P32.12 shows half of this toroid, allowing us to see its cross section. If $R >> r$, the magnetic field in the region enclosed by the wire is essentially the same as the magnetic field of a solenoid that has been bent into a large circle of radius R. Modeling the field as the uniform field of a long solenoid, show that the inductance of such a toroid is approximately

$$L \approx \tfrac{1}{2}\mu_0 N^2 \frac{r^2}{R}$$

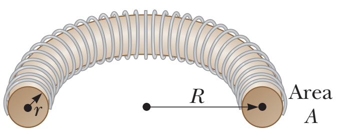


Figure P32.12

13. M A 10.0-mH inductor carries a current $i = I_{max} \sin \omega t$, with $I_{max} = 5.00$ A and $f = \omega/2\pi = 60.0$ Hz. What is the self-induced emf as a function of time?

14. The current in a 4.00 mH-inductor varies in time as shown in Figure P32.14. Construct a graph of the self-induced emf across the inductor over the time interval $t = 0$ to $t = 12.0$ ms.

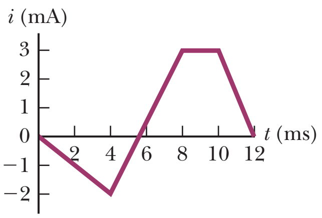


Figure P32.14

Section 32.2 *RL* Circuits

15. A 510-turn solenoid has a radius of 8.00 mm and an overall length of 14.0 cm. (a) What is its inductance? (b) If the solenoid is connected in series with a 2.50-Ω resistor and a battery, what is the time constant of the circuit?

16. M A 12.0-V battery is connected into a series circuit containing a 10.0-Ω resistor and a 2.00-H inductor. In what time interval will the current reach (a) 50.0% and (b) 90.0% of its final value?

17. A series RL circuit with $L = 3.00$ H and a series RC circuit with $C = 3.00\ \mu$F have equal time constants. If the two circuits contain the same resistance R, (a) what is the value of R? (b) What is the time constant?

18. In the circuit diagrammed in Figure P32.18, take $\mathcal{E} = 12.0$ V and $R = 24.0\ \Omega$. Assume the switch is open for $t < 0$ and is closed at $t = 0$. On a single set of axes, sketch graphs of the current in the circuit as a function of time for $t \geq 0$, assuming (a) the inductance in the circuit is essentially zero, (b) the inductance has an intermediate value, and (c) the inductance has a very large value. Label the initial and final values of the current.

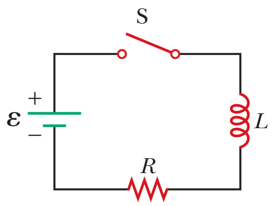


Figure P32.18
Problems 18, 20, 23, 24, and 27.

19. Consider the circuit shown in Figure P32.19. (a) When the switch is in position a, for what value of R will the circuit have a time constant of 15.0 μs? (b) What is the current in the inductor at the instant the switch is thrown to position b?

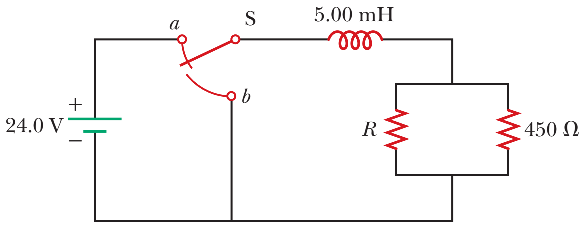


Figure P32.19

20. When the switch in Figure P32.18 is closed, the current takes 3.00 ms to reach 98.0% of its final value. If $R = 10.0\ \Omega$, what is the inductance?

21. A circuit consists of a coil, a switch, and a battery, all in series. The internal resistance of the battery is negligible compared with that of the coil. The switch is originally open. It is thrown closed, and after a time interval Δt, the current in the circuit reaches 80.0%

of its final value. The switch then remains closed for a time interval much longer than Δt. The wires connected to the terminals of the battery are then short-circuited with another wire and removed from the battery, so that the current is uninterrupted. (a) At an instant that is a time interval Δt after the short circuit, the current is what percentage of its maximum value? (b) At the moment $2\Delta t$ after the coil is short-circuited, the current in the coil is what percentage of its maximum value?

22. Show that $i = I_i e^{-t/\tau}$ is a solution of the differential equation

$$iR + L\frac{di}{dt} = 0$$

where I_i is the current at $t = 0$ and $\tau = L/R$.

23. W In the circuit shown in Figure P32.18, let $L = 7.00$ H, $R = 9.00\ \Omega$, and $\boldsymbol{\mathcal{E}} = 120$ V. What is the self-induced emf 0.200 s after the switch is closed?

24. Consider the circuit in Figure P32.18, taking $\boldsymbol{\mathcal{E}} = 6.00$ V, $L = 8.00$ mH, and $R = 4.00\ \Omega$. (a) What is the inductive time constant of the circuit? (b) Calculate the current in the circuit 250 μs after the switch is closed. (c) What is the value of the final steady-state current? (d) After what time interval does the current reach 80.0% of its maximum value?

25. The switch in Figure P32.25 is open for $t < 0$ and is then thrown closed at time $t = 0$. Assume $R = 4.00\ \Omega$, $L = 1.00$ H, and $\boldsymbol{\mathcal{E}} = 10.0$ V. Find (a) the current in the inductor and (b) the current in the switch as functions of time thereafter.

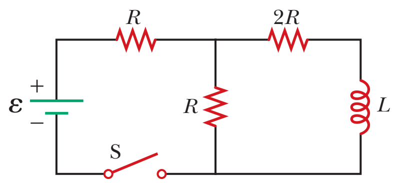


Figure P32.25 Problems 25, 26, and 64.

26. The switch in Figure P32.25 is open for $t < 0$ and is then thrown closed at time $t = 0$. Find (a) the current in the inductor and (b) the current in the switch as functions of time thereafter.

27. For the *RL* circuit shown in Figure P32.18, let the inductance be 3.00 H, the resistance 8.00 Ω, and the battery emf 36.0 V. (a) Calculate $\Delta V_R/\boldsymbol{\mathcal{E}}_L$, that is, the ratio of the potential difference across the resistor to the emf across the inductor when the current is 2.00 A. (b) Calculate the emf across the inductor when the current is 4.50 A.

28. Consider the current pulse $i(t)$ shown in Figure P32.28a. The current begins at zero, becomes 10.0 A between $t = 0$ and $t = 200\ \mu$s, and then is zero once again. This pulse is applied to the input of the partial circuit shown in Figure P32.28b. Determine the current in the inductor as a function of time.

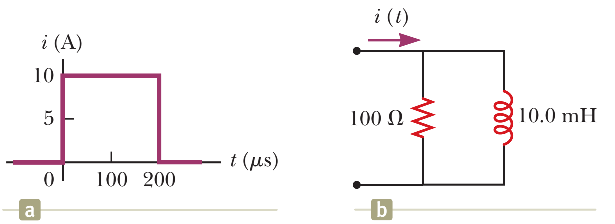


Figure P32.28

29. An inductor that has an inductance of 15.0 H and a resistance of 30.0 Ω is connected across a 100-V battery. What is the rate of increase of the current (a) at $t = 0$ and (b) at $t = 1.50$ s?

30. Two ideal inductors, L_1 and L_2, have *zero* internal resistance and are far apart, so their magnetic fields do not influence each other. (a) Assuming these inductors are connected in series, show that they are equivalent to a single ideal inductor having $L_{eq} = L_1 + L_2$. (b) Assuming these same two inductors are connected in parallel, show that they are equivalent to a single ideal inductor having $1/L_{eq} = 1/L_1 + 1/L_2$. (c) **What If?** Now consider two inductors L_1 and L_2 that have *nonzero* internal resistances R_1 and R_2, respectively. Assume they are still far apart, so their mutual inductance is zero, and assume they are connected in series. Show that they are equivalent to a single inductor having $L_{eq} = L_1 + L_2$ and $R_{eq} = R_1 + R_2$. (d) If these same inductors are now connected in parallel, is it necessarily true that they are equivalent to a single ideal inductor having $1/L_{eq} = 1/L_1 + 1/L_2$ and $1/R_{eq} = 1/R_1 + 1/R_2$? Explain your answer.

31. M A 140-mH inductor and a 4.90-Ω resistor are connected with a switch to a 6.00-V battery as shown in Figure P32.31. (a) After the switch is first thrown to *a* (connecting the battery), what time interval elapses before the current reaches 220 mA? (b) What is the current in the inductor 10.0 s after the switch is closed? (c) Now the switch is quickly thrown from *a* to *b*. What time interval elapses before the current in the inductor falls to 160 mA?

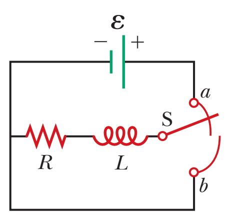


Figure P32.31

Section 32.3 Energy in a Magnetic Field

32. Calculate the energy associated with the magnetic field of a 200-turn solenoid in which a current of 1.75 A produces a magnetic flux of $3.70 \times 10^{-4}\ \text{T} \cdot \text{m}^2$ in each turn.

33. M An air-core solenoid with 68 turns is 8.00 cm long and has a diameter of 1.20 cm. When the solenoid carries a current of 0.770 A, how much energy is stored in its magnetic field?

34. W A 10.0-V battery, a 5.00-Ω resistor, and a 10.0-H inductor are connected in series. After the current in the circuit has reached its maximum value, calculate (a) the power being supplied by the battery, (b) the power being delivered to the resistor, (c) the power being delivered to the inductor, and (d) the energy stored in the magnetic field of the inductor.

35. On a clear day at a certain location, a 100-V/m vertical electric field exists near the Earth's surface. At the same place, the Earth's magnetic field has a magnitude of 0.500×10^{-4} T. Compute the energy densities of (a) the electric field and (b) the magnetic field.

36. Complete the calculation in Example 32.3 by proving that

$$\int_0^\infty e^{-2Rt/L}\, dt = \frac{L}{2R}$$

37. M A 24.0-V battery is connected in series with a resistor and an inductor, with $R = 8.00\ \Omega$ and $L = 4.00$ H, respectively. Find the energy stored in the inductor (a) when the current reaches its maximum value and (b) at an instant that is a time interval of one time constant after the switch is closed.

38. A flat coil of wire has an inductance of 40.0 mH and a resistance of 5.00 Ω. It is connected to a 22.0-V battery at the instant $t = 0$. Consider the moment when the current is 3.00 A. (a) At what rate is energy being delivered by the battery? (b) What is the power being delivered to the resistance of the coil? (c) At what rate is energy being stored in the magnetic field of the coil? (d) What is the relationship among these three power values? (e) Is the relationship described in part (d) true at other instants as well? (f) Explain the relationship at the moment immediately after $t = 0$ and at a moment several seconds later.

39. The magnetic field inside a superconducting solenoid is 4.50 T. The solenoid has an inner diameter of 6.20 cm and a length of 26.0 cm. Determine (a) the magnetic energy density in the field and (b) the energy stored in the magnetic field within the solenoid.

Section 32.4 Mutual Inductance

40. An emf of 96.0 mV is induced in the windings of a coil when the current in a nearby coil is increasing at the rate of 1.20 A/s. What is the mutual inductance of the two coils?

41. Two coils, held in fixed positions, have a mutual inductance of 100 μH. What is the peak emf in one coil when the current in the other coil is $i(t) = 10.0 \sin (1.00 \times 10^3 t)$, where i is in amperes and t is in seconds?

42. Two coils are close to each other. The first coil carries a current given by $i(t) = 5.00\, e^{-0.025\,0t} \sin 120\pi t$, where i is in amperes and t is in seconds. At $t = 0.800$ s, the emf measured across the second coil is −3.20 V. What is the mutual inductance of the coils?

43. M Two solenoids A and B, spaced close to each other and sharing the same cylindrical axis, have 400 and 700 turns, respectively. A current of 3.50 A in solenoid A produces an average flux of 300 μWb through each turn of A and a flux of 90.0 μWb through each turn of B. (a) Calculate the mutual inductance of the two solenoids. (b) What is the inductance of A? (c) What emf is induced in B when the current in A changes at the rate of 0.500 A/s?

44. Solenoid S_1 has N_1 turns, radius R_1, and length ℓ. It is so long that its magnetic field is uniform nearly everywhere inside it and is nearly zero outside. Solenoid S_2 has N_2 turns, radius $R_2 < R_1$, and the same length as S_1. It lies inside S_1, with their axes parallel. (a) Assume S_1 carries variable current i. Compute the mutual inductance characterizing the emf induced in S_2. (b) Now assume S_2 carries current i. Compute the mutual inductance to which the emf in S_1 is proportional. (c) State how the results of parts (a) and (b) compare with each other.

45. On a printed circuit board, a relatively long, straight conductor and a conducting rectangular loop lie in the same plane as shown in Figure P32.45. Taking $h = 0.400$ mm, $w = 1.30$ mm, and $\ell = 2.70$ mm, find their mutual inductance.

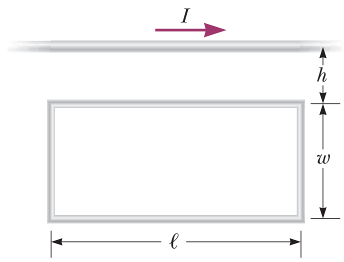


Figure P32.45

46. Two single-turn circular loops of wire have radii R and r, with $R >> r$. The loops lie in the same plane and are concentric. (a) Show that the mutual inductance of the pair is approximately $M = \mu_0 \pi r^2/2R$. (b) Evaluate M for $r = 2.00$ cm and $R = 20.0$ cm.

Section 32.5 Oscillations in an *LC* Circuit

47. In the circuit of Figure P32.47, the battery emf is 50.0 V, the resistance is 250 Ω, and the capacitance is 0.500 μF. The switch S is closed for a long time

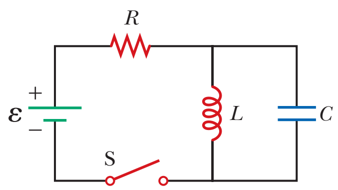


Figure P32.47

interval, and zero potential difference is measured across the capacitor. After the switch is opened, the potential difference across the capacitor reaches a maximum value of 150 V. What is the value of the inductance?

48. A 1.05-μH inductor is connected in series with a variable capacitor in the tuning section of a shortwave radio set. What capacitance tunes the circuit to the signal from a transmitter broadcasting at 6.30 MHz?

49. A 1.00-μF capacitor is charged by a 40.0-V power supply. The fully charged capacitor is then discharged through a 10.0-mH inductor. Find the maximum current in the resulting oscillations.

50. Calculate the inductance of an *LC* circuit that oscillates at 120 Hz when the capacitance is 8.00 μF.

51. An *LC* circuit consists of a 20.0-mH inductor and a 0.500-μF capacitor. If the maximum instantaneous current is 0.100 A, what is the greatest potential difference across the capacitor?

52. *Why is the following situation impossible?* The *LC* circuit shown in Figure CQ32.8 has L = 30.0 mH and C = 50.0 μF. The capacitor has an initial charge of 200 μC. The switch is closed, and the circuit undergoes undamped *LC* oscillations. At periodic instants, the energies stored by the capacitor and the inductor are equal, with each of the two components storing 250 μJ.

53. AMT The switch in Figure P32.53 is connected to position *a* for a long time interval. At $t = 0$, the switch is thrown to position *b*. After this time, what are (a) the frequency of oscillation of the *LC* circuit, (b) the maximum charge that appears on the capacitor, (c) the maximum current in the inductor, and (d) the total energy the circuit possesses at t = 3.00 s?

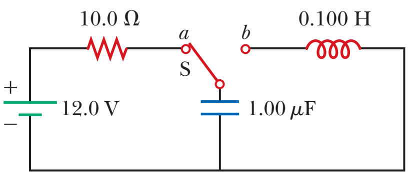


Figure P32.53

54. M An *LC* circuit like that in Figure CQ32.8 consists of a 3.30-H inductor and an 840-pF capacitor that initially carries a 105-μC charge. The switch is open for $t < 0$ and is then thrown closed at $t = 0$. Compute the following quantities at t = 2.00 ms: (a) the energy stored in the capacitor, (b) the energy stored in the inductor, and (c) the total energy in the circuit.

55. AMT An *LC* circuit like the one in Figure CQ32.8 contains an 82.0-mH inductor and a 17.0-μF capacitor that initially carries a 180-μC charge. The switch is open for $t < 0$ and is then thrown closed at $t = 0$. (a) Find the frequency (in hertz) of the resulting oscillations. At t = 1.00 ms, find (b) the charge on the capacitor and (c) the current in the circuit.

Section 32.6 The *RLC* Circuit

56. Show that Equation 32.28 in the text is Kirchhoff's loop rule as applied to the circuit in Figure P32.56 with the switch thrown to position *b*.

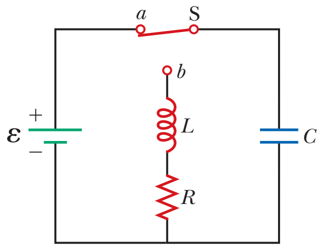


Figure P32.56 Problems 56 and 57.

57. In Figure P32.56, let R = 7.60 Ω, L = 2.20 mH, and C = 1.80 μF. (a) Calculate the frequency of the damped oscillation of the circuit when the switch is thrown to position *b*. (b) What is the critical resistance for damped oscillations?

58. M Consider an *LC* circuit in which L = 500 mH and C = 0.100 μF. (a) What is the resonance frequency ω_0? (b) If a resistance of 1.00 kΩ is introduced into this circuit, what is the frequency of the damped oscillations? (c) By what percentage does the frequency of the damped oscillations differ from the resonance frequency?

59. Electrical oscillations are initiated in a series circuit containing a capacitance C, inductance L, and resistance R. (a) If $R << \sqrt{4L/C}$ (weak damping), what time interval elapses before the amplitude of the current oscillation falls to 50.0% of its initial value? (b) Over what time interval does the energy decrease to 50.0% of its initial value?

Additional Problems

60. **Review.** This problem extends the reasoning of Section 26.4, Problem 38 in Chapter 26, Problem 34 in Chapter 30, and Section 32.3. (a) Consider a capacitor with vacuum between its large, closely spaced, oppositely charged parallel plates. Show that the force on one plate can be accounted for by thinking of the electric field between the plates as exerting a "negative pressure" equal to the energy density of the electric field. (b) Consider two infinite plane sheets carrying electric currents in opposite directions with equal linear current densities J_s. Calculate the force per area acting on one sheet due to the magnetic field, of magnitude $\mu_0 J_s/2$, created by the other sheet. (c) Calculate the net magnetic field between the sheets and the field outside of the volume between them. (d) Calculate the energy density in the magnetic field between the sheets. (e) Show that the force on one sheet can be accounted for by thinking of the magnetic field between the sheets as exerting a positive pressure equal to its energy density. This result for magnetic pressure applies to all current configurations, not only to sheets of current.

61. A 1.00-mH inductor and a 1.00-μF capacitor are connected in series. The current in the circuit increases linearly in time as $i = 20.0t$, where i is in amperes and t is in seconds. The capacitor initially has no charge. Determine (a) the voltage across the inductor as a function of time, (b) the voltage across the capacitor as a function of time, and (c) the time when the energy stored in the capacitor first exceeds that in the inductor.

62. An inductor having inductance L and a capacitor having capacitance C are connected in series. The current in the circuit increases linearly in time as described by $i = Kt$, where K is a constant. The capacitor is initially uncharged. Determine (a) the voltage across the inductor as a function of time, (b) the voltage across the capacitor as a function of time, and (c) the time when the energy stored in the capacitor first exceeds that in the inductor.

63. A capacitor in a series LC circuit has an initial charge Q and is being discharged. When the charge on the capacitor is $Q/2$, find the flux through each of the N turns in the coil of the inductor in terms of Q, N, L, and C.

64. AMT In the circuit diagrammed in Figure P32.25, assume the switch has been closed for a long time interval and is opened at $t = 0$. Also assume $R = 4.00\ \Omega$, $L = 1.00$ H, and $\mathcal{E} = 10.0$ V. (a) Before the switch is opened, does the inductor behave as an open circuit, a short circuit, a resistor of some particular resistance, or none of those choices? (b) What current does the inductor carry? (c) How much energy is stored in the inductor for $t < 0$? (d) After the switch is opened, what happens to the energy previously stored in the inductor? (e) Sketch a graph of the current in the inductor for $t \geq 0$. Label the initial and final values and the time constant.

65. When the current in the portion of the circuit shown in Figure P32.65 is 2.00 A and increases at a rate of 0.500 A/s, the measured voltage is $\Delta V_{ab} = 9.00$ V. When the current is 2.00 A and decreases at the rate of 0.500 A/s, the measured voltage is $\Delta V_{ab} = 5.00$ V. Calculate the values of (a) L and (b) R.

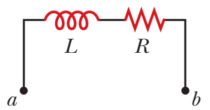


Figure P32.65

66. At the moment $t = 0$, a 24.0-V battery is connected to a 5.00-mH coil and a 6.00-Ω resistor. (a) Immediately thereafter, how does the potential difference across the resistor compare to the emf across the coil? (b) Answer the same question about the circuit several seconds later. (c) Is there an instant at which these two voltages are equal in magnitude? If so, when? Is there more than one such instant? (d) After a 4.00-A current is established in the resistor and coil, the battery is suddenly replaced by a short circuit. Answer parts (a), (b), and (c) again with reference to this new circuit.

67. (a) A flat, circular coil does not actually produce a uniform magnetic field in the area it encloses. Nevertheless, estimate the inductance of a flat, compact, circular coil with radius R and N turns by assuming the field at its center is uniform over its area. (b) A circuit on a laboratory table consists of a 1.50-volt battery, a 270-Ω resistor, a switch, and three 30.0-cm-long patch cords connecting them. Suppose the circuit is arranged to be circular. Think of it as a flat coil with one turn. Compute the order of magnitude of its inductance and (c) of the time constant describing how fast the current increases when you close the switch.

68. *Why is the following situation impossible?* You are working on an experiment involving a series circuit consisting of a charged 500-μF capacitor, a 32.0-mH inductor, and a resistor R. You discharge the capacitor through the inductor and resistor and observe the decaying oscillations of the current in the circuit. When the resistance R is 8.00 Ω, the decay in the oscillations is too slow for your experimental design. To make the decay faster, you double the resistance. As a result, you generate decaying oscillations of the current that are perfect for your needs.

69. A time-varying current i is sent through a 50.0-mH inductor from a source as shown in Figure P32.69a. The current is constant at $i = -1.00$ mA until $t = 0$ and then varies with time afterward as shown in Figure P32.69b. Make a graph of the emf across the inductor as a function of time.

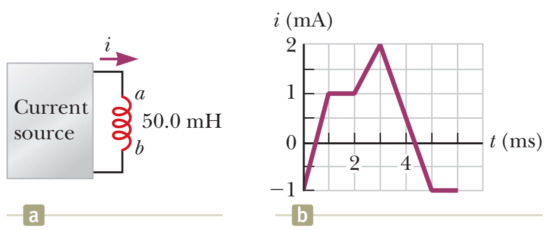


Figure P32.69

70. GP At $t = 0$, the open switch in Figure P32.70 is thrown closed. We wish to find a symbolic expression for the current in the inductor for time $t > 0$. Let this current be called i and choose it to be downward in the inductor in Figure P32.70. Identify i_1 as the current to the right through R_1 and i_2 as the current downward through R_2. (a) Use Kirchhoff's junction rule to find

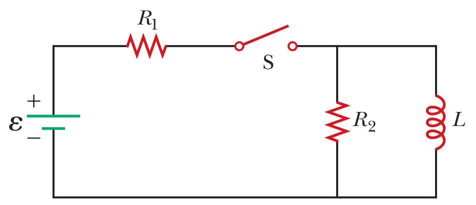


Figure P32.70

a relation among the three currents. (b) Use Kirchhoff's loop rule around the left loop to find another relationship. (c) Use Kirchhoff's loop rule around the outer loop to find a third relationship. (d) Eliminate i_1 and i_2 among the three equations to find an equation involving only the current i. (e) Compare the equation in part (d) with Equation 32.6 in the text. Use this comparison to rewrite Equation 32.7 in the text for the situation in this problem and show that

$$i(t) = \frac{\boldsymbol{\mathcal{E}}}{R_1}\left[1 - e^{-(R'/L)t}\right]$$

where $R' = R_1R_2/(R_1 + R_2)$.

71. The toroid in Figure P32.71 consists of N turns and has a rectangular cross section. Its inner and outer radii are a and b, respectively. The figure shows half of the toroid to allow us to see its cross-section. Compute the inductance of a 500-turn toroid for which $a = 10.0$ cm, $b = 12.0$ cm, and $h = 1.00$ cm.

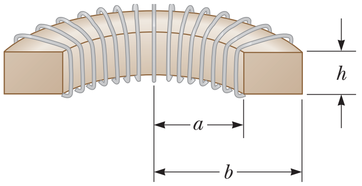


Figure P32.71 Problems 71 and 72.

72. The toroid in Figure P32.71 consists of N turns and has a rectangular cross section. Its inner and outer radii are a and b, respectively. Find the inductance of the toroid.

Problems 73 through 76 apply ideas from this and earlier chapters to some properties of superconductors, which were introduced in Section 27.5.

73. Review. A novel method of storing energy has been proposed. A huge underground superconducting coil, 1.00 km in diameter, would be fabricated. It would carry a maximum current of 50.0 kA through each winding of a 150-turn Nb_3Sn solenoid. (a) If the inductance of this huge coil were 50.0 H, what would be the total energy stored? (b) What would be the compressive force per unit length acting between two adjacent windings 0.250 m apart?

74. Review. In an experiment carried out by S. C. Collins between 1955 and 1958, a current was maintained in a superconducting lead ring for 2.50 yr with no observed loss, even though there was no energy input. If the inductance of the ring were 3.14×10^{-8} H and the sensitivity of the experiment were 1 part in 10^9, what was the maximum resistance of the ring? *Suggestion:* Treat the ring as an *RL* circuit carrying decaying current and recall that the approximation $e^{-x} \approx 1 - x$ is valid for small x.

75. Review. The use of superconductors has been proposed for power transmission lines. A single coaxial cable (Fig. P32.75) could carry a power of 1.00×10^3 MW (the output of a large power plant) at 200 kV, DC, over a distance of 1.00×10^3 km without loss. An inner wire of radius $a = 2.00$ cm, made from the superconductor Nb_3Sn, carries the current I in one direction. A surrounding superconducting cylinder of radius $b = 5.00$ cm would carry the return current I. In such a system, what is the magnetic field (a) at the surface of the inner conductor and (b) at the inner surface of the outer conductor? (c) How much energy would be stored in the magnetic field in the space between the conductors in a 1.00×10^3 km superconducting line? (d) What is the pressure exerted on the outer conductor due to the current in the inner conductor?

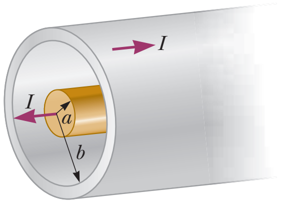


Figure P32.75

76. Review. A fundamental property of a type I superconducting material is *perfect diamagnetism,* or demonstration of the *Meissner effect,* illustrated in Figure 30.27 in Section 30.6 and described as follows. If a sample of superconducting material is placed into an externally produced magnetic field or is cooled to become superconducting while it is in a magnetic field, electric currents appear on the surface of the sample. The currents have precisely the strength and orientation required to make the total magnetic field be zero throughout the interior of the sample. This problem will help you understand the magnetic force that can then act on the sample. Compare this problem with Problem 65 in Chapter 26, pertaining to the force attracting a perfect dielectric into a strong electric field.

A vertical solenoid with a length of 120 cm and a diameter of 2.50 cm consists of 1 400 turns of copper wire carrying a counterclockwise current (when viewed from above) of 2.00 A as shown in Figure P32.76a (page 996). (a) Find the magnetic field in the vacuum inside the solenoid. (b) Find the energy density of the magnetic field. Now a superconducting bar 2.20 cm in diameter is inserted partway into the solenoid. Its upper end is far outside the solenoid, where the magnetic field is negligible. The lower end of the bar is deep inside the solenoid. (c) Explain how you identify the direction required for the current on the curved surface of the bar so that the total magnetic field is zero within the bar. The field created by the supercurrents is sketched in Figure P32.76b, and the total field is sketched in Figure P32.76c. (d) The field of the solenoid exerts a force on the current in the superconductor. Explain how you determine the direction of the force on the bar. (e) Noting that the units J/m³ of energy density are the same as the units N/m^2 of pressure, calculate the magnitude of the force by multiplying the energy density of the solenoid field times the area of the bottom end of the superconducting bar.

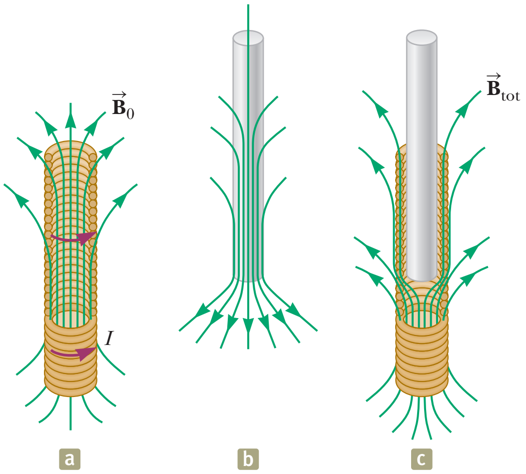


Figure P32.76

77. A wire of nonmagnetic material, with radius R, carries current uniformly distributed over its cross section. The total current carried by the wire is I. Show that the magnetic energy per unit length inside the wire is $\mu_0 I^2/16\pi$.

Challenge Problems

78. In earlier times when many households received nondigital television signals from an antenna, the lead-in wires from the antenna were often constructed in the form of two parallel wires (Fig. P32.78). The two wires carry currents of equal magnitude in opposite directions. The center-to-center separation of the wires is w, and a is their radius. Assume w is large enough compared with a that the wires carry the current uniformly distributed over their surfaces and negligible magnetic field exists inside the wires. (a) Why does this configuration of conductors have an inductance? (b) What constitutes the flux loop for this configuration? (c) Show that the inductance of a length x of this type of lead-in is

$$L = \frac{\mu_0 x}{\pi} \ln\left(\frac{w-a}{a}\right)$$

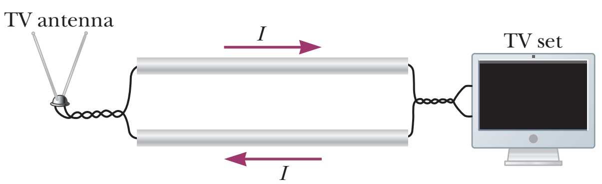


Figure P32.78

79. Assume the magnitude of the magnetic field outside a sphere of radius R is $B = B_0(R/r)^2$, where B_0 is a constant. (a) Determine the total energy stored in the magnetic field outside the sphere. (b) Evaluate your result from part (a) for $B_0 = 5.00 \times 10^{-5}$ T and $R = 6.00 \times 10^6$ m, values appropriate for the Earth's magnetic field.

80. In Figure P32.80, the battery has emf $\mathcal{E} = 18.0$ V and the other circuit elements have values $L = 0.400$ H, $R_1 = 2.00$ kΩ, and $R_2 = 6.00$ kΩ. The switch is closed for $t < 0$, and steady-state conditions are established. The switch is then opened at $t = 0$. (a) Find the emf across L immediately after $t = 0$. (b) Which end of the coil, a or b, is at the higher potential? (c) Make graphs of the currents in R_1 and in R_2 as a function of time, treating the steady-state directions as positive. Show values before and after $t = 0$. (d) At what moment after $t = 0$ does the current in R_2 have the value 2.00 mA?

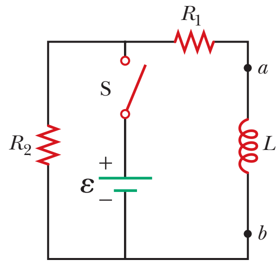


Figure P32.80

81. To prevent damage from arcing in an electric motor, a discharge resistor is sometimes placed in parallel with the armature. If the motor is suddenly unplugged while running, this resistor limits the voltage that appears across the armature coils. Consider a 12.0-V DC motor with an armature that has a resistance of 7.50 Ω and an inductance of 450 mH. Assume the magnitude of the self-induced emf in the armature coils is 10.0 V when the motor is running at normal speed. (The equivalent circuit for the armature is shown in Fig. P32.81.) Calculate the maximum resistance R that limits the voltage across the armature to 80.0 V when the motor is unplugged.

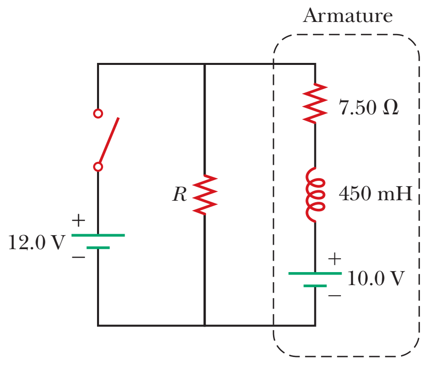


Figure P32.81

82. One application of an RL circuit is the generation of time-varying high voltage from a low-voltage source as shown in Figure P32.82. (a) What is the current in the circuit a long time after the switch has been in position *a*? (b) Now the switch is thrown quickly from *a* to *b*. Compute the initial voltage across each resistor and across the inductor. (c) How much time elapses before the voltage across the inductor drops to 12.0 V?

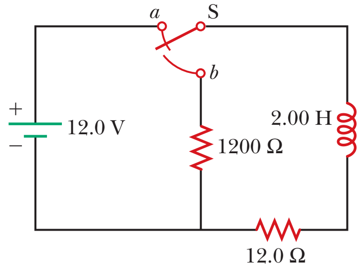


Figure P32.82

83. Two inductors having inductances L_1 and L_2 are connected in parallel as shown in Figure P32.83a. The mutual inductance between the two inductors is M. Determine the equivalent inductance L_{eq} for the system (Fig. P32.83b).

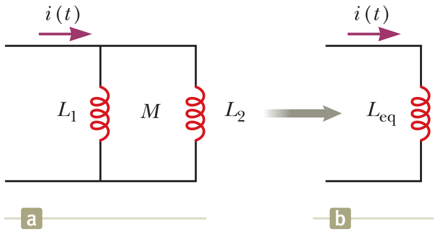


Figure P32.83