# Faraday's Law

CHAPTER
31



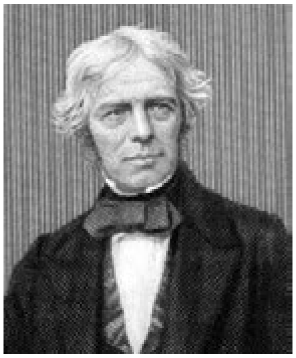
An artist's impression of the Skerries SeaGen Array, a tidal energy generator under development near the island of Anglesey, North Wales. When it is brought online, it will offer 10.5 MW of power from generators turned by tidal streams. The image shows the underwater blades that are driven by the tidal currents. The second blade system has been raised from the water for servicing. We will study generators in this chapter. *(Marine Current Turbines TM Ltd.)*

**So far, our studies in electricity and magnetism have focused on the electric fields** produced by stationary charges and the magnetic fields produced by moving charges. This chapter explores the effects produced by magnetic fields that vary in time.

Experiments conducted by Michael Faraday in England in 1831 and independently by Joseph Henry in the United States that same year showed that an emf can be induced in a circuit by a changing magnetic field. The results of these experiments led to a very basic and important law of electromagnetism known as *Faraday's law of induction.* An emf (and therefore a current as well) can be induced in various processes that involve a change in a magnetic flux.

## 31.1 Faraday's Law of Induction

To see how an emf can be induced by a changing magnetic field, consider the experimental results obtained when a loop of wire is connected to a sensitive ammeter as illustrated in Figure 31.1 (page 936). When a magnet is moved toward the loop, the reading on the ammeter changes from zero to a nonzero value, arbitrarily shown as negative in Figure 31.1a. When the magnet is brought to rest and held stationary relative to the loop (Fig. 31.1b), a reading of zero is observed. When the magnet is moved away from the loop, the reading on the ammeter changes to a positive value as shown in Figure 31.1c. Finally, when the magnet is held stationary and the loop




**Michael Faraday**
*British Physicist and Chemist (1791–1867)*
Faraday is often regarded as the greatest experimental scientist of the 1800s. His many contributions to the study of electricity include the invention of the electric motor, electric generator, and transformer as well as the discovery of electromagnetic induction and the laws of electrolysis. Greatly influenced by religion, he refused to work on the development of poison gas for the British military.

is moved either toward or away from it, the reading changes from zero. From these observations, we conclude that the loop detects that the magnet is moving relative to it and we relate this detection to a change in magnetic field. Therefore, it seems that a relationship exists between a current and a changing magnetic field.

These results are quite remarkable because a current is set up even though no batteries are present in the circuit! We call such a current an *induced current* and say that it is produced by an *induced emf.*

Now let's describe an experiment conducted by Faraday and illustrated in Figure 31.2. A primary coil is wrapped around an iron ring and connected to a switch and a battery. A current in the coil produces a magnetic field when the switch is closed. A secondary coil also is wrapped around the ring and is connected to a sensitive ammeter. No battery is present in the secondary circuit, and the secondary coil is not electrically connected to the primary coil. Any current detected in the secondary circuit must be induced by some external agent.

Initially, you might guess that no current is ever detected in the secondary circuit. Something quite amazing happens when the switch in the primary circuit is either opened or thrown closed, however. At the instant the switch is closed, the ammeter reading changes from zero momentarily and then returns to zero. At the instant the switch is opened, the ammeter changes to a reading with the opposite sign and again returns to zero. Finally, the ammeter reads zero when there is either a steady current or no current in the primary circuit. To understand what happens in this experiment, note that when the switch is closed, the current in the primary circuit produces a magnetic field that penetrates the secondary circuit. Furthermore, when the switch is thrown closed, the magnetic field produced by the current in the primary circuit changes from zero to some value over some finite time, and this changing field induces a current in the secondary circuit. Notice that no current is induced in the secondary coil even when a steady current exists in the primary coil. It is a *change* in the current in the primary coil that induces a current in the secondary coil, not just the *existence* of a current.

As a result of these observations, Faraday concluded that an electric current can be induced in a loop by a changing magnetic field. The induced current exists only while the magnetic field through the loop is changing. Once the magnetic field reaches a steady value, the current in the loop disappears. In effect, the loop behaves as though a source of emf were connected to it for a short time. It is customary to say that an induced emf is produced in the loop by the changing magnetic field.

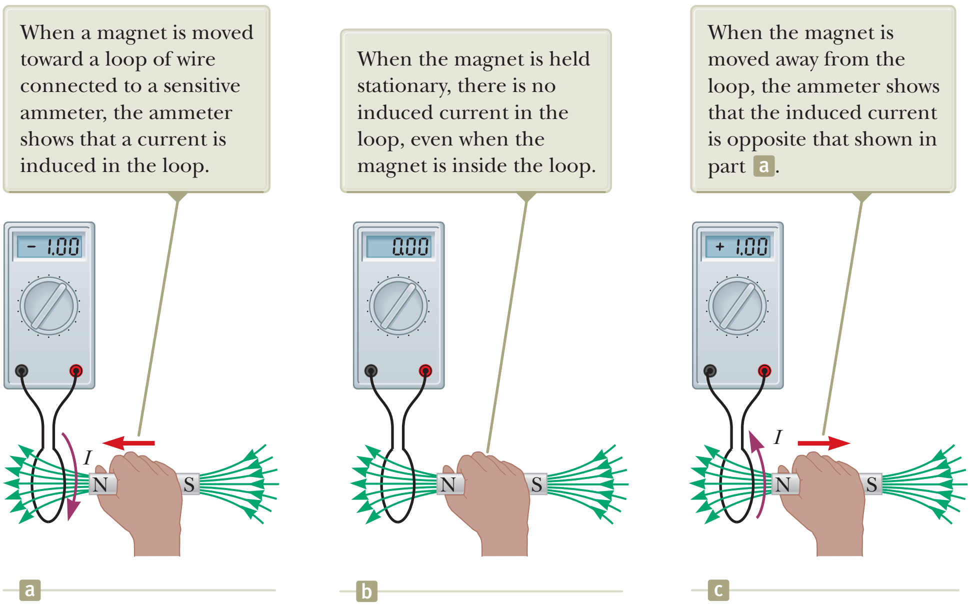



**Figure 31.1** A simple experiment showing that a current is induced in a loop when a magnet is moved toward or away from the loop.

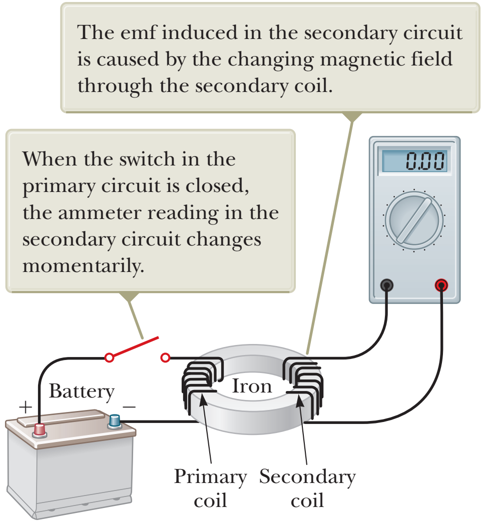



**Figure 31.2** Faraday's experiment.

The experiments shown in Figures 31.1 and 31.2 have one thing in common: in each case, an emf is induced in a loop when the magnetic flux through the loop changes with time. In general, this emf is directly proportional to the time rate of change of the magnetic flux through the loop. This statement can be written mathematically as **Faraday's law of induction:**

$$\mathcal{E} = -\frac{d\Phi_B}{dt} \tag{31.1}$$

◀ **Faraday's law of induction**

where $\Phi_B = \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$ is the magnetic flux through the loop. (See Section 30.5.)

If a coil consists of $N$ loops with the same area and $\Phi_B$ is the magnetic flux through one loop, an emf is induced in every loop. The loops are in series, so their emfs add; therefore, the total induced emf in the coil is given by

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} \tag{31.2}$$

The negative sign in Equations 31.1 and 31.2 is of important physical significance and will be discussed in Section 31.3.

Suppose a loop enclosing an area $A$ lies in a uniform magnetic field $\vec{\mathbf{B}}$ as in Figure 31.3. The magnetic flux through the loop is equal to $BA\cos\theta$, where $\theta$ is the angle between the magnetic field and the normal to the loop; hence, the induced emf can be expressed as

$$\mathcal{E} = -\frac{d}{dt}(BA\cos\theta) \tag{31.3}$$

From this expression, we see that an emf can be induced in the circuit in several ways:

- The magnitude of $\vec{\mathbf{B}}$ can change with time.
- The area enclosed by the loop can change with time.
- The angle $\theta$ between $\vec{\mathbf{B}}$ and the normal to the loop can change with time.
- Any combination of the above can occur.

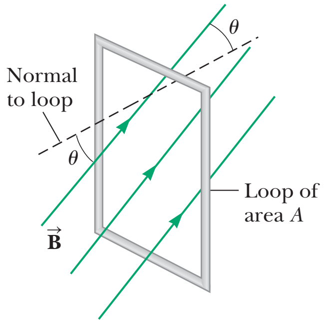



**Figure 31.3** A conducting loop that encloses an area $A$ in the presence of a uniform magnetic field $\vec{\mathbf{B}}$. The angle between $\vec{\mathbf{B}}$ and the normal to the loop is $\theta$.

**Quick Quiz 31.1** A circular loop of wire is held in a uniform magnetic field, with the plane of the loop perpendicular to the field lines. Which of the following will *not* cause a current to be induced in the loop? **(a)** crushing the loop **(b)** rotating the loop about an axis perpendicular to the field lines **(c)** keeping the orientation of the loop fixed and moving it along the field lines **(d)** pulling the loop out of the field

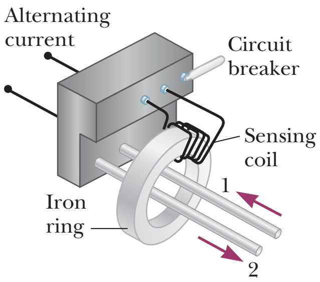



**Figure 31.4** Essential components of a ground fault circuit interrupter.

## Some Applications of Faraday's Law

The ground fault circuit interrupter (GFCI) is an interesting safety device that protects users of electrical appliances against electric shock. Its operation makes use of Faraday's law. In the GFCI shown in Figure 31.4, wire 1 leads from the wall outlet to the appliance to be protected and wire 2 leads from the appliance back to the wall outlet. An iron ring surrounds the two wires, and a sensing coil is wrapped around part of the ring. Because the currents in the wires are in opposite directions and of equal magnitude, there is zero net current flowing through the ring and the net magnetic flux through the sensing coil is zero. Now suppose the return current in wire 2 changes so that the two currents are not equal in magnitude. (That can happen if, for example, the appliance becomes wet, enabling current to leak to ground.) Then the net current through the ring is not zero and the magnetic flux through the sensing coil is no longer zero. Because household current is alternating (meaning that its direction keeps reversing), the magnetic flux through the sensing coil changes with time, inducing an emf in the coil. This induced emf is used to trigger a circuit breaker, which stops the current before it is able to reach a harmful level.

Another interesting application of Faraday's law is the production of sound in an electric guitar. The coil in this case, called the *pickup coil,* is placed near the vibrating guitar string, which is made of a metal that can be magnetized. A permanent magnet inside the coil magnetizes the portion of the string nearest the coil (Fig. 31.5a). When the string vibrates at some frequency, its magnetized segment produces a changing magnetic flux through the coil. The changing flux induces an emf in the coil that is fed to an amplifier. The output of the amplifier is sent to the loudspeakers, which produce the sound waves we hear.

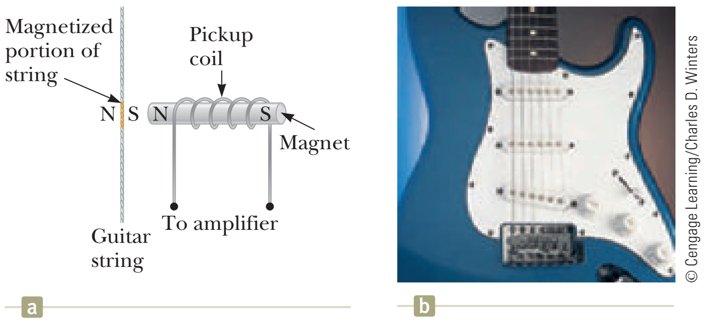



**Figure 31.5** (a) In an electric guitar, a vibrating magnetized string induces an emf in a pickup coil. (b) The pickups (the circles beneath the metallic strings) of this electric guitar detect the vibrations of the strings and send this information through an amplifier and into speakers. (A switch on the guitar allows the musician to select which set of six pickups is used.)

### Example 31.1 Inducing an emf in a Coil

A coil consists of 200 turns of wire. Each turn is a square of side $d = 18$ cm, and a uniform magnetic field directed perpendicular to the plane of the coil is turned on. If the field changes linearly from 0 to 0.50 T in 0.80 s, what is the magnitude of the induced emf in the coil while the field is changing?

**SOLUTION**

**Conceptualize** From the description in the problem, imagine magnetic field lines passing through the coil. Because the magnetic field is changing in magnitude, an emf is induced in the coil.

**Categorize** We will evaluate the emf using Faraday's law from this section, so we categorize this example as a substitution problem.

31.1 continued

Evaluate Equation 31.2 for the situation described here, noting that the magnetic field changes linearly with time:

$$|\mathcal{E}| = N\frac{\Delta\Phi_B}{\Delta t} = N\frac{\Delta(BA)}{\Delta t} = NA\frac{\Delta B}{\Delta t} = Nd^2\frac{B_f - B_i}{\Delta t}$$

Substitute numerical values:

$$|\mathcal{E}| = (200)(0.18\text{ m})^2\frac{(0.50\text{ T} - 0)}{0.80\text{ s}} = 4.0\text{ V}$$

**WHAT IF?** What if you were asked to find the magnitude of the induced current in the coil while the field is changing? Can you answer that question?

**Answer** If the ends of the coil are not connected to a circuit, the answer to this question is easy: the current is zero! (Charges move within the wire of the coil, but they cannot move into or out of the ends of the coil.) For a steady current to exist, the ends of the coil must be connected to an external circuit. Let's assume the coil is connected to a circuit and the total resistance of the coil and the circuit is 2.0 Ω. Then, the magnitude of the induced current in the coil is

$$I = \frac{|\mathcal{E}|}{R} = \frac{4.0\text{ V}}{2.0\ \Omega} = 2.0\text{ A}$$

## Example 31.2 An Exponentially Decaying Magnetic Field

A loop of wire enclosing an area $A$ is placed in a region where the magnetic field is perpendicular to the plane of the loop. The magnitude of $\vec{\mathbf{B}}$ varies in time according to the expression $B = B_{max}e^{-at}$, where $a$ is some constant. That is, at $t = 0$, the field is $B_{max}$, and for $t > 0$, the field decreases exponentially (Fig. 31.6). Find the induced emf in the loop as a function of time.

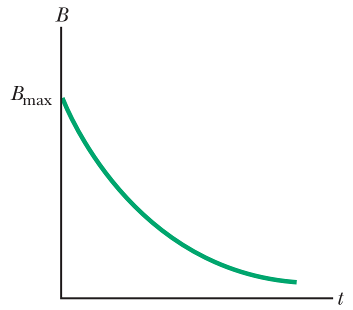



**Figure 31.6** (Example 31.2) Exponential decrease in the magnitude of the magnetic field through a loop with time. The induced emf and induced current in a conducting path attached to the loop vary with time in the same way.

**SOLUTION**

**Conceptualize** The physical situation is similar to that in Example 31.1 except for two things: there is only one loop, and the field varies exponentially with time rather than linearly.

**Categorize** We will evaluate the emf using Faraday's law from this section, so we categorize this example as a substitution problem.

Evaluate Equation 31.1 for the situation described here:

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(AB_{max}\,e^{-at}) = -AB_{max}\frac{d}{dt}e^{-at} = aAB_{max}\,e^{-at}$$

This expression indicates that the induced emf decays exponentially in time. The maximum emf occurs at $t = 0$, where $\mathcal{E}_{max} = aAB_{max}$. The plot of $\mathcal{E}$ versus $t$ is similar to the $B$-versus-$t$ curve shown in Figure 31.6.

# 31.2 Motional emf

In Examples 31.1 and 31.2, we considered cases in which an emf is induced in a stationary circuit placed in a magnetic field when the field changes with time. In this section, we describe **motional emf,** the emf induced in a conductor moving through a constant magnetic field.

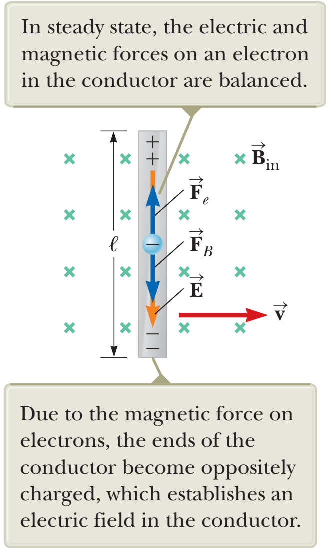



**Figure 31.7** A straight electrical conductor of length $\ell$ moving with a velocity $\vec{v}$ through a uniform magnetic field $\vec{B}$ directed perpendicular to $\vec{v}$.

The straight conductor of length $\ell$ shown in Figure 31.7 is moving through a uniform magnetic field directed into the page. For simplicity, let's assume the conductor is moving in a direction perpendicular to the field with constant velocity under the influence of some external agent. From the magnetic version of the particle in a field model, the electrons in the conductor experience a force $\vec{F}_B = q\vec{v} \times \vec{B}$ (Eq. 29.1) that is directed along the length $\ell$, perpendicular to both $\vec{v}$ and $\vec{B}$. Under the influence of this force, the electrons move to the lower end of the conductor and accumulate there, leaving a net positive charge at the upper end. As a result of this charge separation, an electric field $\vec{E}$ is produced inside the conductor. Therefore, the electrons are also described by the electric version of the particle in a field model. The charges accumulate at both ends until the downward magnetic force $qvB$ on charges remaining in the conductor is balanced by the upward electric force $qE$. The electrons are then described by the particle in equilibrium model. The condition for equilibrium requires that the forces on the electrons balance:

$$qE = qvB \quad \text{or} \quad E = vB$$

The magnitude of the electric field produced in the conductor is related to the potential difference across the ends of the conductor according to the relationship $\Delta V = E\ell$ (Eq. 25.6). Therefore, for the equilibrium condition,

$$\Delta V = E\ell = B\ell v \tag{31.4}$$

where the upper end of the conductor in Figure 31.7 is at a higher electric potential than the lower end. Therefore, a potential difference is maintained between the ends of the conductor as long as the conductor continues to move through the uniform magnetic field. If the direction of the motion is reversed, the polarity of the potential difference is also reversed.

A more interesting situation occurs when the moving conductor is part of a closed conducting path. This situation is particularly useful for illustrating how a changing magnetic flux causes an induced current in a closed circuit. Consider a circuit consisting of a conducting bar of length $\ell$ sliding along two fixed, parallel conducting rails as shown in Figure 31.8a. For simplicity, let's assume the bar has zero resistance and the stationary part of the circuit has a resistance $R$. A uniform and constant magnetic field $\vec{B}$ is applied perpendicular to the plane of the circuit. As the bar is pulled to the right with a velocity $\vec{v}$ under the influence of an applied force $\vec{F}_{app}$, free charges in the bar are moving particles in a magnetic field that experience a magnetic force directed along the length of the bar. This force sets up an induced current because the charges are free to move in the closed conducting path. In this case, the rate of change of magnetic flux through the circuit and the corresponding

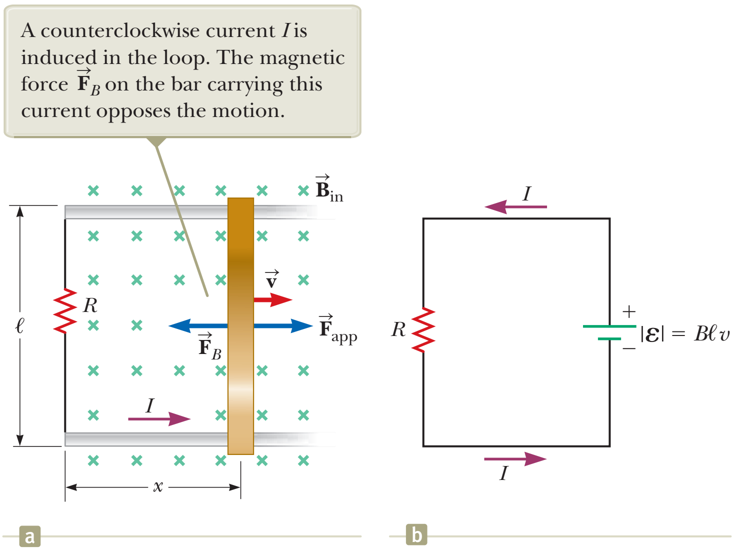



**Figure 31.8** (a) A conducting bar sliding with a velocity $\vec{v}$ along two conducting rails under the action of an applied force $\vec{F}_{app}$. (b) The equivalent circuit diagram for the setup shown in (a).

induced motional emf across the moving bar are proportional to the change in area of the circuit.

Because the area enclosed by the circuit at any instant is $\ell x$, where $x$ is the position of the bar, the magnetic flux through that area is

$$\Phi_B = B\ell x$$

Using Faraday's law and noting that $x$ changes with time at a rate $dx/dt = v$, we find that the induced motional emf is

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(B\ell x) = -B\ell\frac{dx}{dt}$$

$$\mathcal{E} = -B\ell v \qquad (31.5)$$

◀ **Motional emf**

Because the resistance of the circuit is $R$, the magnitude of the induced current is

$$I = \frac{|\mathcal{E}|}{R} = \frac{B\ell v}{R} \qquad (31.6)$$

The equivalent circuit diagram for this example is shown in Figure 31.8b.

Let's examine the system using energy considerations. Because no battery is in the circuit, you might wonder about the origin of the induced current and the energy delivered to the resistor. We can understand the source of this current and energy by noting that the applied force does work on the conducting bar. Therefore, we model the circuit as a nonisolated system. The movement of the bar through the field causes charges to move along the bar with some average drift velocity; hence, a current is established. The change in energy in the system during some time interval must be equal to the transfer of energy into the system by work, consistent with the general principle of conservation of energy described by Equation 8.2. The appropriate reduction of Equation 8.2 is $W = \Delta E_{\text{int}}$, because the input energy appears as internal energy in the resistor.

Let's verify this equality mathematically. As the bar moves through the uniform magnetic field $\vec{\mathbf{B}}$, it experiences a magnetic force $\vec{\mathbf{F}}_B$ of magnitude $I\ell B$ (see Section 29.4). Because the bar moves with constant velocity, it is modeled as a particle in equilibrium and the magnetic force must be equal in magnitude and opposite in direction to the applied force, or to the left in Figure 31.8a. (If $\vec{\mathbf{F}}_B$ acted in the direction of motion, it would cause the bar to accelerate, violating the principle of conservation of energy.) Using Equation 31.6 and $F_{\text{app}} = F_B = I\ell B$, the power delivered by the applied force is

$$P = F_{\text{app}}v = (I\ell B)v = \frac{B^2\ell^2v^2}{R} = \frac{\mathcal{E}^2}{R} \qquad (31.7)$$

From Equation 27.22, we see that this power input is equal to the rate at which energy is delivered to the resistor, consistent with the principle of conservation of energy.

**Quick Quiz 31.2** In Figure 31.8a, a given applied force of magnitude $F_{\text{app}}$ results in a constant speed $v$ and a power input $P$. Imagine that the force is increased so that the constant speed of the bar is doubled to $2v$. Under these conditions, what are the new force and the new power input? **(a)** $2F$ and $2P$ **(b)** $4F$ and $2P$ **(c)** $2F$ and $4P$ **(d)** $4F$ and $4P$

### Example 31.3 Magnetic Force Acting on a Sliding Bar AM

The conducting bar illustrated in Figure 31.9 (page 942) moves on two frictionless, parallel rails in the presence of a uniform magnetic field directed into the page. The bar has mass $m$, and its length is $\ell$. The bar is given an initial velocity $\vec{\mathbf{v}}_i$ to the right and is released at $t = 0$.

*continued*

**31.3** continued

**(A)** Using Newton's laws, find the velocity of the bar as a function of time.

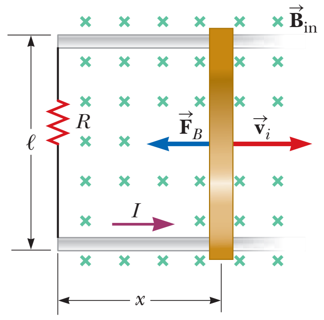



**Figure 31.9** (Example 31.3) A conducting bar of length $\ell$ on two fixed conducting rails is given an initial velocity $\vec{v}_i$ to the right.

**SOLUTION**

**Conceptualize** As the bar slides to the right in Figure 31.9, a counterclockwise current is established in the circuit consisting of the bar, the rails, and the resistor. The upward current in the bar results in a magnetic force to the left on the bar as shown in the figure. Therefore, the bar must slow down, so our mathematical solution should demonstrate that.

**Categorize** The text already categorizes this problem as one that uses Newton's laws. We model the bar as a *particle under a net force.*

**Analyze** From Equation 29.10, the magnetic force is $F_B = -I\ell B$, where the negative sign indicates that the force is to the left. The magnetic force is the *only* horizontal force acting on the bar.

Using the particle under a net force model, apply Newton's second law to the bar in the horizontal direction:

$$F_x = ma \;\rightarrow\; -I\ell B = m\frac{dv}{dt}$$

Substitute $I = B\ell v/R$ from Equation 31.6:

$$m\frac{dv}{dt} = -\frac{B^2\ell^2}{R}v$$

Rearrange the equation so that all occurrences of the variable $v$ are on the left and those of $t$ are on the right:

$$\frac{dv}{v} = -\left(\frac{B^2\ell^2}{mR}\right)dt$$

Integrate this equation using the initial condition that $v = v_i$ at $t = 0$ and noting that $(B^2\ell^2/mR)$ is a constant:

$$\int_{v_i}^{v}\frac{dv}{v} = -\frac{B^2\ell^2}{mR}\int_0^t dt$$

$$\ln\left(\frac{v}{v_i}\right) = -\left(\frac{B^2\ell^2}{mR}\right)t$$

Define the constant $\tau = mR/B^2\ell^2$ and solve for the velocity:

$$(1)\quad v = v_i e^{-t/\tau}$$

**Finalize** This expression for $v$ indicates that the velocity of the bar decreases with time under the action of the magnetic force as expected from our conceptualization of the problem.

**(B)** Show that the same result is found by using an energy approach.

**SOLUTION**

**Categorize** The text of this part of the problem tells us to use an energy approach for the same situation. We model the entire circuit in Figure 31.9 as an *isolated system.*

**Analyze** Consider the sliding bar as one system component possessing kinetic energy, which decreases because energy is transferring *out* of the bar by electrical transmission through the rails. The resistor is another system component possessing internal energy, which rises because energy is transferring *into* the resistor. Because energy is not leaving the system, the rate of energy transfer out of the bar equals the rate of energy transfer into the resistor.

Equate the power entering the resistor to that leaving the bar:

$$P_{\text{resistor}} = -P_{\text{bar}}$$

Substitute for the electrical power delivered to the resistor and the time rate of change of kinetic energy for the bar:

$$I^2R = -\frac{d}{dt}\left(\tfrac{1}{2}mv^2\right)$$

Use Equation 31.6 for the current and carry out the derivative:

$$\frac{B^2\ell^2v^2}{R} = -mv\frac{dv}{dt}$$

31.3 continued

Rearrange terms:

$$\frac{dv}{v} = -\left(\frac{B^2\ell^2}{mR}\right) dt$$

**Finalize** This result is the same expression to be integrated that we found in part (A).

**WHAT IF?** Suppose you wished to increase the distance through which the bar moves between the time it is initially projected and the time it essentially comes to rest. You can do so by changing one of three variables—$v_i$, $R$, or $B$—by a factor of 2 or $\frac{1}{2}$. Which variable should you change to maximize the distance, and would you double it or halve it?

**Answer** Increasing $v_i$ would make the bar move farther. Increasing $R$ would decrease the current and therefore the magnetic force, making the bar move farther. Decreasing $B$ would decrease the magnetic force and make the bar move farther. Which method is most effective, though?

Use Equation (1) to find the distance the bar moves by integration:

$$v = \frac{dx}{dt} = v_i e^{-t/\tau}$$

$$x = \int_0^\infty v_i e^{-t/\tau}\, dt = -v_i \tau e^{-t/\tau}\Big|_0^\infty$$

$$= -v_i \tau(0 - 1) = v_i \tau = v_i\left(\frac{mR}{B^2\ell^2}\right)$$

This expression shows that doubling $v_i$ or $R$ will double the distance. Changing $B$ by a factor of $\frac{1}{2}$, however, causes the distance to be four times as great!

## Example 31.4 Motional emf Induced in a Rotating Bar

A conducting bar of length $\ell$ rotates with a constant angular speed $\omega$ about a pivot at one end. A uniform magnetic field $\vec{\mathbf{B}}$ is directed perpendicular to the plane of rotation as shown in Figure 31.10. Find the motional emf induced between the ends of the bar.

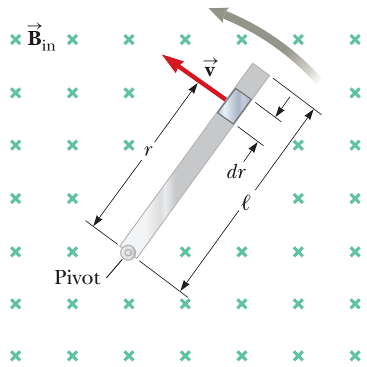



**Figure 31.10** (Example 31.4) A conducting bar rotating around a pivot at one end in a uniform magnetic field that is perpendicular to the plane of rotation. A motional emf is induced between the ends of the bar.

**SOLUTION**

**Conceptualize** The rotating bar is different in nature from the sliding bar in Figure 31.8. Consider a small segment of the bar, however. It is a short length of conductor moving in a magnetic field and has an emf generated in it like the sliding bar. By thinking of each small segment as a source of emf, we see that all segments are in series and the emfs add.

**Categorize** Based on the conceptualization of the problem, we approach this example as we did in the discussion leading to Equation 31.5, with the added feature that the short segments of the bar are traveling in circular paths.

**Analyze** Evaluate the magnitude of the emf induced in a segment of the bar of length $dr$ having a velocity $\vec{\mathbf{v}}$ from Equation 31.5:

$$d\mathcal{E} = Bv\, dr$$

Find the total emf between the ends of the bar by adding the emfs induced across all segments:

$$\mathcal{E} = \int Bv\, dr$$

The tangential speed $v$ of an element is related to the angular speed $\omega$ through the relationship $v = r\omega$ (Eq. 10.10); use that fact and integrate:

$$\mathcal{E} = B\int v\, dr = B\omega\int_0^\ell r\, dr = \frac{1}{2}B\omega\ell^2$$

*continued*

31.4 continued

**Finalize** In Equation 31.5 for a sliding bar, we can increase $\mathcal{E}$ by increasing $B$, $\ell$, or $v$. Increasing any one of these variables by a given factor increases $\mathcal{E}$ by the same factor. Therefore, you would choose whichever of these three variables is most convenient to increase. For the rotating rod, however, there is an advantage to increasing the length of the rod to raise the emf because $\ell$ is squared. Doubling the length gives four times the emf, whereas doubling the angular speed only doubles the emf.

**WHAT IF?** Suppose, after reading through this example, you come up with a brilliant idea. A Ferris wheel has radial metallic spokes between the hub and the circular rim. These spokes move in the magnetic field of the Earth, so each spoke acts like the bar in Figure 31.10. You plan to use the emf generated by the rotation of the Ferris wheel to power the lightbulbs on the wheel. Will this idea work?

**Answer** Let's estimate the emf that is generated in this situation. We know the magnitude of the magnetic field of the Earth from Table 29.1: $B = 0.5 \times 10^{-4}$ T. A typical spoke on a Ferris wheel might have a length on the order of 10 m. Suppose the period of rotation is on the order of 10 s.

Determine the angular speed of the spoke:

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{10\text{ s}} = 0.63\text{ s}^{-1} \sim 1\text{ s}^{-1}$$

Assume the magnetic field lines of the Earth are horizontal at the location of the Ferris wheel and perpendicular to the spokes. Find the emf generated:

$$\mathcal{E} = \tfrac{1}{2}B\omega\ell^2 = \tfrac{1}{2}(0.5 \times 10^{-4}\text{ T})(1\text{ s}^{-1})(10\text{ m})^2$$
$$= 2.5 \times 10^{-3}\text{ V} \sim 1\text{ mV}$$

This value is a tiny emf, far smaller than that required to operate lightbulbs.

An additional difficulty is related to energy. Even assuming you could find lightbulbs that operate using a potential difference on the order of millivolts, a spoke must be part of a circuit to provide a voltage to the lightbulbs. Consequently, the spoke must carry a current. Because this current-carrying spoke is in a magnetic field, a magnetic force is exerted on the spoke in the direction opposite its direction of motion. As a result, the motor of the Ferris wheel must supply more energy to perform work against this magnetic drag force. The motor must ultimately provide the energy that is operating the lightbulbs, and you have not gained anything for free!

## 31.3 Lenz's Law

Faraday's law (Eq. 31.1) indicates that the induced emf and the change in flux have opposite algebraic signs. This feature has a very real physical interpretation that has come to be known as **Lenz's law:**[1]

Lenz's law ▶

> The induced current in a loop is in the direction that creates a magnetic field that opposes the change in magnetic flux through the area enclosed by the loop.

That is, the induced current tends to keep the original magnetic flux through the loop from changing. We shall show that this law is a consequence of the law of conservation of energy.

To understand Lenz's law, let's return to the example of a bar moving to the right on two parallel rails in the presence of a uniform magnetic field (the *external* magnetic field, shown by the green crosses in Fig. 31.11a). As the bar moves to the right, the magnetic flux through the area enclosed by the circuit increases with time because the area increases. Lenz's law states that the induced current must be directed so that the magnetic field it produces opposes the change in the external magnetic flux. Because the magnetic flux due to an external field directed into the page is increasing, the induced current—if it is to oppose this change—must

[1]Developed by German physicist Heinrich Lenz (1804–1865).

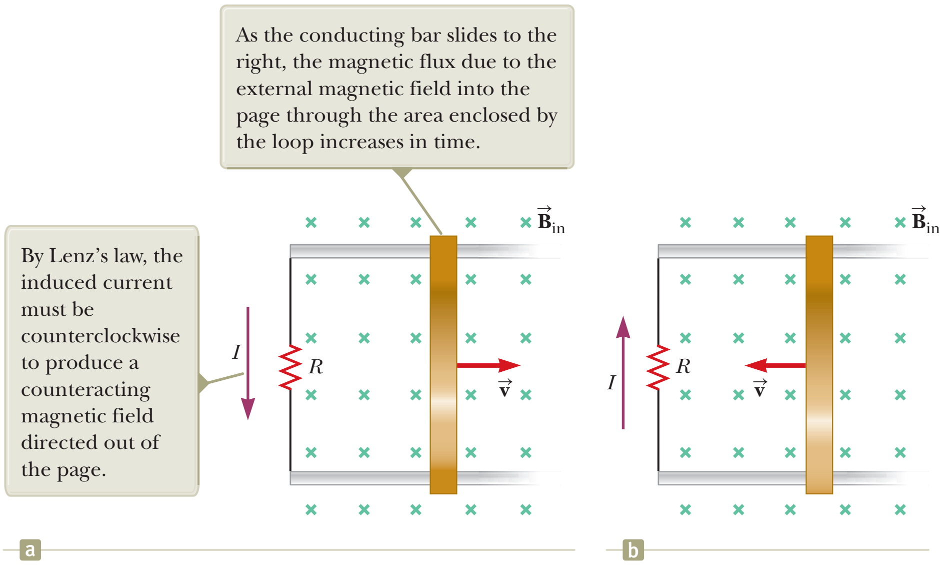



**Figure 31.11** (a) Lenz's law can be used to determine the direction of the induced current. (b) When the bar moves to the left, the induced current must be clockwise. Why?

produce a field directed out of the page. Hence, the induced current must be directed counterclockwise when the bar moves to the right. (Use the right-hand rule to verify this direction.) If the bar is moving to the left as in Figure 31.11b, the external magnetic flux through the area enclosed by the loop decreases with time. Because the field is directed into the page, the direction of the induced current must be clockwise if it is to produce a field that also is directed into the page. In either case, the induced current attempts to maintain the original flux through the area enclosed by the current loop.

Let's examine this situation using energy considerations. Suppose the bar is given a slight push to the right. In the preceding analysis, we found that this motion sets up a counterclockwise current in the loop. What happens if we assume the current is clockwise such that the direction of the magnetic force exerted on the bar is to the right? This force would accelerate the rod and increase its velocity, which in turn would cause the area enclosed by the loop to increase more rapidly. The result would be an increase in the induced current, which would cause an increase in the force, which would produce an increase in the current, and so on. In effect, the system would acquire energy with no input of energy. This behavior is clearly inconsistent with all experience and violates the law of conservation of energy. Therefore, the current must be counterclockwise.

**Quick Quiz 31.3** Figure 31.12 shows a circular loop of wire falling toward a wire carrying a current to the left. What is the direction of the induced current in the loop of wire? **(a)** clockwise **(b)** counterclockwise **(c)** zero **(d)** impossible to determine

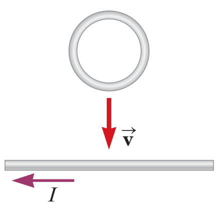



**Figure 31.12** (Quick Quiz 31.3)

## Conceptual Example 31.5 Application of Lenz's Law

A magnet is placed near a metal loop as shown in Figure 31.13a (page 946).

**(A)** Find the direction of the induced current in the loop when the magnet is pushed toward the loop.

**SOLUTION**

As the magnet moves to the right toward the loop, the external magnetic flux through the loop increases with time. To counteract this increase in flux due to a field toward the right, the induced current produces its own magnetic field to the left as illustrated in Figure 31.13b; hence, the induced current is in the direction shown. Knowing that like

*continued*

31.5 continued

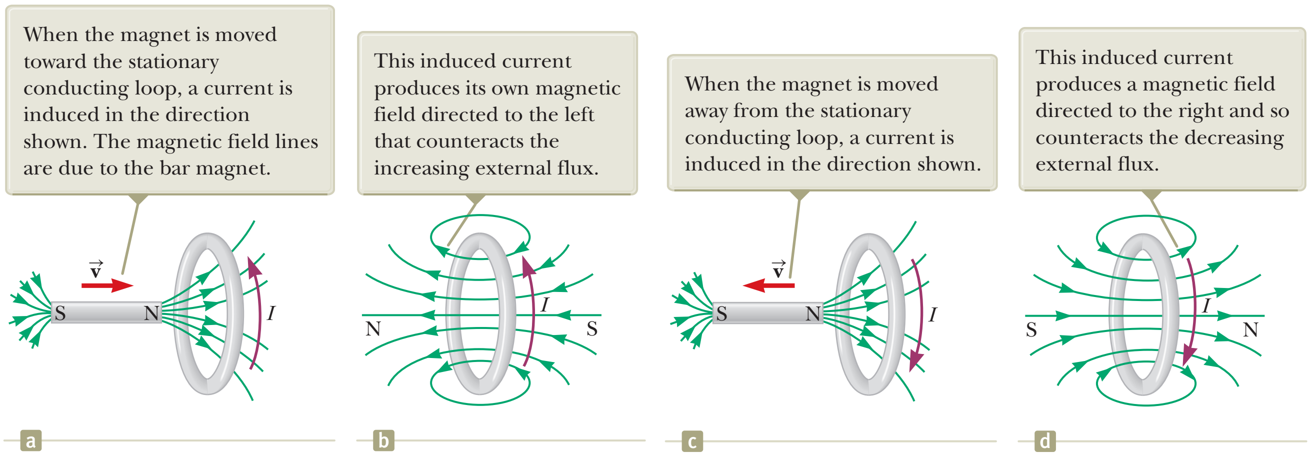



**Figure 31.13** (Conceptual Example 31.5) A moving bar magnet induces a current in a conducting loop.

magnetic poles repel each other, we conclude that the left face of the current loop acts like a north pole and the right face acts like a south pole.

**(B)** Find the direction of the induced current in the loop when the magnet is pulled away from the loop.

**SOLUTION**

If the magnet moves to the left as in Figure 31.13c, its flux through the area enclosed by the loop decreases in time. Now the induced current in the loop is in the direction shown in Figure 31.13d because this current direction produces a magnetic field in the same direction as the external field. In this case, the left face of the loop is a south pole and the right face is a north pole.

## Conceptual Example 31.6 A Loop Moving Through a Magnetic Field

A rectangular metallic loop of dimensions $\ell$ and $w$ and resistance $R$ moves with constant speed $v$ to the right as in Figure 31.14a. The loop passes through a uniform magnetic field $\vec{\mathbf{B}}$ directed into the page and extending a distance $3w$ along the $x$ axis. Define $x$ as the position of the right side of the loop along the $x$ axis.

**(A)** Plot the magnetic flux through the area enclosed by the loop as a function of $x$.

**SOLUTION**

Figure 31.14b shows the flux through the area enclosed by the loop as a function of $x$. Before the loop enters the field, the flux through the loop is zero. As the loop enters the field, the flux increases linearly with position until the left edge of the loop is just inside the field. Finally, the flux through the loop decreases linearly to zero as the loop leaves the field.

**(B)** Plot the induced motional emf in the loop as a function of $x$.

**SOLUTION**

Before the loop enters the field, no motional emf is induced in it because no field is present (Fig. 31.14c). As the right side of the loop enters the field, the magnetic flux directed into the page increases. Hence, according to Lenz's law, the induced current is counterclockwise because it must produce its own magnetic field directed out of the page. The motional emf $-B\ell v$ (from Eq. 31.5) arises from the magnetic force experienced by charges in the right side of the loop. When the loop is entirely in the field, the change in magnetic flux through the loop is zero; hence, the motional emf vanishes. That happens because once the left side of the loop enters the field, the motional emf induced in it

31.6 continued

cancels the motional emf present in the right side of the loop. As the right side of the loop leaves the field, the flux through the loop begins to decrease, a clockwise current is induced, and the induced emf is $B\ell v$. As soon as the left side leaves the field, the emf decreases to zero.

**(C)** Plot the external applied force necessary to counter the magnetic force and keep $v$ constant as a function of $x$.

**SOLUTION**

The external force that must be applied to the loop to maintain this motion is plotted in Figure 31.14d. Before the loop enters the field, no magnetic force acts on it; hence, the applied force must be zero if $v$ is constant. When the right side of the loop enters the field, the applied force necessary to maintain constant speed must be equal in magnitude and opposite in direction to the magnetic force exerted on that side, so that the loop is a particle in equilibrium. When the loop is entirely in the field, the flux through the loop is not changing with time. Hence, the net emf induced in the loop is zero and the current also is zero. Therefore, no external force is needed to maintain the motion. Finally, as the right side leaves the field, the applied force must be equal in magnitude and opposite in direction to the magnetic force acting on the left side of the loop.

From this analysis, we conclude that power is supplied only when the loop is either entering or leaving the field. Furthermore, this example shows that the motional emf induced in the loop can be zero even when there is motion through the field! A motional emf is induced *only* when the magnetic flux through the loop *changes in time.*

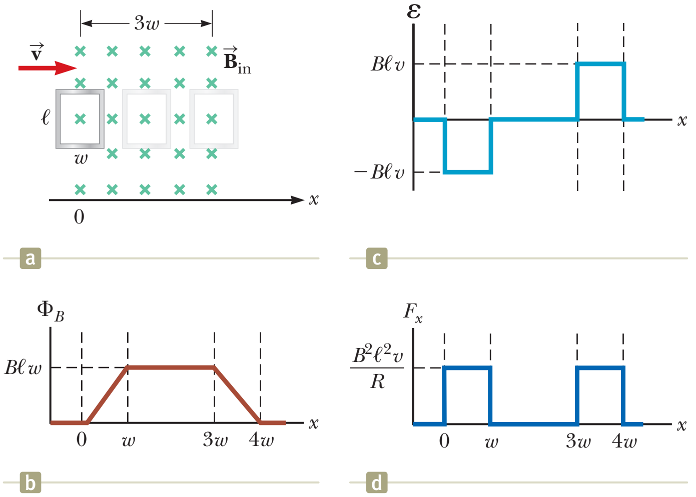



**Figure 31.14** (Conceptual Example 31.6) (a) A conducting rectangular loop of width $w$ and length $\ell$ moving with a velocity $\vec{v}$ through a uniform magnetic field extending a distance $3w$. (b) Magnetic flux through the area enclosed by the loop as a function of loop position. (c) Induced emf as a function of loop position. (d) Applied force required for constant velocity as a function of loop position.

## 31.4 Induced emf and Electric Fields

We have seen that a changing magnetic flux induces an emf and a current in a conducting loop. In our study of electricity, we related a current to an electric field that applies electric forces on charged particles. In the same way, we can relate an induced current in a conducting loop to an electric field by claiming that an electric field is created in the conductor as a result of the changing magnetic flux.

We also noted in our study of electricity that the existence of an electric field is independent of the presence of any test charges. This independence suggests that even in the absence of a conducting loop, a changing magnetic field generates an electric field in empty space.

This induced electric field is *nonconservative,* unlike the electrostatic field produced by stationary charges. To illustrate this point, consider a conducting loop of radius $r$ situated in a uniform magnetic field that is perpendicular to the plane of the loop as in Figure 31.15. If the magnetic field changes with time, an emf $\mathcal{E} = -d\Phi_B/dt$ is, according to Faraday's law (Eq. 31.1), induced in the loop. The induction of a current in the loop implies the presence of an induced electric field $\vec{E}$, which must be tangent to the loop because that is the direction in which the charges in the wire move in response to the electric force. The work done by the electric field in moving a charge $q$ once around the loop is equal to $q\mathcal{E}$. Because the electric force acting on the charge is $q\vec{E}$, the work done by the electric field in

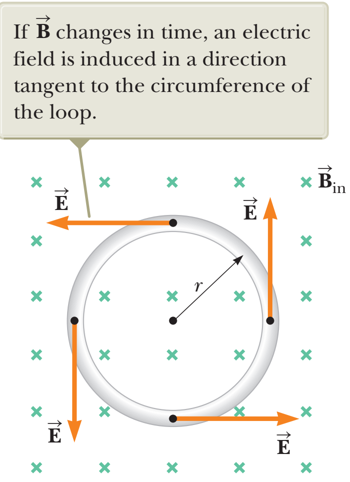



**Figure 31.15** A conducting loop of radius $r$ in a uniform magnetic field perpendicular to the plane of the loop.

> **Pitfall Prevention 31.1**
> **Induced Electric Fields** The changing magnetic field does *not* need to exist at the location of the induced electric field. In Figure 31.15, even a loop outside the region of magnetic field experiences an induced electric field.

moving the charge once around the loop is $qE(2\pi r)$, where $2\pi r$ is the circumference of the loop. These two expressions for the work done must be equal; therefore,

$$q\mathcal{E} = qE(2\pi r)$$

$$E = \frac{\mathcal{E}}{2\pi r}$$

Using this result along with Equation 31.1 and that $\Phi_B = BA = B\pi r^2$ for a circular loop, the induced electric field can be expressed as

$$E = -\frac{1}{2\pi r}\frac{d\Phi_B}{dt} = -\frac{r}{2}\frac{dB}{dt} \qquad \textbf{(31.8)}$$

If the time variation of the magnetic field is specified, the induced electric field can be calculated from Equation 31.8.

The emf for any closed path can be expressed as the line integral of $\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$ over that path: $\mathcal{E} = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$. In more general cases, $E$ may not be constant and the path may not be a circle. Hence, Faraday's law of induction, $\mathcal{E} = -d\Phi_B/dt$, can be written in the general form

**Faraday's law in general form** ▶

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt} \qquad \textbf{(31.9)}$$

The induced electric field $\vec{\mathbf{E}}$ in Equation 31.9 is a nonconservative field that is generated by a changing magnetic field. The field $\vec{\mathbf{E}}$ that satisfies Equation 31.9 cannot possibly be an electrostatic field because were the field electrostatic and hence conservative, the line integral of $\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$ over a closed loop would be zero (Section 25.1), which would be in contradiction to Equation 31.9.

## Example 31.7 Electric Field Induced by a Changing Magnetic Field in a Solenoid

A long solenoid of radius $R$ has $n$ turns of wire per unit length and carries a time-varying current that varies sinusoidally as $I = I_{max} \cos \omega t$, where $I_{max}$ is the maximum current and $\omega$ is the angular frequency of the alternating current source (Fig. 31.16).

**(A)** Determine the magnitude of the induced electric field outside the solenoid at a distance $r > R$ from its long central axis.

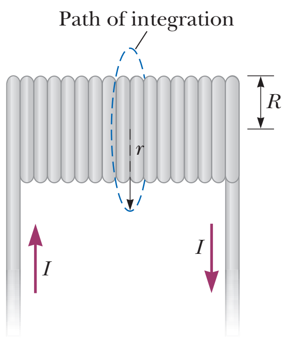



**Figure 31.16** (Example 31.7) A long solenoid carrying a time-varying current given by $I = I_{max} \cos \omega t$. An electric field is induced both inside and outside the solenoid.

**SOLUTION**

**Conceptualize** Figure 31.16 shows the physical situation. As the current in the coil changes, imagine a changing magnetic field at all points in space as well as an induced electric field.

**Categorize** In this analysis problem, because the current varies in time, the magnetic field is changing, leading to an induced electric field as opposed to the electrostatic electric fields due to stationary electric charges.

**Analyze** First consider an external point and take the path for the line integral to be a circle of radius $r$ centered on the solenoid as illustrated in Figure 31.16.

Evaluate the right side of Equation 31.9, noting that the magnetic field $\vec{\mathbf{B}}$ inside the solenoid is perpendicular to the circle bounded by the path of integration:

$$(1) \quad -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(B\pi R^2) = -\pi R^2 \frac{dB}{dt}$$

Evaluate the magnetic field inside the solenoid from Equation 30.17:

$$(2) \quad B = \mu_0 n I = \mu_0 n I_{max} \cos \omega t$$

**31.7 continued**

Substitute Equation (2) into Equation (1):

$$(3)\quad -\frac{d\Phi_B}{dt} = -\pi R^2 \mu_0 n I_{\max} \frac{d}{dt}(\cos \omega t) = \pi R^2 \mu_0 n I_{\max} \omega \sin \omega t$$

Evaluate the left side of Equation 31.9, noting that the magnitude of $\vec{\mathbf{E}}$ is constant on the path of integration and $\vec{\mathbf{E}}$ is tangent to it:

$$(4)\quad \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = E(2\pi r)$$

Substitute Equations (3) and (4) into Equation 31.9:

$$E(2\pi r) = \pi R^2 \mu_0 n I_{\max} \omega \sin \omega t$$

Solve for the magnitude of the electric field:

$$E = \frac{\mu_0 n I_{\max} \omega R^2}{2r} \sin \omega t \quad (\text{for } r > R)$$

**Finalize** This result shows that the amplitude of the electric field outside the solenoid falls off as $1/r$ and varies sinusoidally with time. It is proportional to the current $I$ as well as to the frequency $\omega$, consistent with the fact that a larger value of $\omega$ means more change in magnetic flux per unit time. As we will learn in Chapter 34, the time-varying electric field creates an additional contribution to the magnetic field. The magnetic field can be somewhat stronger than we first stated, both inside and outside the solenoid. The correction to the magnetic field is small if the angular frequency $\omega$ is small. At high frequencies, however, a new phenomenon can dominate: The electric and magnetic fields, each re-creating the other, constitute an electromagnetic wave radiated by the solenoid as we will study in Chapter 34.

**(B)** What is the magnitude of the induced electric field inside the solenoid, a distance $r$ from its axis?

**SOLUTION**

**Analyze** For an interior point ($r < R$), the magnetic flux through an integration loop is given by $\Phi_B = B\pi r^2$.

Evaluate the right side of Equation 31.9:

$$(5)\quad -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(B\pi r^2) = -\pi r^2 \frac{dB}{dt}$$

Substitute Equation (2) into Equation (5):

$$(6)\quad -\frac{d\Phi_B}{dt} = -\pi r^2 \mu_0 n I_{\max} \frac{d}{dt}(\cos \omega t) = \pi r^2 \mu_0 n I_{\max} \omega \sin \omega t$$

Substitute Equations (4) and (6) into Equation 31.9:

$$E(2\pi r) = \pi r^2 \mu_0 n I_{\max} \omega \sin \omega t$$

Solve for the magnitude of the electric field:

$$E = \frac{\mu_0 n I_{\max} \omega}{2} r \sin \omega t \quad (\text{for } r < R)$$

**Finalize** This result shows that the amplitude of the electric field induced inside the solenoid by the changing magnetic flux through the solenoid increases linearly with $r$ and varies sinusoidally with time. As with the field outside the solenoid, the field inside is proportional to the current $I$ and the frequency $\omega$.

## 31.5 Generators and Motors

Electric generators are devices that take in energy by work and transfer it out by electrical transmission. To understand how they operate, let us consider the **alternating-current (AC) generator.** In its simplest form, it consists of a loop of wire rotated by some external means in a magnetic field (Fig. 31.17a, page 950).

In commercial power plants, the energy required to rotate the loop can be derived from a variety of sources. For example, in a hydroelectric plant, falling water directed against the blades of a turbine produces the rotary motion; in a coal-fired plant, the energy released by burning coal is used to convert water to steam, and this steam is directed against the turbine blades.

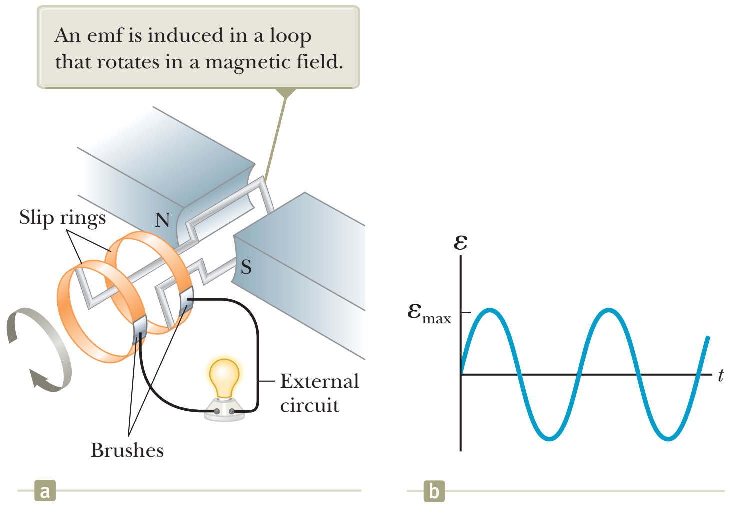



**Figure 31.17** (a) Schematic diagram of an AC generator. (b) The alternating emf induced in the loop plotted as a function of time.

As a loop rotates in a magnetic field, the magnetic flux through the area enclosed by the loop changes with time, and this change induces an emf and a current in the loop according to Faraday's law. The ends of the loop are connected to slip rings that rotate with the loop. Connections from these slip rings, which act as output terminals of the generator, to the external circuit are made by stationary metallic brushes in contact with the slip rings.

Instead of a single turn, suppose a coil with $N$ turns (a more practical situation), with the same area $A$, rotates in a magnetic field with a constant angular speed $\omega$. If $\theta$ is the angle between the magnetic field and the normal to the plane of the coil as in Figure 31.18, the magnetic flux through the coil at any time $t$ is

$$\Phi_B = BA\cos\theta = BA\cos\omega t$$

where we have used the relationship $\theta = \omega t$ between angular position and angular speed (see Eq. 10.3). (We have set the clock so that $t = 0$ when $\theta = 0$.) Hence, the induced emf in the coil is

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -NBA\frac{d}{dt}(\cos\omega t) = NBA\omega\sin\omega t \quad \textbf{(31.10)}$$

This result shows that the emf varies sinusoidally with time as plotted in Figure 31.17b. Equation 31.10 shows that the maximum emf has the value

$$\mathcal{E}_{max} = NBA\omega \quad \textbf{(31.11)}$$

which occurs when $\omega t = 90°$ or $270°$. In other words, $\mathcal{E} = \mathcal{E}_{max}$ when the magnetic field is in the plane of the coil and the time rate of change of flux is a maximum. Furthermore, the emf is zero when $\omega t = 0$ or $180°$, that is, when $\vec{\mathbf{B}}$ is perpendicular to the plane of the coil and the time rate of change of flux is zero.

The frequency for commercial generators in the United States and Canada is 60 Hz, whereas in some European countries it is 50 Hz. (Recall that $\omega = 2\pi f$, where $f$ is the frequency in hertz.)

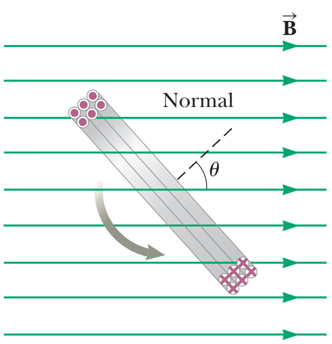



**Figure 31.18** A cutaway view of a loop enclosing an area $A$ and containing $N$ turns, rotating with constant angular speed $\omega$ in a magnetic field. The emf induced in the loop varies sinusoidally in time.

**Quick Quiz 31.4** In an AC generator, a coil with $N$ turns of wire spins in a magnetic field. Of the following choices, which does *not* cause an increase in the emf generated in the coil? **(a)** replacing the coil wire with one of lower resistance **(b)** spinning the coil faster **(c)** increasing the magnetic field **(d)** increasing the number of turns of wire on the coil

## Example 31.8 emf Induced in a Generator

The coil in an AC generator consists of 8 turns of wire, each of area $A = 0.090\,0\ \text{m}^2$, and the total resistance of the wire is 12.0 Ω. The coil rotates in a 0.500-T magnetic field at a constant frequency of 60.0 Hz.

**(A)** Find the maximum induced emf in the coil.

**SOLUTION**

**Conceptualize** Study Figure 31.17 to make sure you understand the operation of an AC generator.

**Categorize** We evaluate parameters using equations developed in this section, so we categorize this example as a substitution problem.

Use Equation 31.11 to find the maximum induced emf:

$$\mathcal{E}_{\max} = NBA\omega = NBA(2\pi f)$$

Substitute numerical values:

$$\mathcal{E}_{\max} = 8(0.500\ \text{T})(0.090\,0\ \text{m}^2)(2\pi)(60.0\ \text{Hz}) = \boxed{136\ \text{V}}$$

**(B)** What is the maximum induced current in the coil when the output terminals are connected to a low-resistance conductor?

**SOLUTION**

Use Equation 27.7 and the result to part (A):

$$I_{\max} = \frac{\mathcal{E}_{\max}}{R} = \frac{136\ \text{V}}{12.0\ \Omega} = \boxed{11.3\ \text{A}}$$

The **direct-current (DC) generator** is illustrated in Figure 31.19a. Such generators are used, for instance, in older cars to charge the storage batteries. The components are essentially the same as those of the AC generator except that the contacts to the rotating coil are made using a split ring called a *commutator.*

In this configuration, the output voltage always has the same polarity and pulsates with time as shown in Figure 31.19b. We can understand why by noting that the contacts to the split ring reverse their roles every half cycle. At the same time, the polarity of the induced emf reverses; hence, the polarity of the split ring (which is the same as the polarity of the output voltage) remains the same.

A pulsating DC current is not suitable for most applications. To obtain a steadier DC current, commercial DC generators use many coils and commutators distributed so that the sinusoidal pulses from the various coils are out of phase. When these pulses are superimposed, the DC output is almost free of fluctuations.

A **motor** is a device into which energy is transferred by electrical transmission while energy is transferred out by work. A motor is essentially a generator operating

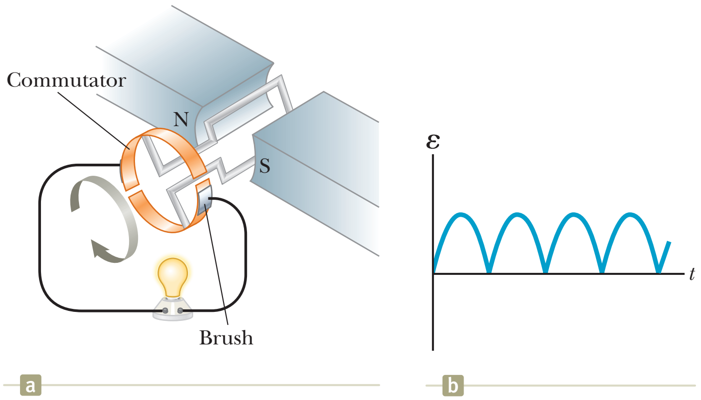



**Figure 31.19** (a) Schematic diagram of a DC generator. (b) The magnitude of the emf varies in time, but the polarity never changes.


**Figure 31.20** The engine compartment of a Toyota Prius, a hybrid vehicle.

in reverse. Instead of generating a current by rotating a coil, a current is supplied to the coil by a battery, and the torque acting on the current-carrying coil (Section 29.5) causes it to rotate.

Useful mechanical work can be done by attaching the rotating coil of a motor to some external device. As the coil rotates in a magnetic field, however, the changing magnetic flux induces an emf in the coil; consistent with Lenz's law, this induced emf always acts to reduce the current in the coil. The back emf increases in magnitude as the rotational speed of the coil increases. (The phrase *back emf* is used to indicate an emf that tends to reduce the supplied current.) Because the voltage available to supply current equals the difference between the supply voltage and the back emf, the current in the rotating coil is limited by the back emf.

When a motor is turned on, there is initially no back emf, and the current is very large because it is limited only by the resistance of the coil. As the coil begins to rotate, the induced back emf opposes the applied voltage and the current in the coil decreases. If the mechanical load increases, the motor slows down, which causes the back emf to decrease. This reduction in the back emf increases the current in the coil and therefore also increases the power needed from the external voltage source. For this reason, the power requirements for running a motor are greater for heavy loads than for light ones. If the motor is allowed to run under no mechanical load, the back emf reduces the current to a value just large enough to overcome energy losses due to internal energy and friction. If a very heavy load jams the motor so that it cannot rotate, the lack of a back emf can lead to dangerously high current in the motor's wire. This dangerous situation is explored in the What If? section of Example 31.9.

A modern application of motors in automobiles is seen in the development of *hybrid drive systems.* In these automobiles, a gasoline engine and an electric motor are combined to increase the fuel economy of the vehicle and reduce its emissions. Figure 31.20 shows the engine compartment of a Toyota Prius, one of the hybrids available in the United States. In this automobile, power to the wheels can come from either the gasoline engine or the electric motor. In normal driving, the electric motor accelerates the vehicle from rest until it is moving at a speed of about 15 mi/h (24 km/h). During this acceleration period, the engine is not running, so gasoline is not used and there is no emission. At higher speeds, the motor and engine work together so that the engine always operates at or near its most efficient speed. The result is a significantly higher gasoline mileage than that obtained by a traditional gasoline-powered automobile. When a hybrid vehicle brakes, the motor acts as a generator and returns some of the vehicle's kinetic energy back to the battery as stored energy. In a normal vehicle, this kinetic energy is not recovered because it is transformed to internal energy in the brakes and roadway.

## Example 31.9 The Induced Current in a Motor

A motor contains a coil with a total resistance of 10 Ω and is supplied by a voltage of 120 V. When the motor is running at its maximum speed, the back emf is 70 V.

**(A)** Find the current in the coil at the instant the motor is turned on.

**SOLUTION**

**Conceptualize** Think about the motor just after it is turned on. It has not yet moved, so there is no back emf generated. As a result, the current in the motor is high. After the motor begins to turn, a back emf is generated and the current decreases.

**Categorize** We need to combine our new understanding about motors with the relationship between current, voltage, and resistance in this substitution problem.

31.9 continued

Evaluate the current in the coil from Equation 27.7 with no back emf generated:

$$I = \frac{\mathcal{E}}{R} = \frac{120\text{ V}}{10\ \Omega} = 12\text{ A}$$

**(B)** Find the current in the coil when the motor has reached maximum speed.

**SOLUTION**

Evaluate the current in the coil with the maximum back emf generated:

$$I = \frac{\mathcal{E} - \mathcal{E}_{\text{back}}}{R} = \frac{120\text{ V} - 70\text{ V}}{10\ \Omega} = \frac{50\text{ V}}{10\ \Omega} = 5.0\text{ A}$$

The current drawn by the motor when operating at its maximum speed is significantly less than that drawn before it begins to turn.

**WHAT IF?** Suppose this motor is in a circular saw. When you are operating the saw, the blade becomes jammed in a piece of wood and the motor cannot turn. By what percentage does the power input to the motor increase when it is jammed?

**Answer** You may have everyday experiences with motors becoming warm when they are prevented from turning. That is due to the increased power input to the motor. The higher rate of energy transfer results in an increase in the internal energy of the coil, an undesirable effect.

Set up the ratio of power input to the motor when jammed, using the current calculated in part (A), to that when it is not jammed, part (B):

$$\frac{P_{\text{jammed}}}{P_{\text{not jammed}}} = \frac{I_A^2 R}{I_B^2 R} = \frac{I_A^2}{I_B^2}$$

Substitute numerical values:

$$\frac{P_{\text{jammed}}}{P_{\text{not jammed}}} = \frac{(12\text{ A})^2}{(5.0\text{ A})^2} = 5.76$$

That represents a 476% increase in the input power! Such a high power input can cause the coil to become so hot that it is damaged.

## 31.6 Eddy Currents

As we have seen, an emf and a current are induced in a circuit by a changing magnetic flux. In the same manner, circulating currents called **eddy currents** are induced in bulk pieces of metal moving through a magnetic field. This phenomenon can be demonstrated by allowing a flat copper or aluminum plate attached at the end of a rigid bar to swing back and forth through a magnetic field (Fig. 31.21). As the plate enters the field, the changing magnetic flux induces an emf in the plate, which in turn causes the free electrons in the plate to move, producing the swirling eddy currents. According to Lenz's law, the direction of the eddy currents is such that they create magnetic fields that oppose the change that causes the currents. For this reason, the eddy currents must produce effective magnetic poles on the plate, which are repelled by the poles of the magnet; this situation gives rise to a repulsive force that opposes the motion of the plate. (If the opposite were true, the plate would accelerate and its energy would increase after each swing, in violation of the law of conservation of energy.)

As indicated in Figure 31.22a (page 954), with $\vec{\mathbf{B}}$ directed into the page, the induced eddy current is counterclockwise as the swinging plate enters the field at position 1 because the flux due to the external magnetic field into the page through the plate is increasing. Hence, by Lenz's law, the induced current must provide its own magnetic field out of the page. The opposite is true as the plate

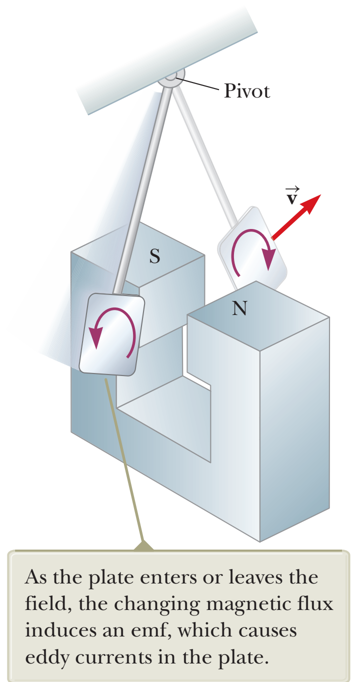



**Figure 31.21** Formation of eddy currents in a conducting plate moving through a magnetic field.

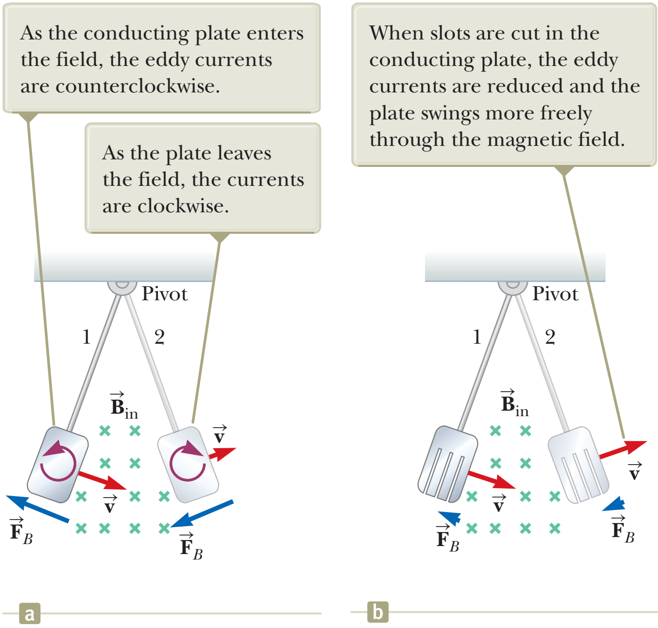



**Figure 31.22** When a conducting plate swings through a magnetic field, eddy currents are induced and the magnetic force $\vec{F}_B$ on the plate opposes its velocity, causing it to eventually come to rest.

leaves the field at position 2, where the current is clockwise. Because the induced eddy current always produces a magnetic retarding force $\vec{F}_B$ when the plate enters or leaves the field, the swinging plate eventually comes to rest.

If slots are cut in the plate as shown in Figure 31.22b, the eddy currents and the corresponding retarding force are greatly reduced. We can understand this reduction in force by realizing that the cuts in the plate prevent the formation of any large current loops.

The braking systems on many subway and rapid-transit cars make use of electromagnetic induction and eddy currents. An electromagnet attached to the train is positioned near the steel rails. (An electromagnet is essentially a solenoid with an iron core.) The braking action occurs when a large current is passed through the electromagnet. The relative motion of the magnet and rails induces eddy currents in the rails, and the direction of these currents produces a drag force on the moving train. Because the eddy currents decrease steadily in magnitude as the train slows down, the braking effect is quite smooth. As a safety measure, some power tools use eddy currents to stop rapidly spinning blades once the device is turned off.

Eddy currents are often undesirable because they represent a transformation of mechanical energy to internal energy. To reduce this energy loss, conducting parts are often laminated; that is, they are built up in thin layers separated by a nonconducting material such as lacquer or a metal oxide. This layered structure prevents large current loops and effectively confines the currents to small loops in individual layers. Such a laminated structure is used in transformer cores (see Section 33.8) and motors to minimize eddy currents and thereby increase the efficiency of these devices.

**Quick Quiz 31.5** In an equal-arm balance from the early 20th century (Fig. 31.23), an aluminum sheet hangs from one of the arms and passes between the poles of a magnet, causing the oscillations of the balance to decay rapidly. In the absence of such magnetic braking, the oscillation might continue for a long time, and the experimenter would have to wait to take a reading. Why do the oscillations decay? **(a)** because the aluminum sheet is attracted to the magnet

**(b)** because currents in the aluminum sheet set up a magnetic field that opposes the oscillations **(c)** because aluminum is paramagnetic

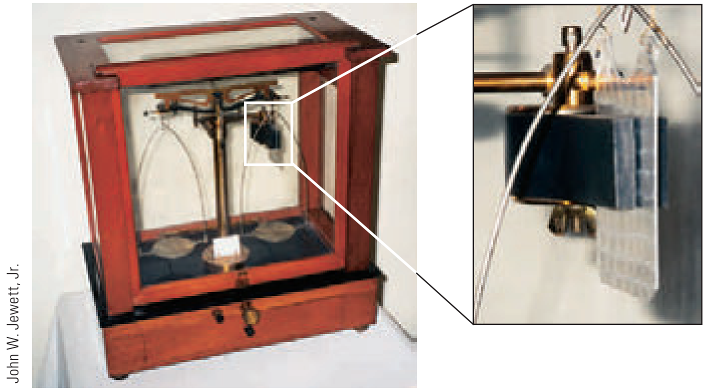



**Figure 31.23** (Quick Quiz 31.5) In an old-fashioned equal-arm balance, an aluminum sheet hangs between the poles of a magnet.

# Summary

## Concepts and Principles

**Faraday's law of induction** states that the emf induced in a loop is directly proportional to the time rate of change of magnetic flux through the loop, or

$$\mathcal{E} = -\frac{d\Phi_B}{dt} \tag{31.1}$$

where $\Phi_B = \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$ is the magnetic flux through the loop.

When a conducting bar of length $\ell$ moves at a velocity $\vec{\mathbf{v}}$ through a magnetic field $\vec{\mathbf{B}}$, where $\vec{\mathbf{B}}$ is perpendicular to the bar and to $\vec{\mathbf{v}}$, the **motional emf** induced in the bar is

$$\mathcal{E} = -B\ell v \tag{31.5}$$

**Lenz's law** states that the induced current and induced emf in a conductor are in such a direction as to set up a magnetic field that opposes the change that produced them.

A general form of **Faraday's law of induction** is

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt} \tag{31.9}$$

where $\vec{\mathbf{E}}$ is the nonconservative electric field that is produced by the changing magnetic flux.

## Objective Questions

**1.** denotes answer available in *Student Solutions Manual/Study Guide*

**1.** Figure OQ31.1 is a graph of the magnetic flux through a certain coil of wire as a function of time during an interval while the radius of the coil is increased, the coil is rotated through 1.5 revolutions, and the external source of the magnetic field is turned off, in that order. Rank the emf induced in the coil at the instants marked A through E from the largest positive value to the largest-magnitude negative value. In your ranking,

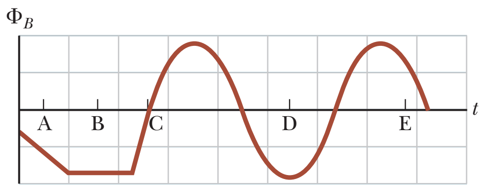



**Figure OQ31.1**

note any cases of equality and also any instants when the emf is zero.

**2.** A flat coil of wire is placed in a uniform magnetic field that is in the $y$ direction. **(i)** The magnetic flux through the coil is a maximum if the plane of the coil is where? More than one answer may be correct. (a) in the $xy$ plane (b) in the $yz$ plane (c) in the $xz$ plane (d) in any orientation, because it is a constant **(ii)** For what orientation is the flux zero? Choose from the same possibilities as in part (i).

**3.** A rectangular conducting loop is placed near a long wire carrying a current $I$ as shown in Figure OQ31.3. If $I$ decreases in time, what can be said of the current induced in the loop? (a) The direction of the current depends on the size of the loop. (b) The current is clockwise. (c) The current is counterclockwise. (d) The current is zero. (e) Nothing can be said about the current in the loop without more information.


**Figure OQ31.3**

**4.** A circular loop of wire with a radius of 4.0 cm is in a uniform magnetic field of magnitude 0.060 T. The plane of the loop is perpendicular to the direction of the magnetic field. In a time interval of 0.50 s, the magnetic field changes to the opposite direction with a magnitude of 0.040 T. What is the magnitude of the average emf induced in the loop? (a) 0.20 V (b) 0.025 V (c) 5.0 mV (d) 1.0 mV (e) 0.20 mV

**5.** A square, flat loop of wire is pulled at constant velocity through a region of uniform magnetic field directed perpendicular to the plane of the loop as shown in Figure OQ31.5. Which of the following statements are correct? More than one statement may be correct. (a) Current is induced in the loop in the clockwise direction. (b) Current is induced in the loop in the counterclockwise direction. (c) No current is induced in the loop. (d) Charge separation occurs in the loop, with the top edge positive. (e) Charge separation occurs in the loop, with the top edge negative.

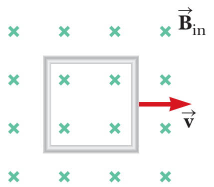



**Figure OQ31.5**

**6.** The bar in Figure OQ31.6 moves on rails to the right with a velocity $\vec{v}$, and a uniform, constant magnetic field is directed out of the page. Which of the following statements are correct? More than one statement may be correct. (a) The induced current in the loop is zero. (b) The induced current in the loop is clockwise. (c) The induced current in the loop is counterclockwise. (d) An external force is required to keep the bar moving at constant speed. (e) No force is required to keep the bar moving at constant speed.

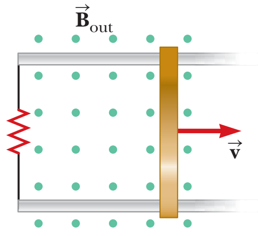



**Figure OQ31.6**

**7.** A bar magnet is held in a vertical orientation above a loop of wire that lies in the horizontal plane as shown in Figure OQ31.7. The south end of the magnet is toward the loop. After the magnet is dropped, what is true of the induced current in the loop as viewed from above? (a) It is clockwise as the magnet falls toward the loop. (b) It is counterclockwise as the magnet falls toward the loop. (c) It is clockwise after the magnet has moved through the loop and moves away from it. (d) It is always clockwise. (e) It is first counterclockwise as the magnet approaches the loop and then clockwise after it has passed through the loop.

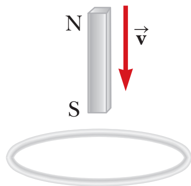



**Figure OQ31.7**

**8.** What happens to the amplitude of the induced emf when the rate of rotation of a generator coil is doubled? (a) It becomes four times larger. (b) It becomes two times larger. (c) It is unchanged. (d) It becomes one-half as large. (e) It becomes one-fourth as large.

**9.** Two coils are placed near each other as shown in Figure OQ31.9. The coil on the left is connected to a battery and a switch, and the coil on the right is connected to a resistor. What is the direction of the cur-

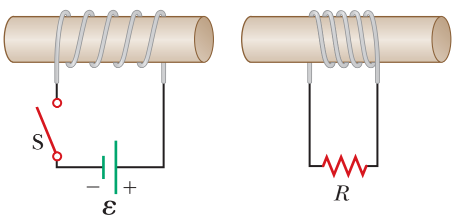



**Figure OQ31.9**

rent in the resistor **(i)** at an instant immediately after the switch is thrown closed, **(ii)** after the switch has been closed for several seconds, and **(iii)** at an instant after the switch has then been thrown open? Choose each answer from the possibilities (a) left, (b) right, or (c) the current is zero.

**10.** A circuit consists of a conducting movable bar and a lightbulb connected to two conducting rails as shown in Figure OQ31.10. An external magnetic field is directed perpendicular to the plane of the circuit. Which of the following actions will make the bulb light up? More than one statement may be correct. (a) The bar is moved to the left. (b) The bar is moved to the right. (c) The magnitude of the magnetic field is increased. (d) The magnitude of the magnetic field is decreased. (e) The bar is lifted off the rails.

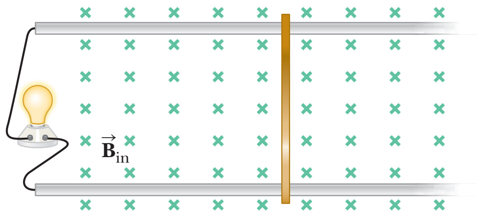



Figure OQ31.10

**11.** Two rectangular loops of wire lie in the same plane as shown in Figure OQ31.11. If the current $I$ in the outer loop is counterclockwise and increases with time, what is true of the current induced in the inner loop? More than one statement may be correct. (a) It is zero. (b) It is clockwise. (c) It is counterclockwise. (d) Its magnitude depends on the dimensions of the loops. (e) Its direction depends on the dimensions of the loops.


Figure OQ31.11

## Conceptual Questions

**1.** denotes answer available in *Student Solutions Manual/Study Guide*

**1.** In Section 7.7, we defined conservative and nonconservative forces. In Chapter 23, we stated that an electric charge creates an electric field that produces a conservative force. Argue now that induction creates an electric field that produces a nonconservative force.

**2.** A spacecraft orbiting the Earth has a coil of wire in it. An astronaut measures a small current in the coil, although there is no battery connected to it and there are no magnets in the spacecraft. What is causing the current?

**3.** In a hydroelectric dam, how is energy produced that is then transferred out by electrical transmission? That is, how is the energy of motion of the water converted to energy that is transmitted by AC electricity?

**4.** A bar magnet is dropped toward a conducting ring lying on the floor. As the magnet falls toward the ring, does it move as a freely falling object? Explain.

**5.** A circular loop of wire is located in a uniform and constant magnetic field. Describe how an emf can be induced in the loop in this situation.

**6.** A piece of aluminum is dropped vertically downward between the poles of an electromagnet. Does the magnetic field affect the velocity of the aluminum?

**7.** What is the difference between magnetic flux and magnetic field?

**8.** When the switch in Figure CQ31.8a is closed, a current is set up in the coil and the metal ring springs upward (Fig. CQ31.8b). Explain this behavior.

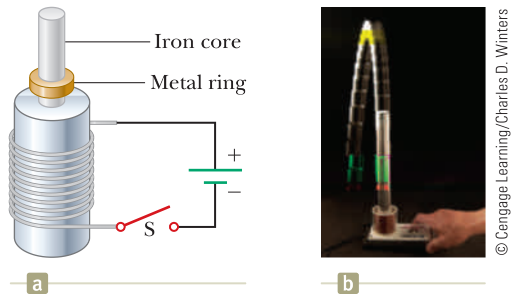



Figure CQ31.8 Conceptual Questions 8 and 9.

**9.** Assume the battery in Figure CQ31.8a is replaced by an AC source and the switch is held closed. If held down, the metal ring on top of the solenoid becomes hot. Why?

**10.** A loop of wire is moving near a long, straight wire carrying a constant current $I$ as shown in Figure CQ31.10. (a) Determine the direction of the induced current in the loop as it moves away from the wire. (b) What would be the direction of the induced current in the loop if it were moving toward the wire?

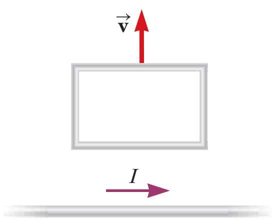



Figure CQ31.10

## Problems

The problems found in this chapter may be assigned online in Enhanced WebAssign

**1.** straightforward; **2.** intermediate; **3.** challenging

**1.** full solution available in the *Student Solutions Manual/Study Guide*

AMT Analysis Model tutorial available in Enhanced WebAssign

GP Guided Problem

M Master It tutorial available in Enhanced WebAssign

W Watch It video solution available in Enhanced WebAssign

### Section 31.1 Faraday's Law of Induction

**1.** A flat loop of wire consisting of a single turn of cross-sectional area 8.00 cm$^2$ is perpendicular to a magnetic field that increases uniformly in magnitude from 0.500 T to 2.50 T in 1.00 s. What is the resulting induced current if the loop has a resistance of 2.00 Ω?

**2.** An instrument based on induced emf has been used to measure projectile speeds up to 6 km/s. A small magnet is imbedded in the projectile as shown in Figure P31.2. The projectile passes through two coils separated by a distance *d*. As the projectile passes through each coil, a pulse of emf is induced in the coil. The time interval between pulses can be measured accurately with an oscilloscope, and thus the speed can be determined. (a) Sketch a graph of $\Delta V$ versus $t$ for the arrangement shown. Consider a current that flows counterclockwise as viewed from the starting point of the projectile as positive. On your graph, indicate which pulse is from coil 1 and which is from coil 2. (b) If the pulse separation is 2.40 ms and $d = 1.50$ m, what is the projectile speed?

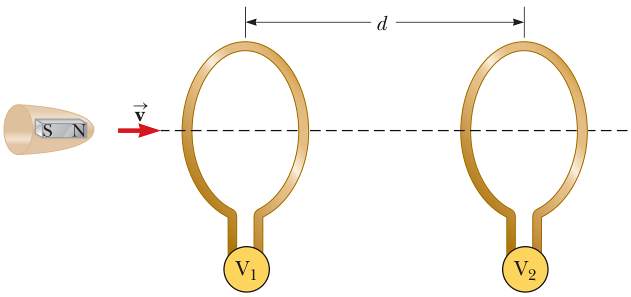



Figure P31.2

**3.** Transcranial magnetic stimulation (TMS) is a noninvasive technique used to stimulate regions of the human brain (Figure P31.3). In TMS, a small coil is placed on the scalp and a brief burst of current in the coil produces a rapidly changing magnetic field inside the brain. The induced emf can stimulate neuronal activity. (a) One such device generates an upward magnetic field within the brain that rises from zero to 1.50 T in 120 ms. Determine the induced emf around a horizontal circle of tissue of radius 1.60 mm. (b) **What If?** The field next changes to 0.500 T downward in 80.0 ms. How does the emf induced in this process compare with that in part (a)?

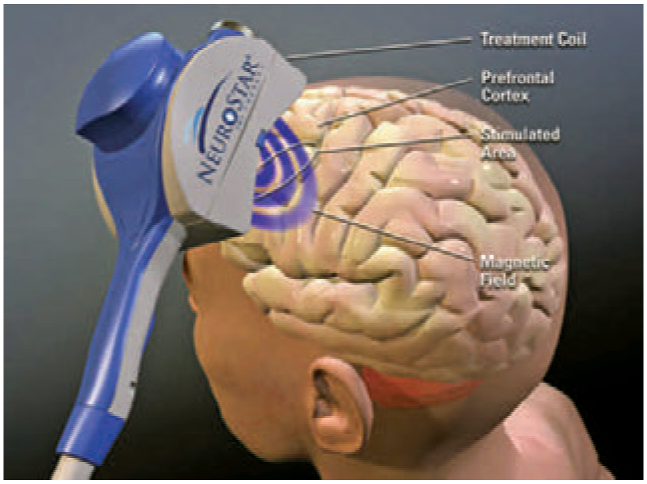





**Figure P31.3** Problems 3 and 51. The magnetic coil of a Neurostar TMS apparatus is held near the head of a patient.

**4.** W A 25-turn circular coil of wire has diameter 1.00 m. It is placed with its axis along the direction of the Earth's magnetic field of 50.0 $\mu$T and then in 0.200 s is flipped 180°. An average emf of what magnitude is generated in the coil?

**5.** The flexible loop in Figure P31.5 has a radius of 12.0 cm and is in a magnetic field of magnitude 0.150 T. The loop is grasped at points *A* and *B* and stretched until its area is nearly zero. If it takes 0.200 s to close the loop, what is the magnitude of the average induced emf in it during this time interval?

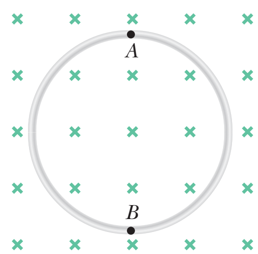



**Figure P31.5** Problems 5 and 6.

**6.** A circular loop of wire of radius 12.0 cm is placed in a magnetic field directed perpendicular to the plane of the loop as in Figure P31.5. If the field decreases at the rate of 0.050 0 T/s in some time interval, find the magnitude of the emf induced in the loop during this interval.

**7.** To monitor the breathing of a hospital patient, a thin belt is girded around the patient's chest. The belt is a 200-turn coil. When the patient inhales, the area encircled by the coil increases by 39.0 cm$^2$. The magnitude of the Earth's magnetic field is 50.0 $\mu$T and makes an angle of 28.0° with the plane of the coil. Assuming a patient takes 1.80 s to inhale, find the average induced emf in the coil during this time interval.

**8.** A strong electromagnet produces a uniform magnetic field of 1.60 T over a cross-sectional area of 0.200 m$^2$. A coil having 200 turns and a total resistance of 20.0 Ω is placed around the electromagnet. The current in the electromagnet is then smoothly reduced until it reaches zero in 20.0 ms. What is the current induced in the coil?

**9.** A 30-turn circular coil of radius 4.00 cm and resistance 1.00 Ω is placed in a magnetic field directed perpendicular to the plane of the coil. The magnitude of the magnetic field varies in time according to the expression $B = 0.010\,0t + 0.040\,0t^2$, where $B$ is in teslas and $t$ is in seconds. Calculate the induced emf in the coil at $t = 5.00$ s.

**10.** Scientific work is currently under way to determine whether weak oscillating magnetic fields can affect human health. For example, one study found that drivers of trains had a higher incidence of blood cancer than other railway workers, possibly due to long exposure to mechanical devices in the train engine cab. Consider a magnetic field of magnitude $1.00 \times 10^{-3}$ T, oscillating sinusoidally at 60.0 Hz. If the diameter of a red blood cell is 8.00 $\mu$m, determine the maximum emf that can be generated around the perimeter of a cell in this field.

**11.** An aluminum ring of radius $r_1 = 5.00$ cm and resistance $3.00 \times 10^{-4}$ Ω is placed around one end of a long air-core solenoid with 1 000 turns per meter and radius $r_2 =$ 3.00 cm as shown in Figure P31.11. Assume the axial component of the field produced by the solenoid is one-half as strong over the area of the end of the solenoid as at the center of the solenoid. Also assume the solenoid produces negligible field outside its cross-sectional area. The current in the solenoid is increasing at a rate of 270 A/s. (a) What is the induced current in the ring? At the center of the ring, what are (b) the magnitude and (c) the direction of the magnetic field produced by the induced current in the ring?

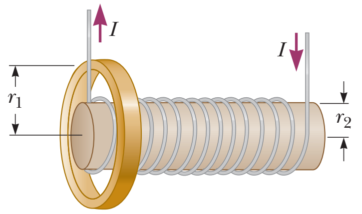



**Figure P31.11** Problems 11 and 12.

**12.** An aluminum ring of radius $r_1$ and resistance $R$ is placed around one end of a long air-core solenoid with $n$ turns per meter and smaller radius $r_2$ as shown in Figure P31.11. Assume the axial component of the field produced by the solenoid over the area of the end of the solenoid is one-half as strong as at the center of the solenoid. Also assume the solenoid produces negligible field outside its cross-sectional area. The current in the solenoid is increasing at a rate of $\Delta I/\Delta t$. (a) What is the induced current in the ring? (b) At the center of the ring, what is the magnetic field produced by the induced current in the ring? (c) What is the direction of this field?

**13.** A loop of wire in the shape of a rectangle of width $w$ and length $L$ and a long, straight wire carrying a current $I$ lie on a tabletop as shown in Figure P31.13. (a) Determine the magnetic flux through the loop due to the current $I$. (b) Suppose the current is changing with time according to $I = a + bt$, where $a$ and $b$ are constants. Determine the emf that is induced in the loop if $b = 10.0$ A/s, $h = 1.00$ cm, $w = 10.0$ cm, and $L = 1.00$ m. (c) What is the direction of the induced current in the rectangle?

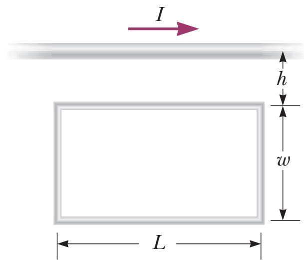



**Figure P31.13**

**14.** A coil of 15 turns and radius 10.0 cm surrounds a long solenoid of radius 2.00 cm and $1.00 \times 10^3$ turns/meter (Fig. P31.14). The current in the solenoid changes as $I = 5.00 \sin 120t$, where $I$ is in amperes and $t$ is in seconds. Find the induced emf in the 15-turn coil as a function of time.

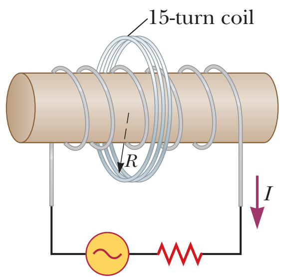



**Figure P31.14**

**15.** A square, single-turn wire loop $\ell = 1.00$ cm on a side is placed inside a solenoid that has a circular cross section of radius $r = 3.00$ cm as shown in the end view of Figure P31.15 (page 960). The solenoid is 20.0 cm long and wound with 100 turns of wire. (a) If the current in the solenoid is 3.00 A, what is the magnetic flux through the square loop? (b) If the current in the solenoid is reduced to zero in 3.00 s, what is the magnitude of the average induced emf in the square loop?

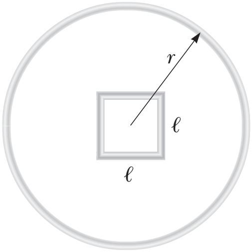



Figure P31.15

**16.** M A long solenoid has $n = 400$ turns per meter and carries a current given by $I = 30.0(1 - e^{-1.60t})$, where $I$ is in amperes and $t$ is in seconds. Inside the solenoid and coaxial with it is a coil that has a radius of $R = 6.00$ cm and consists of a total of $N = 250$ turns of fine wire (Fig. P31.16). What emf is induced in the coil by the changing current?

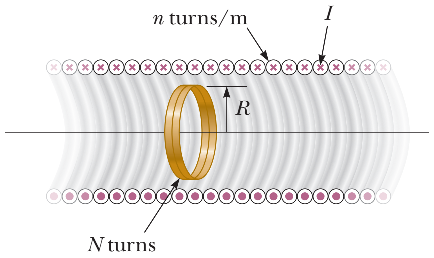



Figure P31.16

**17.** A coil formed by wrapping 50 turns of wire in the shape of a square is positioned in a magnetic field so that the normal to the plane of the coil makes an angle of 30.0° with the direction of the field. When the magnetic field is increased uniformly from 200 $\mu$T to 600 $\mu$T in 0.400 s, an emf of magnitude 80.0 mV is induced in the coil. What is the total length of the wire in the coil?

**18.** When a wire carries an AC current with a known frequency, you can use a *Rogowski coil* to determine the amplitude $I_{max}$ of the current without disconnecting the wire to shunt the current through a meter. The Rogowski coil, shown in Figure P31.18, simply clips around the wire. It consists of a toroidal conductor wrapped around a circular return cord. Let $n$ represent the number of turns in the toroid per unit distance along it. Let $A$ represent the cross-sectional area of the toroid. Let $I(t) = I_{max} \sin \omega t$ represent the current to be measured. (a) Show that the amplitude of the emf induced in the Rogowski coil is $\mathcal{E}_{max} = \mu_0 n A \omega I_{max}$. (b) Explain why the wire carrying the unknown current need not be at the center of the Rogowski coil and why the coil will not respond to nearby currents that it does not enclose.

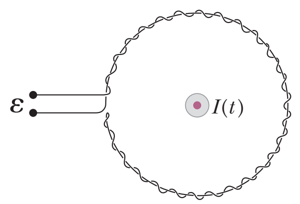



Figure P31.18

**19.** A toroid having a rectangular cross section ($a = 2.00$ cm by $b = 3.00$ cm) and inner radius $R = 4.00$ cm consists of $N = 500$ turns of wire that carry a sinusoidal current $I = I_{max} \sin \omega t$, with $I_{max} = 50.0$ A and a frequency $f = \omega/2\pi = 60.0$ Hz. A coil that consists of $N' = 20$ turns of wire is wrapped around one section of the toroid as shown in Figure P31.19. Determine the emf induced in the coil as a function of time.

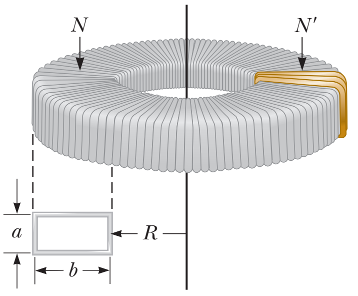



Figure P31.19

**20.** A piece of insulated wire is shaped into a figure eight as shown in Figure P31.20. For simplicity, model the two halves of the figure eight as circles. The radius of the upper circle is 5.00 cm and that of the lower circle is 9.00 cm. The wire has a uniform resistance per unit length of 3.00 Ω/m. A uniform magnetic field is applied perpendicular to the plane of the two circles, in the direction shown. The magnetic field is increasing at a constant rate of 2.00 T/s. Find (a) the magnitude and (b) the direction of the induced current in the wire.

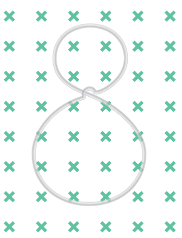


Figure P31.20

## Section 31.2 Motional emf

## Section 31.3 Lenz's Law

Problem 72 in Chapter 29 can be assigned with this section.

**21.** A helicopter (Fig. P31.21) has blades of length 3.00 m, extending out from a central hub and rotating at 2.00 rev/s. If the vertical component of the Earth's magnetic field is 50.0 $\mu$T, what is the emf induced between the blade tip and the center hub?



**Figure P31.21**

**22.** **W** Use Lenz's law to answer the following questions concerning the direction of induced currents. Express your answers in terms of the letter labels $a$ and $b$ in each part of Figure P31.22. (a) What is the direction of the induced current in the resistor $R$ in Figure P31.22a when the bar magnet is moved to the left? (b) What is the direction of the current induced in the resistor $R$ immediately after the switch S in Figure P31.22b is closed? (c) What is the direction of the induced current in the resistor $R$ when the current $I$ in Figure P31.22c decreases rapidly to zero?

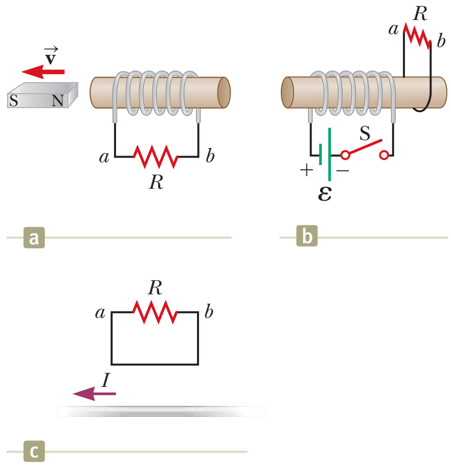



**Figure P31.22**

**23.** A truck is carrying a steel beam of length 15.0 m on a freeway. An accident causes the beam to be dumped off the truck and slide horizontally along the ground at a speed of 25.0 m/s. The velocity of the center of mass of the beam is northward while the length of the beam maintains an east–west orientation. The vertical component of the Earth's magnetic field at this location has a magnitude of 35.0 $\mu$T. What is the magnitude of the induced emf between the ends of the beam?

**24.** A small airplane with a wingspan of 14.0 m is flying due north at a speed of 70.0 m/s over a region where the vertical component of the Earth's magnetic field is 1.20 $\mu$T downward. (a) What potential difference is developed between the airplane's wingtips? (b) Which wingtip is at higher potential? (c) **What If?** How would the answers to parts (a) and (b) change if the plane turned to fly due east? (d) Can this emf be used to power a lightbulb in the passenger compartment? Explain your answer.

**25.** A 2.00-m length of wire is held in an east–west direction and moves horizontally to the north with a speed of 0.500 m/s. The Earth's magnetic field in this region is of magnitude 50.0 $\mu$T and is directed northward and 53.0° below the horizontal. (a) Calculate the magnitude of the induced emf between the ends of the wire and (b) determine which end is positive.

**26.** Consider the arrangement shown in Figure P31.26. Assume that $R = 6.00\ \Omega$, $\ell = 1.20$ m, and a uniform 2.50-T magnetic field is directed into the page. At what speed should the bar be moved to produce a current of 0.500 A in the resistor?

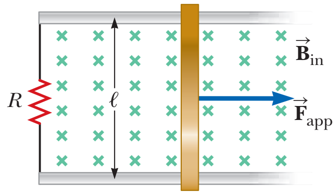



**Figure P31.26** Problems 26 through 29.

**27.** **M** Figure P31.26 shows a top view of a bar that can slide on two frictionless rails. The resistor is $R = 6.00\ \Omega$, and a 2.50-T magnetic field is directed perpendicularly downward, into the paper. Let $\ell = 1.20$ m. (a) Calculate the applied force required to move the bar to the right at a constant speed of 2.00 m/s. (b) At what rate is energy delivered to the resistor?

**28.** A metal rod of mass $m$ slides without friction along two parallel horizontal rails, separated by a distance $\ell$ and connected by a resistor $R$, as shown in Figure P31.26. A uniform vertical magnetic field of magnitude $B$ is applied perpendicular to the plane of the paper. The applied force shown in the figure acts only for a moment, to give the rod a speed $v$. In terms of $m$, $\ell$, $R$, $B$, and $v$, find the distance the rod will then slide as it coasts to a stop.

**29.** A conducting rod of length $\ell$ moves on two horizontal, frictionless rails as shown in Figure P31.26. If a constant force of 1.00 N moves the bar at 2.00 m/s through a magnetic field $\vec{B}$ that is directed into the page, (a) what is the current through the 8.00-$\Omega$ resistor $R$? (b) What is the rate at which energy is delivered to the resistor? (c) What is the mechanical power delivered by the force $\vec{F}_{app}$?

**30.** *Why is the following situation impossible?* An automobile has a vertical radio antenna of length $\ell = 1.20$ m. The automobile travels on a curvy, horizontal road where the Earth's magnetic field has a magnitude of $B = 50.0\ \mu$T and is directed toward the north and downward at an angle of $\theta = 65.0°$ below the horizontal. The motional emf developed between the top and bottom of the antenna varies with the speed and direction of the automobile's travel and has a maximum value of 4.50 mV.

**31.** AMT **Review.** Figure P31.31 shows a bar of mass $m = 0.200$ kg that can slide without friction on a pair of rails separated by a distance $\ell = 1.20$ m and located on an inclined plane that makes an angle $\theta = 25.0°$ with respect to the ground. The resistance of the resistor is $R = 1.00\ \Omega$ and a uniform magnetic field of magnitude $B = 0.500$ T is directed downward, perpendicular to the ground, over the entire region through which the bar moves. With what constant speed $v$ does the bar slide along the rails?

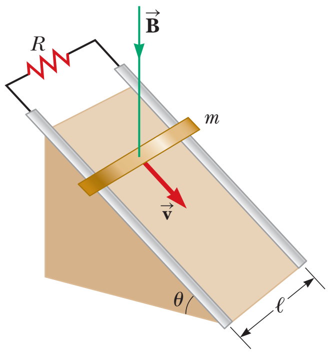



**Figure P31.31** Problems 31 and 32.

**32.** **Review.** Figure P31.31 shows a bar of mass $m$ that can slide without friction on a pair of rails separated by a distance $\ell$ and located on an inclined plane that makes an angle $\theta$ with respect to the ground. The resistance of the resistor is $R$, and a uniform magnetic field of magnitude $B$ is directed downward, perpendicular to the ground, over the entire region through which the bar moves. With what constant speed $v$ does the bar slide along the rails?

**33.** M The *homopolar generator,* also called the *Faraday disk,* is a low-voltage, high-current electric generator. It consists of a rotating conducting disk with one stationary brush (a sliding electrical contact) at its axle and another at a point on its circumference as shown in Figure P31.33. A uniform magnetic field is applied perpendicular to the plane of the disk. Assume the field is 0.900 T, the angular speed is $3.20 \times 10^3$ rev/min, and the radius of the disk is 0.400 m. Find the emf generated between the brushes. When superconducting coils are used to produce a large magnetic field, a homopolar generator can have a power output of several megawatts. Such a generator is useful, for example, in purifying metals by electrolysis. If a voltage is applied to the output terminals of the generator, it runs in reverse as a *homopolar motor* capable of providing great torque, useful in ship propulsion.

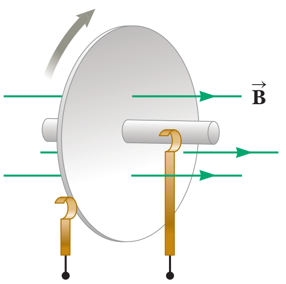



**Figure P31.33**

**34.** A conducting bar of length $\ell$ moves to the right on two frictionless rails as shown in Figure P31.34. A uniform magnetic field directed into the page has a magnitude of 0.300 T. Assume $R = 9.00\ \Omega$ and $\ell = 0.350$ m. (a) At what constant speed should the bar move to produce an 8.50-mA current in the resistor? (b) What is the direction of the induced current? (c) At what rate is energy delivered to the resistor? (d) Explain the origin of the energy being delivered to the resistor.

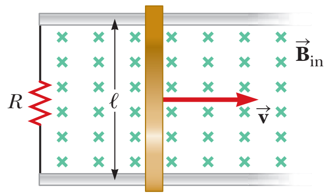



**Figure P31.34**

**35.** AMT **Review.** After removing one string while restringing his acoustic guitar, a student is distracted by a video game. His experimentalist roommate notices his inattention and attaches one end of the string, of linear density $\mu = 3.00 \times 10^{-3}$ kg/m, to a rigid support. The other end passes over a pulley, a distance $\ell = 64.0$ cm from the fixed end, and an object of mass $m = 27.2$ kg is attached to the hanging end of the string. The roommate places a magnet across the string as shown in Figure P31.35. The magnet does not touch the string, but produces a uniform field of 4.50 mT over a 2.00-cm length of the string and negligible field elsewhere. Strumming the string sets it vibrating vertically at its fundamental (lowest) frequency. The section of the string in the magnetic field moves perpendicular to the field with a uniform amplitude of 1.50 cm. Find (a) the frequency and (b) the amplitude of the emf induced between the ends of the string.

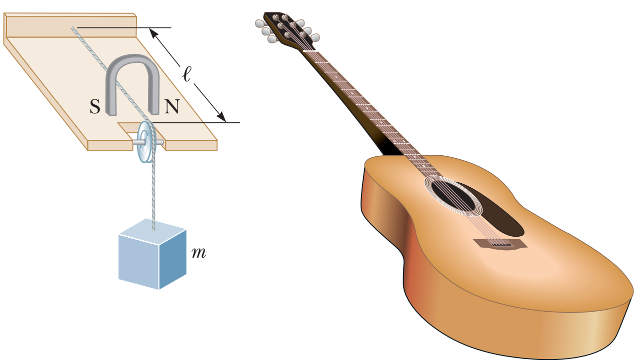



**Figure P31.35**

**36.** A rectangular coil with resistance $R$ has $N$ turns, each of length $\ell$ and width $w$ as shown in Figure P31.36. The coil moves into a uniform magnetic field $\vec{\mathbf{B}}$ with constant velocity $\vec{\mathbf{v}}$. What are the magnitude and direction of the total magnetic force on the coil (a) as it enters the magnetic field, (b) as it moves within the field, and (c) as it leaves the field?

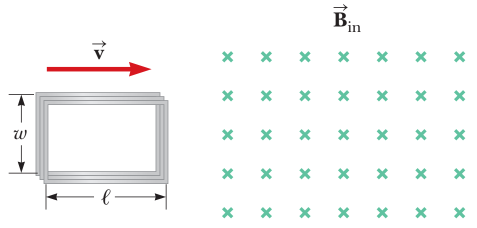



Figure P31.36

**37.** Two parallel rails with negligible resistance are 10.0 cm apart and are connected by a resistor of resistance $R_3 = 5.00\ \Omega$. The circuit also contains two metal rods having resistances of $R_1 = 10.0\ \Omega$ and $R_2 = 15.0\ \Omega$ sliding along the rails (Fig. P31.37). The rods are pulled away from the resistor at constant speeds of $v_1 = 4.00$ m/s and $v_2 = 2.00$ m/s, respectively. A uniform magnetic field of magnitude $B = 0.010\ 0$ T is applied perpendicular to the plane of the rails. Determine the current in $R_3$.

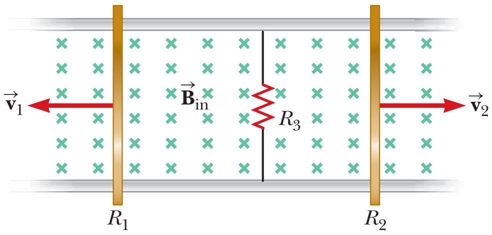



Figure P31.37

**38.** An astronaut is connected to her spacecraft by a 25.0-m-long tether cord as she and the spacecraft orbit the Earth in a circular path at a speed of $7.80 \times 10^3$ m/s. At one instant, the emf between the ends of a wire embedded in the cord is measured to be 1.17 V. Assume the long dimension of the cord is perpendicular to the Earth's magnetic field at that instant. Assume also the tether's center of mass moves with a velocity perpendicular to the Earth's magnetic field. (a) What is the magnitude of the Earth's field at this location? (b) Does the emf change as the system moves from one location to another? Explain. (c) Provide two conditions under which the emf would be zero even though the magnetic field is not zero.

## Section 31.4 Induced emf and Electric Fields

**39.** Within the green dashed circle shown in Figure P31.39, the magnetic field changes with time according to the expression $B = 2.00t^3 - 4.00t^2 + 0.800$, where $B$ is in teslas, $t$ is in seconds, and $R = 2.50$ cm. When $t = 2.00$ s, calculate (a) the magnitude and (b) the direction of the force exerted on an electron located at point $P_1$, which is at a distance $r_1 = 5.00$ cm from the center of the circular field region. (c) At what instant is this force equal to zero?

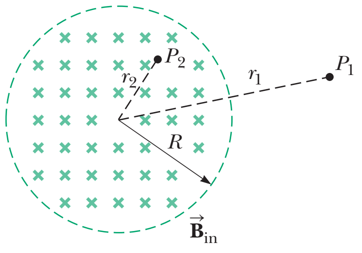



Figure P31.39 Problems 39 and 40.

**40.** M A magnetic field directed into the page changes with time according to $B = 0.030\ 0t^2 + 1.40$, where $B$ is in teslas and $t$ is in seconds. The field has a circular cross section of radius $R = 2.50$ cm (see Fig. P31.39). When $t = 3.00$ s and $r_2 = 0.020\ 0$ m, what are (a) the magnitude and (b) the direction of the electric field at point $P_2$?

**41.** A long solenoid with $1.00 \times 10^3$ turns per meter and radius 2.00 cm carries an oscillating current $I = 5.00 \sin 100\pi t$, where $I$ is in amperes and $t$ is in seconds. (a) What is the electric field induced at a radius $r = 1.00$ cm from the axis of the solenoid? (b) What is the direction of this electric field when the current is increasing counterclockwise in the solenoid?

## Section 31.5 Generators and Motors

Problems 50 and 68 in Chapter 29 can be assigned with this section.

**42.** A 100-turn square coil of side 20.0 cm rotates about a vertical axis at $1.50 \times 10^3$ rev/min as indicated in Figure P31.42. The horizontal component of the Earth's magnetic field at the coil's location is equal to $2.00 \times 10^{-5}$ T. (a) Calculate the maximum emf induced in the coil by this field. (b) What is the orientation of the coil with respect to the magnetic field when the maximum emf occurs?

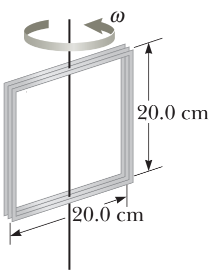



Figure P31.42

**43.** A generator produces 24.0 V when turning at 900 rev/min. What emf does it produce when turning at 500 rev/min?

**44.** Figure P31.44 (page 964) is a graph of the induced emf versus time for a coil of $N$ turns rotating with angular speed $\omega$ in a uniform magnetic field directed perpendicular to the coil's axis of rotation. **What If?** Copy this sketch (on a larger scale) and on the same set of axes show the graph of emf versus $t$ (a) if the number of turns in the coil is doubled, (b) if instead the angular speed is doubled, and (c) if the angular speed is doubled while the number of turns in the coil is halved.

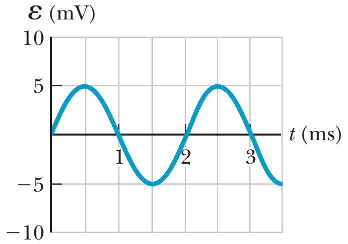



Figure P31.44

**45.** In a 250-turn automobile alternator, the magnetic flux in each turn is $\Phi_B = 2.50 \times 10^{-4} \cos \omega t$, where $\Phi_B$ is in webers, $\omega$ is the angular speed of the alternator, and $t$ is in seconds. The alternator is geared to rotate three times for each engine revolution. When the engine is running at an angular speed of $1.00 \times 10^3$ rev/min, determine (a) the induced emf in the alternator as a function of time and (b) the maximum emf in the alternator.

**46.** In Figure P31.46, a semicircular conductor of radius $R = 0.250$ m is rotated about the axis $AC$ at a constant rate of 120 rev/min. A uniform magnetic field of magnitude 1.30 T fills the entire region below the axis and is directed out of the page. (a) Calculate the maximum value of the emf induced between the ends of the conductor. (b) What is the value of the average induced emf for each complete rotation? (c) **What If?** How would your answers to parts (a) and (b) change if the magnetic field were allowed to extend a distance $R$ above the axis of rotation? Sketch the emf versus time (d) when the field is as drawn in Figure P31.46 and (e) when the field is extended as described in part (c).

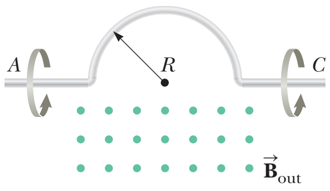



Figure P31.46

**47.** A long solenoid, with its axis along the $x$ axis, consists of 200 turns per meter of wire that carries a steady current of 15.0 A. A coil is formed by wrapping 30 turns of thin wire around a circular frame that has a radius of 8.00 cm. The coil is placed inside the solenoid and mounted on an axis that is a diameter of the coil and coincides with the $y$ axis. The coil is then rotated with an angular speed of $4.00\pi$ rad/s. The plane of the coil is in the $yz$ plane at $t = 0$. Determine the emf generated in the coil as a function of time.

**48.** A motor in normal operation carries a direct current of 0.850 A when connected to a 120-V power supply. The resistance of the motor windings is 11.8 Ω. While in normal operation, (a) what is the back emf generated by the motor? (b) At what rate is internal energy produced in the windings? (c) **What If?** Suppose a malfunction stops the motor shaft from rotating. At what rate will internal energy be produced in the windings in this case? (Most motors have a thermal switch that will turn off the motor to prevent overheating when this stalling occurs.)

**49.** The rotating loop in an AC generator is a square 10.0 cm on each side. It is rotated at 60.0 Hz in a uniform field of 0.800 T. Calculate (a) the flux through the loop as a function of time, (b) the emf induced in the loop, (c) the current induced in the loop for a loop resistance of 1.00 Ω, (d) the power delivered to the loop, and (e) the torque that must be exerted to rotate the loop.

## Section 31.6 Eddy Currents

**50.** Figure P31.50 represents an electromagnetic brake that uses eddy currents. An electromagnet hangs from a railroad car near one rail. To stop the car, a large current is sent through the coils of the electromagnet. The moving electromagnet induces eddy currents in the rails, whose fields oppose the change in the electromagnet's field. The magnetic fields of the eddy currents exert force on the current in the electromagnet, thereby slowing the car. The direction of the car's motion and the direction of the current in the electromagnet are shown correctly in the picture. Determine which of the eddy currents shown on the rails is correct. Explain your answer.

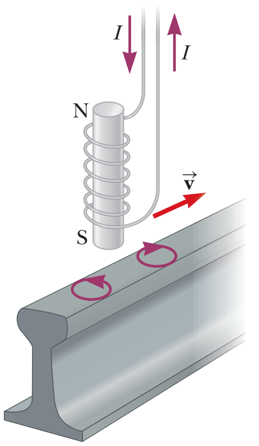



Figure P31.50

## Additional Problems

**51.** Consider a transcranial magnetic stimulation (TMS) device (Figure P31.3) containing a coil with several turns of wire, each of radius 6.00 cm. In a circular area of the brain of radius 6.00 cm directly below and coaxial with the coil, the magnetic field changes at the rate of $1.00 \times 10^4$ T/s. Assume that this rate of change is the same everywhere inside the circular area. (a) What is the emf induced around the circumference of this circular area in the brain? (b) What electric field is induced on the circumference of this circular area?

**52.** Suppose you wrap wire onto the core from a roll of cellophane tape to make a coil. Describe how you can use a bar magnet to produce an induced voltage in the coil. What is the order of magnitude of the emf you generate? State the quantities you take as data and their values.

**53.** M A circular coil enclosing an area of 100 $cm^2$ is made of 200 turns of copper wire (Figure P31.53). The wire making up the coil has no resistance; the ends of the wire are connected across a 5.00-Ω resistor to form a closed circuit. Initially, a 1.10-T uniform magnetic field points perpendicularly upward through the plane of the coil. The direction of the field then reverses so that the final magnetic field has a magnitude of 1.10 T and points downward through the coil. If the time interval required for the field to reverse directions is 0.100 s, what is the average current in the coil during that time?

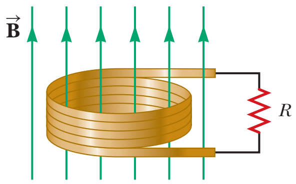


Figure P31.53

**54.** A circular loop of wire of resistance $R$ = 0.500 Ω and radius $r$ = 8.00 cm is in a uniform magnetic field directed out of the page as in Figure P31.54. If a clockwise current of $I$ = 2.50 mA is induced in the loop, (a) is the magnetic field increasing or decreasing in time? (b) Find the rate at which the field is changing with time.

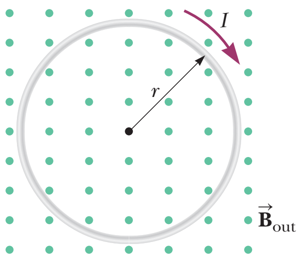


Figure P31.54

**55.** A rectangular loop of area $A$ = 0.160 $m^2$ is placed in a region where the magnetic field is perpendicular to the plane of the loop. The magnitude of the field is allowed to vary in time according to $B = 0.350\ e^{-t/2.00}$, where $B$ is in teslas and $t$ is in seconds. The field has the constant value 0.350 T for $t < 0$. What is the value for $\mathcal{E}$ at $t$ = 4.00 s?

**56.** A rectangular loop of area $A$ is placed in a region where the magnetic field is perpendicular to the plane of the loop. The magnitude of the field is allowed to vary in time according to $B = B_{max}e^{-t/\tau}$, where $B_{max}$ and $\tau$ are constants. The field has the constant value $B_{max}$ for $t < 0$. Find the emf induced in the loop as a function of time.

**57.** Strong magnetic fields are used in such medical procedures as magnetic resonance imaging, or MRI. A technician wearing a brass bracelet enclosing area 0.005 00 $m^2$ places her hand in a solenoid whose magnetic field is 5.00 T directed perpendicular to the plane of the bracelet. The electrical resistance around the bracelet's circumference is 0.020 0 Ω. An unexpected power failure causes the field to drop to 1.50 T in a time interval of 20.0 ms. Find (a) the current induced in the bracelet and (b) the power delivered to the bracelet. *Note:* As this problem implies, you should not wear any metal objects when working in regions of strong magnetic fields.

**58.** GP Consider the apparatus shown in Figure P31.58 in which a conducting bar can be moved along two rails connected to a lightbulb. The whole system is immersed in a magnetic field of magnitude $B$ = 0.400 T perpendicular and into the page. The distance between the horizontal rails is $\ell$ = 0.800 m. The resistance of the lightbulb is $R$ = 48.0 Ω, assumed to be constant. The bar and rails have negligible resistance. The bar is moved toward the right by a constant force of magnitude $F$ = 0.600 N. We wish to find the maximum power delivered to the lightbulb. (a) Find an expression for the current in the lightbulb as a function of $B$, $\ell$, $R$, and $v$, the speed of the bar. (b) When the maximum power is delivered to the lightbulb, what analysis model properly describes the moving bar? (c) Use the analysis model in part (b) to find a numerical value for the speed $v$ of the bar when the maximum power is being delivered to the lightbulb. (d) Find the current in the lightbulb when maximum power is being delivered to it. (e) Using $P = I^2R$, what is the maximum power delivered to the lightbulb? (f) What is the maximum mechanical input power delivered to the bar by the force $F$? (g) We have assumed the resistance of the lightbulb is constant. In reality, as the power delivered to the lightbulb increases, the filament temperature increases and the resistance increases. Does the speed found in part (c) change if the resistance increases and all other quantities are held constant? (h) If so, does the speed found in part (c) increase or decrease? If not, explain. (i) With the assumption that the resistance of the lightbulb increases as the current increases, does the power found in part (f) change? (j) If so, is the power found in part (f) larger or smaller? If not, explain.

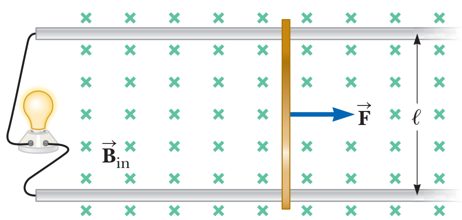


Figure P31.58

**59.** A guitar's steel string vibrates (see Fig. 31.5). The component of magnetic field perpendicular to the area of a pickup coil nearby is given by

$$B = 50.0 + 3.20 \sin 1\,046\pi t$$

where $B$ is in milliteslas and $t$ is in seconds. The circular pickup coil has 30 turns and radius 2.70 mm. Find the emf induced in the coil as a function of time.

**60.** AMT *Why is the following situation impossible?* A conducting rectangular loop of mass $M = 0.100$ kg, resistance $R = 1.00\ \Omega$, and dimensions $w = 50.0$ cm by $\ell = 90.0$ cm is held with its lower edge just above a region with a uniform magnetic field of magnitude $B = 1.00$ T as shown in Figure P31.60. The loop is released from rest. Just as the top edge of the loop reaches the region containing the field, the loop moves with a speed 4.00 m/s.

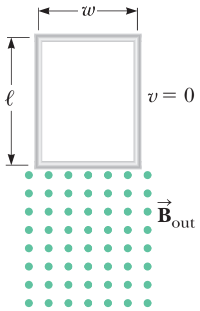


Figure P31.60

**61.** W The circuit in Figure P31.61 is located in a magnetic field whose magnitude varies with time according to the expression $B = 1.00 \times 10^{-3}\ t$, where $B$ is in teslas and $t$ is in seconds. Assume the resistance per length of the wire is 0.100 Ω/m. Find the current in section $PQ$ of length $a = 65.0$ cm.

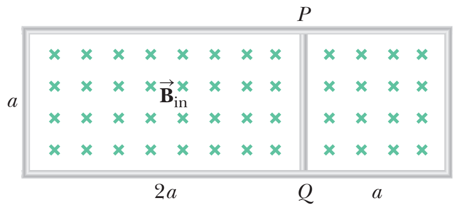


Figure P31.61

**62.** Magnetic field values are often determined by using a device known as a *search coil.* This technique depends on the measurement of the total charge passing through a coil in a time interval during which the magnetic flux linking the windings changes either because of the coil's motion or because of a change in the value of $B$. (a) Show that as the flux through the coil changes from $\Phi_1$ to $\Phi_2$, the charge transferred through the coil is given by $Q = N(\Phi_2 - \Phi_1)/R$, where $R$ is the resistance of the coil and $N$ is the number of turns. (b) As a specific example, calculate $B$ when a total charge of $5.00 \times 10^{-4}$ C passes through a 100-turn coil of resistance 200 Ω and cross-sectional area 40.0 cm$^2$ as it is rotated in a uniform field from a position where the plane of the coil is perpendicular to the field to a position where it is parallel to the field.

**63.** A conducting rod of length $\ell = 35.0$ cm is free to slide on two parallel conducting bars as shown in Figure P31.63. Two resistors $R_1 = 2.00\ \Omega$ and $R_2 = 5.00\ \Omega$ are connected across the ends of the bars to form a loop. A constant magnetic field $B = 2.50$ T is directed perpendicularly into the page. An external agent pulls the rod to the left with a constant speed of $v = 8.00$ m/s. Find (a) the currents in both resistors, (b) the total power delivered to the resistance of the circuit, and (c) the magnitude of the applied force that is needed to move the rod with this constant velocity.

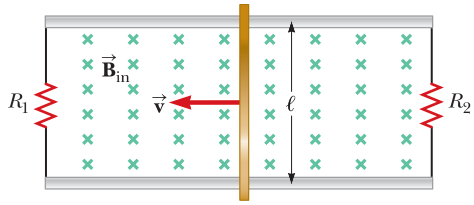


Figure P31.63

**64.** **Review.** A particle with a mass of $2.00 \times 10^{-16}$ kg and a charge of 30.0 nC starts from rest, is accelerated through a potential difference $\Delta V$, and is fired from a small source in a region containing a uniform, constant magnetic field of magnitude 0.600 T. The particle's velocity is perpendicular to the magnetic field lines. The circular orbit of the particle as it returns to the location of the source encloses a magnetic flux of 15.0 μWb. (a) Calculate the particle's speed. (b) Calculate the potential difference through which the particle was accelerated inside the source.

**65.** M The plane of a square loop of wire with edge length $a = 0.200$ m is oriented vertically and along an east–west axis. The Earth's magnetic field at this point is of magnitude $B = 35.0\ \mu$T and is directed northward at 35.0° below the horizontal. The total resistance of the loop and the wires connecting it to a sensitive ammeter is 0.500 Ω. If the loop is suddenly collapsed by horizontal forces as shown in Figure P31.65, what total charge enters one terminal of the ammeter?

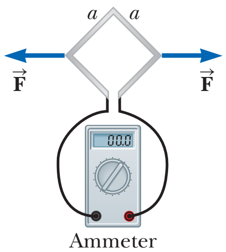


Figure P31.65

**66.** In Figure P31.66, the rolling axle, 1.50 m long, is pushed along horizontal rails at a constant speed $v = 3.00$ m/s. A resistor $R = 0.400\ \Omega$ is connected to the rails at points $a$ and $b$, directly opposite each other.

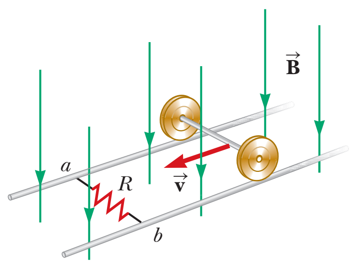


Figure P31.66

The wheels make good electrical contact with the rails, so the axle, rails, and $R$ form a closed-loop circuit. The only significant resistance in the circuit is $R$. A uniform magnetic field $B = 0.080\ 0$ T is vertically downward. (a) Find the induced current $I$ in the resistor. (b) What horizontal force $F$ is required to keep the axle rolling at constant speed? (c) Which end of the resistor, $a$ or $b$, is at the higher electric potential? (d) **What If?** After the axle rolls past the resistor, does the current in $R$ reverse direction? Explain your answer.

**67.** Figure P31.67 shows a stationary conductor whose shape is similar to the letter e. The radius of its circular portion is $a = 50.0$ cm. It is placed in a constant magnetic field of 0.500 T directed out of the page. A straight conducting rod, 50.0 cm long, is pivoted about point $O$ and rotates with a constant angular speed of 2.00 rad/s. (a) Determine the induced emf in the loop $POQ$. Note that the area of the loop is $\theta a^2/2$. (b) If all the conducting material has a resistance per length of 5.00 Ω/m, what is the induced current in the loop $POQ$ at the instant 0.250 s after point $P$ passes point $Q$?

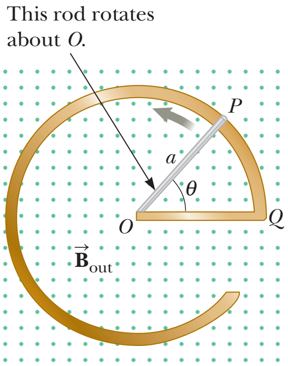



Figure P31.67

**68.** A conducting rod moves with a constant velocity in a direction perpendicular to a long, straight wire carrying a current $I$ as shown in Figure P31.68. Show that the magnitude of the emf generated between the ends of the rod is

$$|\mathcal{E}| = \frac{\mu_0 v I \ell}{2\pi r}$$

In this case, note that the emf decreases with increasing $r$ as you might expect.

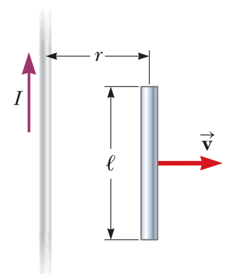



Figure P31.68

**69.** AMT A small, circular washer of radius $a = 0.500$ cm is held directly below a long, straight wire carrying a current of $I = 10.0$ A. The washer is located $h = 0.500$ m above the top of a table (Fig. P31.69). Assume the magnetic field is nearly constant over the area of the washer and equal to the magnetic field at the center of the washer. (a) If the washer is dropped from rest, what is the magnitude of the average induced emf in the washer over the time interval between its release and the moment it hits the tabletop? (b) What is the direction of the induced current in the washer?

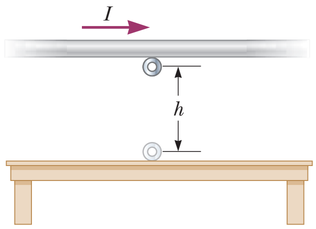



Figure P31.69

**70.** Figure P31.70 shows a compact, circular coil with 220 turns and radius 12.0 cm immersed in a uniform magnetic field parallel to the axis of the coil. The rate of change of the field has the constant magnitude 20.0 mT/s. (a) What additional information is necessary to determine whether the coil is carrying clockwise or counterclockwise current? (b) The coil overheats if more than 160 W of power is delivered to it. What resistance would the coil have at this critical point? (c) To run cooler, should it have lower resistance or higher resistance?

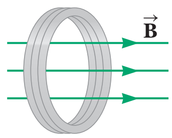



Figure P31.70

**71.** A rectangular coil of 60 turns, dimensions 0.100 m by 0.200 m, and total resistance 10.0 Ω rotates with angular speed 30.0 rad/s about the $y$ axis in a region where a 1.00-T magnetic field is directed along the $x$ axis. The time $t = 0$ is chosen to be at an instant when the plane of the coil is perpendicular to the direction of $\vec{\mathbf{B}}$. Calculate (a) the maximum induced emf in the coil, (b) the maximum rate of change of magnetic flux through the coil, (c) the induced emf at $t = 0.050\ 0$ s, and (d) the torque exerted by the magnetic field on the coil at the instant when the emf is a maximum.

**72.** **Review.** In Figure P31.72, a uniform magnetic field decreases at a constant rate $dB/dt = -K$, where $K$ is a positive constant. A circular loop of wire of radius $a$ containing a resistance $R$ and a capacitance $C$ is placed

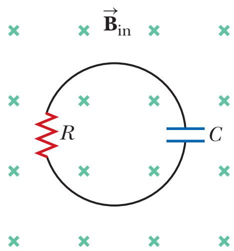



Figure P31.72

with its plane normal to the field. (a) Find the charge $Q$ on the capacitor when it is fully charged. (b) Which plate, upper or lower, is at the higher potential? (c) Discuss the force that causes the separation of charges.

**73.** An $N$-turn square coil with side $\ell$ and resistance $R$ is pulled to the right at constant speed $v$ in the presence of a uniform magnetic field $B$ acting perpendicular to the coil as shown in Figure P31.73. At $t = 0$, the right side of the coil has just departed the right edge of the field. At time $t$, the left side of the coil enters the region where $B = 0$. In terms of the quantities $N$, $B$, $\ell$, $v$, and $R$, find symbolic expressions for (a) the magnitude of the induced emf in the loop during the time interval from $t = 0$ to $t$, (b) the magnitude of the induced current in the coil, (c) the power delivered to the coil, and (d) the force required to remove the coil from the field. (e) What is the direction of the induced current in the loop? (f) What is the direction of the magnetic force on the loop while it is being pulled out of the field?

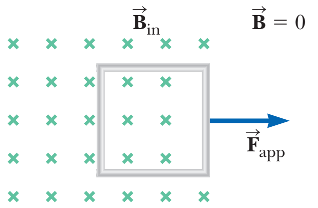



Figure P31.73

**74.** A conducting rod of length $\ell$ moves with velocity $\vec{\mathbf{v}}$ parallel to a long wire carrying a steady current $I$. The axis of the rod is maintained perpendicular to the wire with the near end a distance $r$ away (Fig. P31.74). Show that the magnitude of the emf induced in the rod is

$$|\mathcal{E}| = \frac{\mu_0 I v}{2\pi} \ln\left(1 + \frac{\ell}{r}\right)$$

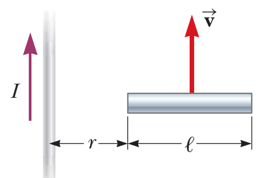



Figure P31.74

**75.** [M] The magnetic flux through a metal ring varies with time $t$ according to $\Phi_B = at^3 - bt^2$, where $\Phi_B$ is in webers, $a = 6.00\ \text{s}^{-3}$, $b = 18.0\ \text{s}^{-2}$, and $t$ is in seconds. The resistance of the ring is $3.00\ \Omega$. For the interval from $t = 0$ to $t = 2.00$ s, determine the maximum current induced in the ring.

**76.** A rectangular loop of dimensions $\ell$ and $w$ moves with a constant velocity $\vec{\mathbf{v}}$ away from a long wire that carries a current $I$ in the plane of the loop (Fig. P31.76). The total resistance of the loop is $R$. Derive an expression that gives the current in the loop at the instant the near side is a distance $r$ from the wire.

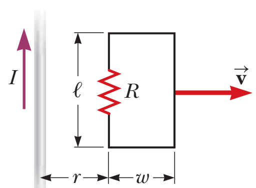



Figure P31.76

**77.** [M] A long, straight wire carries a current given by $I = I_{\text{max}} \sin(\omega t + \phi)$. The wire lies in the plane of a rectangular coil of $N$ turns of wire as shown in Figure P31.77. The quantities $I_{\text{max}}$, $\omega$, and $\phi$ are all constants. Assume $I_{\text{max}} = 50.0$ A, $\omega = 200\pi\ \text{s}^{-1}$, $N = 100$, $h = w = 5.00$ cm, and $L = 20.0$ cm. Determine the emf induced in the coil by the magnetic field created by the current in the straight wire.

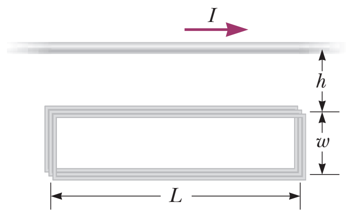



Figure P31.77

**78.** A thin wire $\ell = 30.0$ cm long is held parallel to and $d = 80.0$ cm above a long, thin wire carrying $I = 200$ A and fixed in position (Fig. P31.78). The 30.0-cm wire is released at the instant $t = 0$ and falls, remaining parallel to the current-carrying wire as it falls. Assume the falling wire accelerates at $9.80\ \text{m/s}^2$. (a) Derive an equation for the emf induced in it as a function of time. (b) What is the minimum value of the emf? (c) What is the maximum value? (d) What is the induced emf 0.300 s after the wire is released?

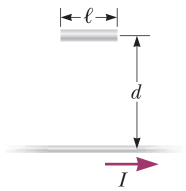



Figure P31.78

## Challenge Problems

**79.** Two infinitely long solenoids (seen in cross section) pass through a circuit as shown in Figure P31.79. The magnitude of $\vec{\mathbf{B}}$ inside each is the same and is increasing at the rate of 100 T/s. What is the current in each resistor?

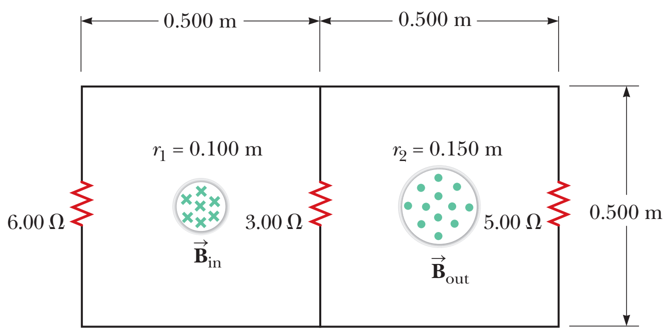



Figure P31.79

**80.** An *induction furnace* uses electromagnetic induction to produce eddy currents in a conductor, thereby raising the conductor's temperature. Commercial units operate at frequencies ranging from 60 Hz to about 1 MHz and deliver powers from a few watts to several megawatts. Induction heating can be used for warming a metal pan on a kitchen stove. It can be used to avoid oxidation and contamination of the metal when welding in a vacuum enclosure. To explore induction heating, consider a flat conducting disk of radius $R$, thickness $b$, and resistivity $\rho$. A sinusoidal magnetic field $B_{\max} \cos \omega t$ is applied perpendicular to the disk. Assume the eddy currents occur in circles concentric with the disk. (a) Calculate the average power delivered to the disk. (b) **What If?** By what factor does the power change when the amplitude of the field doubles? (c) When the frequency doubles? (d) When the radius of the disk doubles?

**81.** A bar of mass $m$ and resistance $R$ slides without friction in a horizontal plane, moving on parallel rails as shown in Figure P31.81. The rails are separated by a distance $d$. A battery that maintains a constant emf $\mathcal{E}$ is connected between the rails, and a constant magnetic field $\vec{\mathbf{B}}$ is directed perpendicularly out of the page. Assuming the bar starts from rest at time $t = 0$, show that at time $t$ it moves with a speed

$$v = \frac{\mathcal{E}}{Bd}\left(1 - e^{-B^2 d^2 t/mR}\right)$$

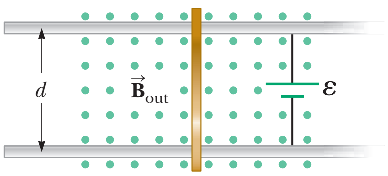



Figure P31.81

**82.** A *betatron* is a device that accelerates electrons to energies in the MeV range by means of electromagnetic induction. Electrons in a vacuum chamber are held in a circular orbit by a magnetic field perpendicular to the orbital plane. The magnetic field is gradually increased to induce an electric field around the orbit. (a) Show that the electric field is in the correct direction to make the electrons speed up. (b) Assume the radius of the orbit remains constant. Show that the average magnetic field over the area enclosed by the orbit must be twice as large as the magnetic field at the circle's circumference.

**83.** **Review.** The bar of mass $m$ in Figure P31.83 is pulled horizontally across parallel, frictionless rails by a massless string that passes over a light, frictionless pulley and is attached to a suspended object of mass $M$. The uniform upward magnetic field has a magnitude $B$, and the distance between the rails is $\ell$. The only significant electrical resistance is the load resistor $R$ shown connecting the rails at one end. Assuming the suspended object is released with the bar at rest at $t = 0$, derive an expression that gives the bar's horizontal speed as a function of time.

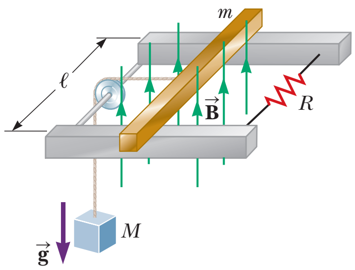



Figure P31.83