CHAPTER

27

Current and Resistance

These two lightbulbs provide similar power output by visible light (electromagnetic radiation). The compact fluorescent bulb on the left, however, produces this light output with far less input by electrical transmission than the incandescent bulb on the right. The fluorescent bulb, therefore, is less costly to operate and saves valuable resources needed to generate electricity. *(Christina Richards/Shutterstock.com)*

We now consider situations involving electric charges that are in motion through some region of space. We use the term *electric current,* or simply *current,* to describe the rate of flow of charge. Most practical applications of electricity deal with electric currents, including a variety of home appliances. For example, the voltage from a wall plug produces a current in the coils of a toaster when it is turned on. In these common situations, current exists in a conductor such as a copper wire. Currents can also exist outside a conductor. For instance, a beam of electrons in a particle accelerator constitutes a current.

This chapter begins with the definition of current. A microscopic description of current is given, and some factors that contribute to the opposition to the flow of charge in conductors are discussed. A classical model is used to describe electrical conduction in metals, and some limitations of this model are cited. We also define electrical resistance and introduce a new circuit element, the resistor. We conclude by discussing the rate at which energy is transferred to a device in an electric circuit. The energy transfer mechanism in Equation 8.2 that corresponds to this process is electrical transmission T_{ET}.

27.1 Electric Current

In this section, we study the flow of electric charges through a piece of material. The amount of flow depends on both the material through which the charges are passing and the potential difference across the material. Whenever there is a net flow of charge through some region, an electric *current* is said to exist.

It is instructive to draw an analogy between water flow and current. The flow of water in a plumbing pipe can be quantified by specifying the amount of water that emerges from a faucet during a given time interval, often measured in liters per minute. A river current can be characterized by describing the rate at which the water flows past a particular location. For example, the flow over the brink at Niagara Falls is maintained at rates between 1 400 m^3/s and 2 800 m^3/s.

There is also an analogy between thermal conduction and current. In Section 20.7, we discussed the flow of energy by heat through a sample of material. The rate of energy flow is determined by the material as well as the temperature difference across the material as described by Equation 20.15.

To define current quantitatively, suppose charges are moving perpendicular to a surface of area A as shown in Figure 27.1. (This area could be the cross-sectional area of a wire, for example.) The **current** is defined as the rate at which charge flows through this surface. If ΔQ is the amount of charge that passes through this surface in a time interval Δt, the **average current** I_{avg} is equal to the charge that passes through A per unit time:

$$I_{avg} = \frac{\Delta Q}{\Delta t} \quad (27.1)$$

If the rate at which charge flows varies in time, the current varies in time; we define the **instantaneous current** I as the limit of the average current as $\Delta t \to 0$:

$$I \equiv \frac{dQ}{dt} \quad (27.2)$$

◀ Electric current

The SI unit of current is the **ampere** (A):

$$1 \text{ A} = 1 \text{ C/s} \quad (27.3)$$

That is, 1 A of current is equivalent to 1 C of charge passing through a surface in 1 s.

The charged particles passing through the surface in Figure 27.1 can be positive, negative, or both. It is conventional to assign to the current the same direction as the flow of positive charge. In electrical conductors such as copper or aluminum, the current results from the motion of negatively charged electrons. Therefore, in an ordinary conductor, the direction of the current is opposite the direction of flow of electrons. For a beam of positively charged protons in an accelerator, however, the current is in the direction of motion of the protons. In some cases—such as those involving gases and electrolytes, for instance—the current is the result of the flow of both positive and negative charges. It is common to refer to a moving charge (positive or negative) as a mobile **charge carrier.**

If the ends of a conducting wire are connected to form a loop, all points on the loop are at the same electric potential; hence, the electric field is zero within and at the surface of the conductor. Because the electric field is zero, there is no net transport of charge through the wire; therefore, there is no current. If the ends of the conducting wire are connected to a battery, however, all points on the loop are not at the same potential. The battery sets up a potential difference between the ends of the loop, creating an electric field within the wire. The electric field exerts forces on the electrons in the wire, causing them to move in the wire and therefore creating a current.

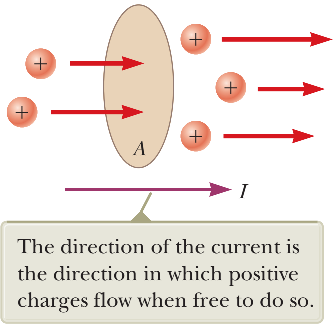


Figure 27.1 Charges in motion through an area A. The time rate at which charge flows through the area is defined as the current I.

Pitfall Prevention 27.1

"Current Flow" Is Redundant
The phrase *current flow* is commonly used, although it is technically incorrect because current *is* a flow (of charge). This wording is similar to the phrase *heat transfer,* which is also redundant because heat *is* a transfer (of energy). We will avoid this phrase and speak of *flow of charge* or *charge flow.*

Pitfall Prevention 27.2

Batteries Do Not Supply Electrons
A battery does not supply electrons to the circuit. It establishes the electric field that exerts a force on electrons already in the wires and elements of the circuit.

Microscopic Model of Current

We can relate current to the motion of the charge carriers by describing a microscopic model of conduction in a metal. Consider the current in a cylindrical

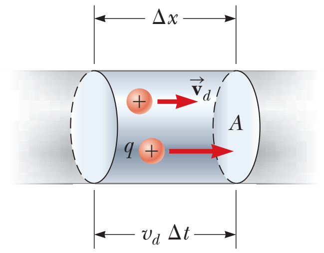


Figure 27.2 A segment of a uniform conductor of cross-sectional area A.

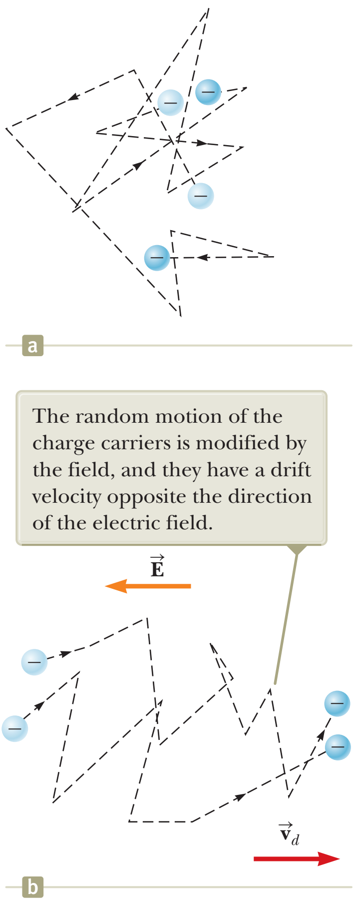


Figure 27.3 (a) A schematic diagram of the random motion of two charge carriers in a conductor in the absence of an electric field. The drift velocity is zero. (b) The motion of the charge carriers in a conductor in the presence of an electric field. Because of the acceleration of the charge carriers due to the electric force, the paths are actually parabolic. The drift speed, however, is much smaller than the average speed, so the parabolic shape is not visible on this scale.

conductor of cross-sectional area A (Fig. 27.2). The volume of a segment of the conductor of length Δx (between the two circular cross sections shown in Fig. 27.2) is $A\,\Delta x$. If n represents the number of mobile charge carriers per unit volume (in other words, the charge carrier density), the number of carriers in the segment is $nA\,\Delta x$. Therefore, the total charge ΔQ in this segment is

$$\Delta Q = (nA\,\Delta x)q$$

where q is the charge on each carrier. If the carriers move with a velocity $\vec{\mathbf{v}}_d$ parallel to the axis of the cylinder, the magnitude of the displacement they experience in the x direction in a time interval Δt is $\Delta x = v_d\,\Delta t$. Let Δt be the time interval required for the charge carriers in the segment to move through a displacement whose magnitude is equal to the length of the segment. This time interval is also the same as that required for all the charge carriers in the segment to pass through the circular area at one end. With this choice, we can write ΔQ as

$$\Delta Q = (nAv_d\,\Delta t)q$$

Dividing both sides of this equation by Δt, we find that the average current in the conductor is

$$I_{\text{avg}} = \frac{\Delta Q}{\Delta t} = nqv_d A \qquad \textbf{(27.4)}$$

In reality, the speed of the charge carriers v_d is an average speed called the **drift speed.** To understand the meaning of drift speed, consider a conductor in which the charge carriers are free electrons. If the conductor is isolated—that is, the potential difference across it is zero—these electrons undergo random motion that is analogous to the motion of gas molecules. The electrons collide repeatedly with the metal atoms, and their resultant motion is complicated and zigzagged as in Figure 27.3a. As discussed earlier, when a potential difference is applied across the conductor (for example, by means of a battery), an electric field is set up in the conductor; this field exerts an electric force on the electrons, producing a current. In addition to the zigzag motion due to the collisions with the metal atoms, the electrons move slowly along the conductor (in a direction opposite that of $\vec{\mathbf{E}}$) at the **drift velocity** $\vec{\mathbf{v}}_d$ as shown in Figure 27.3b.

You can think of the atom–electron collisions in a conductor as an effective internal friction (or drag force) similar to that experienced by a liquid's molecules flowing through a pipe stuffed with steel wool. The energy transferred from the electrons to the metal atoms during collisions causes an increase in the atom's vibrational energy and a corresponding increase in the conductor's temperature.

Quick Quiz 27.1 Consider positive and negative charges moving horizontally through the four regions shown in Figure 27.4. Rank the current in these four regions from highest to lowest.

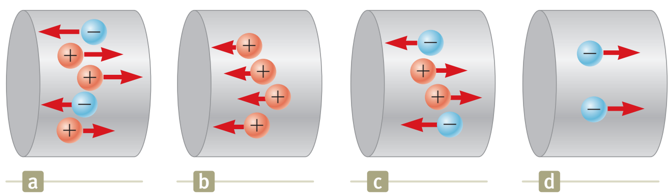


Figure 27.4 (Quick Quiz 27.1) Charges move through four regions.

Example 27.1 Drift Speed in a Copper Wire

The 12-gauge copper wire in a typical residential building has a cross-sectional area of 3.31×10^{-6} m^2. It carries a constant current of 10.0 A. What is the drift speed of the electrons in the wire? Assume each copper atom contributes one free electron to the current. The density of copper is 8.92 g/cm^3.

SOLUTION

Conceptualize Imagine electrons following a zigzag motion such as that in Figure 27.3a, with a drift velocity parallel to the wire superimposed on the motion as in Figure 27.3b. As mentioned earlier, the drift speed is small, and this example helps us quantify the speed.

Categorize We evaluate the drift speed using Equation 27.4. Because the current is constant, the average current during any time interval is the same as the constant current: $I_{avg} = I$.

Analyze The periodic table of the elements in Appendix C shows that the molar mass of copper is $M = 63.5$ g/mol. Recall that 1 mol of any substance contains Avogadro's number of atoms ($N_A = 6.02 \times 10^{23}$ mol^{-1}).

Use the molar mass and the density of copper to find the volume of 1 mole of copper:

$$V = \frac{M}{\rho}$$

From the assumption that each copper atom contributes one free electron to the current, find the electron density in copper:

$$n = \frac{N_A}{V} = \frac{N_A \rho}{M}$$

Solve Equation 27.4 for the drift speed and substitute for the electron density:

$$v_d = \frac{I_{avg}}{nqA} = \frac{I}{nqA} = \frac{IM}{qAN_A\rho}$$

Substitute numerical values:

$$v_d = \frac{(10.0\text{ A})(0.063\,5\text{ kg/mol})}{(1.60 \times 10^{-19}\text{ C})(3.31 \times 10^{-6}\text{ m}^2)(6.02 \times 10^{23}\text{ mol}^{-1})(8\,920\text{ kg/m}^3)}$$

$$= \boxed{2.23 \times 10^{-4}\text{ m/s}}$$

Finalize This result shows that typical drift speeds are very small. For instance, electrons traveling with a speed of 2.23×10^{-4} m/s would take about 75 min to travel 1 m! You might therefore wonder why a light turns on almost instantaneously when its switch is thrown. In a conductor, changes in the electric field that drives the free electrons according to the particle in a field model travel through the conductor with a speed close to that of light. So, when you flip on a light switch, electrons already in the filament of the lightbulb experience electric forces and begin moving after a time interval on the order of nanoseconds.

27.2 Resistance

In Section 24.4, we argued that the electric field inside a conductor is zero. This statement is true, however, *only* if the conductor is in static equilibrium as stated in that discussion. The purpose of this section is to describe what happens when there is a nonzero electric field in the conductor. As we saw in Section 27.1, a current exists in the wire in this case.

Consider a conductor of cross-sectional area A carrying a current I. The **current density** J in the conductor is defined as the current per unit area. Because the current $I = nqv_dA$, the current density is

$$J \equiv \frac{I}{A} = nqv_d \qquad (27.5)$$

◀ Current density

Georg Simon Ohm
German physicist (1789–1854)
Ohm, a high school teacher and later a professor at the University of Munich, formulated the concept of resistance and discovered the proportionalities expressed in Equations 27.6 and 27.7.

where J has SI units of amperes per meter squared. This expression is valid only if the current density is uniform and only if the surface of cross-sectional area A is perpendicular to the direction of the current.

A current density and an electric field are established in a conductor whenever a potential difference is maintained across the conductor. In some materials, the current density is proportional to the electric field:

$$J = \sigma E \quad \textbf{(27.6)}$$

where the constant of proportionality σ is called the **conductivity** of the conductor.[1] Materials that obey Equation 27.6 are said to follow **Ohm's law,** named after Georg Simon Ohm. More specifically, Ohm's law states the following:

> For many materials (including most metals), the ratio of the current density to the electric field is a constant σ that is independent of the electric field producing the current.

Materials and devices that obey Ohm's law and hence demonstrate this simple relationship between E and J are said to be *ohmic*. Experimentally, however, it is found that not all materials and devices have this property. Those that do not obey Ohm's law are said to be *nonohmic.* Ohm's law is not a fundamental law of nature; rather, it is an empirical relationship valid only for certain situations.

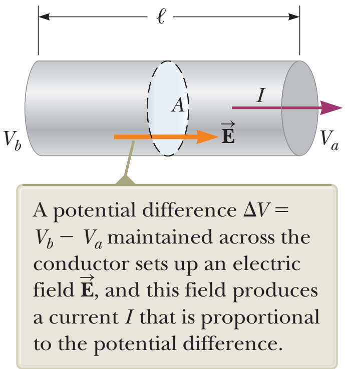


Figure 27.5 A uniform conductor of length ℓ and cross-sectional area A.

We can obtain an equation useful in practical applications by considering a segment of straight wire of uniform cross-sectional area A and length ℓ as shown in Figure 27.5. A potential difference $\Delta V = V_b - V_a$ is maintained across the wire, creating in the wire an electric field and a current. If the field is assumed to be uniform, the magnitude of the potential difference across the wire is related to the field within the wire through Equation 25.6,

$$\Delta V = E\ell$$

Therefore, we can express the current density (Eq. 27.6) in the wire as

$$J = \sigma \frac{\Delta V}{\ell}$$

Because $J = I/A$, the potential difference across the wire is

$$\Delta V = \frac{\ell}{\sigma} J = \left(\frac{\ell}{\sigma A}\right) I = R\, I$$

The quantity $R = \ell/\sigma A$ is called the **resistance** of the conductor. We define the resistance as the ratio of the potential difference across a conductor to the current in the conductor:

$$R \equiv \frac{\Delta V}{I} \quad \textbf{(27.7)}$$

We will use this equation again and again when studying electric circuits. This result shows that resistance has SI units of volts per ampere. One volt per ampere is defined to be one **ohm** (Ω):

$$1\ \Omega \equiv 1\ \text{V/A} \quad \textbf{(27.8)}$$

Equation 27.7 shows that if a potential difference of 1 V across a conductor causes a current of 1 A, the resistance of the conductor is 1 Ω. For example, if an electrical appliance connected to a 120-V source of potential difference carries a current of 6 A, its resistance is 20 Ω.

Most electric circuits use circuit elements called **resistors** to control the current in the various parts of the circuit. As with capacitors in Chapter 26, many resistors are built into integrated circuit chips, but stand-alone resistors are still available and

Pitfall Prevention 27.3

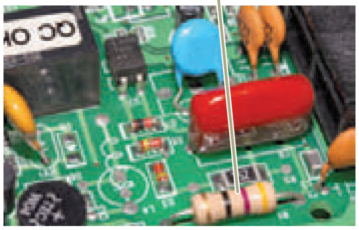
Equation 27.7 Is Not Ohm's Law
Many individuals call Equation 27.7 Ohm's law, but that is incorrect. This equation is simply the definition of resistance, and it provides an important relationship between voltage, current, and resistance. Ohm's law is related to a proportionality of J to E (Eq. 27.6) or, equivalently, of I to ΔV, which, from Equation 27.7, indicates that the resistance is constant, independent of the applied voltage. We will see some devices for which Equation 27.7 correctly describes their resistance, but that do *not* obey Ohm's law.

[1]Do not confuse conductivity σ with surface charge density, for which the same symbol is used.

Table 27.1 Color Coding for Resistors

Color	Number	Multiplier	Tolerance
Black	0	1	
Brown	1	10^1	
Red	2	10^2	
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Gray	8	10^8	
White	9	10^9	
Gold		10^{-1}	5%
Silver		10^{-2}	10%
Colorless			20%

The colored bands on this resistor are yellow, violet, black, and gold.



dexns/Shutterstock.com

Figure 27.6 A close-up view of a circuit board shows the color coding on a resistor. The gold band on the left tells us that the resistor is oriented "backward" in this view and we need to read the colors from right to left.

widely used. Two common types are the *composition resistor,* which contains carbon, and the *wire-wound resistor,* which consists of a coil of wire. Values of resistors in ohms are normally indicated by color coding as shown in Figure 27.6 and Table 27.1. The first two colors on a resistor give the first two digits in the resistance value, with the decimal place to the right of the second digit. The third color represents the power of 10 for the multiplier of the resistance value. The last color is the tolerance of the resistance value. As an example, the four colors on the resistor at the bottom of Figure 27.6 are yellow (= 4), violet (= 7), black (= 10^0), and gold (= 5%), and so the resistance value is $47 \times 10^0 = 47\ \Omega$ with a tolerance value of 5% = 2 Ω.

The inverse of conductivity is **resistivity**[2] ρ:

$$\rho = \frac{1}{\sigma} \tag{27.9}$$

◀ Resistivity is the inverse of conductivity

where ρ has the units ohm · meters (Ω · m). Because $R = \ell/\sigma A$, we can express the resistance of a uniform block of material along the length ℓ as

$$R = \rho \frac{\ell}{A} \tag{27.10}$$

◀ Resistance of a uniform material along the length ℓ

Every ohmic material has a characteristic resistivity that depends on the properties of the material and on temperature. In addition, as you can see from Equation 27.10, the resistance of a sample of the material depends on the geometry of the sample as well as on the resistivity of the material. Table 27.2 (page 814) gives the resistivities of a variety of materials at 20°C. Notice the enormous range, from very low values for good conductors such as copper and silver to very high values for good insulators such as glass and rubber. An ideal conductor would have zero resistivity, and an ideal insulator would have infinite resistivity.

Equation 27.10 shows that the resistance of a given cylindrical conductor such as a wire is proportional to its length and inversely proportional to its cross-sectional area. If the length of a wire is doubled, its resistance doubles. If its cross-sectional area is doubled, its resistance decreases by one half. The situation is analogous to the flow of a liquid through a pipe. As the pipe's length is increased, the resistance to flow increases. As the pipe's cross-sectional area is increased, more liquid crosses a given cross section of the pipe per unit time interval. Therefore, more liquid flows for the same pressure differential applied to the pipe, and the resistance to flow decreases.

Ohmic materials and devices have a linear current–potential difference relationship over a broad range of applied potential differences (Fig. 27.7a, page 814). The slope of the I-versus-ΔV curve in the linear region yields a value for $1/R$. Nonohmic

Pitfall Prevention 27.4

Resistance and Resistivity Resistivity is a property of a *substance,* whereas resistance is a property of an *object.* We have seen similar pairs of variables before. For example, density is a property of a substance, whereas mass is a property of an object. Equation 27.10 relates resistance to resistivity, and Equation 1.1 relates mass to density.

[2]Do not confuse resistivity ρ with mass density or charge density, for which the same symbol is used.

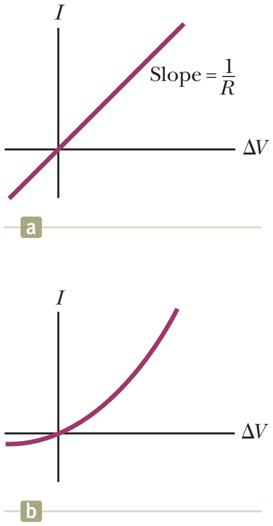


Figure 27.7 (a) The current–potential difference curve for an ohmic material. The curve is linear, and the slope is equal to the inverse of the resistance of the conductor. (b) A nonlinear current–potential difference curve for a junction diode. This device does not obey Ohm's law.

Table 27.2 Resistivities and Temperature Coefficients of Resistivity for Various Materials

Material	Resistivity[a] ($\Omega \cdot$ m)	Temperature Coefficient[b] α [(°C)$^{-1}$]
Silver	1.59×10^{-8}	3.8×10^{-3}
Copper	1.7×10^{-8}	3.9×10^{-3}
Gold	2.44×10^{-8}	3.4×10^{-3}
Aluminum	2.82×10^{-8}	3.9×10^{-3}
Tungsten	5.6×10^{-8}	4.5×10^{-3}
Iron	10×10^{-8}	5.0×10^{-3}
Platinum	11×10^{-8}	3.92×10^{-3}
Lead	22×10^{-8}	3.9×10^{-3}
Nichrome[c]	1.00×10^{-6}	0.4×10^{-3}
Carbon	3.5×10^{-5}	-0.5×10^{-3}
Germanium	0.46	-48×10^{-3}
Silicon[d]	2.3×10^{3}	-75×10^{-3}
Glass	10^{10} to 10^{14}	
Hard rubber	$\sim 10^{13}$	
Sulfur	10^{15}	
Quartz (fused)	75×10^{16}	

[a] All values at 20°C. All elements in this table are assumed to be free of impurities.
[b] See Section 27.4.
[c] A nickel–chromium alloy commonly used in heating elements. The resistivity of Nichrome varies with composition and ranges between 1.00×10^{-6} and 1.50×10^{-6} $\Omega \cdot$ m.
[d] The resistivity of silicon is very sensitive to purity. The value can be changed by several orders of magnitude when it is doped with other atoms.

materials have a nonlinear current–potential difference relationship. One common semiconducting device with nonlinear I-versus-ΔV characteristics is the *junction diode* (Fig. 27.7b). The resistance of this device is low for currents in one direction (positive ΔV) and high for currents in the reverse direction (negative ΔV). In fact, most modern electronic devices, such as transistors, have nonlinear current–potential difference relationships; their proper operation depends on the particular way they violate Ohm's law.

Quick Quiz 27.2 A cylindrical wire has a radius r and length ℓ. If both r and ℓ are doubled, does the resistance of the wire **(a)** increase, **(b)** decrease, or **(c)** remain the same?

Quick Quiz 27.3 In Figure 27.7b, as the applied voltage increases, does the resistance of the diode **(a)** increase, **(b)** decrease, or **(c)** remain the same?

Example 27.2 The Resistance of Nichrome Wire

The radius of 22-gauge Nichrome wire is 0.32 mm.

(A) Calculate the resistance per unit length of this wire.

SOLUTION

Conceptualize Table 27.2 shows that Nichrome has a resistivity two orders of magnitude larger than the best conductors in the table. Therefore, we expect it to have some special practical applications that the best conductors may not have.

Categorize We model the wire as a cylinder so that a simple geometric analysis can be applied to find the resistance.

Analyze Use Equation 27.10 and the resistivity of Nichrome from Table 27.2 to find the resistance per unit length:

$$\frac{R}{\ell} = \frac{\rho}{A} = \frac{\rho}{\pi r^2} = \frac{1.0 \times 10^{-6}\ \Omega \cdot \text{m}}{\pi (0.32 \times 10^{-3}\ \text{m})^2} = 3.1\ \Omega/\text{m}$$

27.2 continued

(B) If a potential difference of 10 V is maintained across a 1.0-m length of the Nichrome wire, what is the current in the wire?

SOLUTION

Analyze Use Equation 27.7 to find the current:

$$I = \frac{\Delta V}{R} = \frac{\Delta V}{(R/\ell)\ell} = \frac{10\text{ V}}{(3.1\ \Omega/\text{m})(1.0\text{ m})} = 3.2\text{ A}$$

Finalize Because of its high resistivity and resistance to oxidation, Nichrome is often used for heating elements in toasters, irons, and electric heaters.

WHAT IF? What if the wire were composed of copper instead of Nichrome? How would the values of the resistance per unit length and the current change?

Answer Table 27.2 shows us that copper has a resistivity two orders of magnitude smaller than that for Nichrome. Therefore, we expect the answer to part (A) to be smaller and the answer to part (B) to be larger. Calculations show that a copper wire of the same radius would have a resistance per unit length of only 0.053 Ω/m. A 1.0-m length of copper wire of the same radius would carry a current of 190 A with an applied potential difference of 10 V.

Example 27.3 The Radial Resistance of a Coaxial Cable

Coaxial cables are used extensively for cable television and other electronic applications. A coaxial cable consists of two concentric cylindrical conductors. The region between the conductors is completely filled with polyethylene plastic as shown in Figure 27.8a. Current leakage through the plastic, in the *radial* direction, is unwanted. (The cable is designed to conduct current along its length, but that is *not* the current being considered here.) The radius of the inner conductor is $a = 0.500$ cm, the radius of the outer conductor is $b = 1.75$ cm, and the length is $L = 15.0$ cm. The resistivity of the plastic is $1.0 \times 10^{13}\ \Omega \cdot \text{m}$. Calculate the resistance of the plastic between the two conductors.

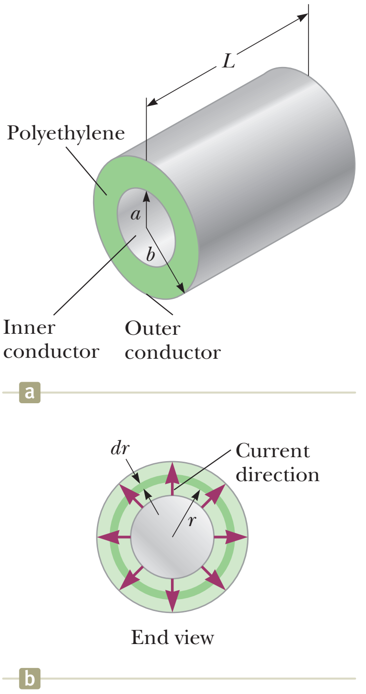


Figure 27.8 (Example 27.3) A coaxial cable. (a) Polyethylene plastic fills the gap between the two conductors. (b) End view, showing current leakage.

SOLUTION

Conceptualize Imagine two currents as suggested in the text of the problem. The desired current is along the cable, carried within the conductors. The undesired current corresponds to leakage through the plastic, and its direction is radial.

Categorize Because the resistivity and the geometry of the plastic are known, we categorize this problem as one in which we find the resistance of the plastic from these parameters. Equation 27.10, however, represents the resistance of a block of material. We have a more complicated geometry in this situation. Because the area through which the charges pass depends on the radial position, we must use integral calculus to determine the answer.

Analyze We divide the plastic into concentric cylindrical shells of infinitesimal thickness dr (Fig. 27.8b). Any charge passing from the inner to the outer conductor must move radially through this shell. Use a differential form of Equation 27.10, replacing ℓ with dr for the length variable: $dR = \rho\, dr/A$, where dR is the resistance of a shell of plastic of thickness dr and surface area A.

Write an expression for the resistance of our hollow cylindrical shell of plastic representing the area as the surface area of the shell:

$$dR = \frac{\rho\, dr}{A} = \frac{\rho}{2\pi r L}\, dr$$

continued

27.3 continued

Integrate this expression from $r = a$ to $r = b$:

$$(1)\quad R = \int dR = \frac{\rho}{2\pi L}\int_a^b \frac{dr}{r} = \frac{\rho}{2\pi L}\ln\left(\frac{b}{a}\right)$$

Substitute the values given:

$$R = \frac{1.0 \times 10^{13}\ \Omega\cdot\text{m}}{2\pi(0.150\ \text{m})}\ln\left(\frac{1.75\ \text{cm}}{0.500\ \text{cm}}\right) = 1.33 \times 10^{13}\ \Omega$$

Finalize Let's compare this resistance to that of the inner copper conductor of the cable along the 15.0-cm length.

Use Equation 27.10 to find the resistance of the copper cylinder:

$$R_{\text{Cu}} = \rho\frac{\ell}{A} = (1.7 \times 10^{-8}\ \Omega\cdot\text{m})\left[\frac{0.150\ \text{m}}{\pi(5.00 \times 10^{-3}\ \text{m})^2}\right]$$
$$= 3.2 \times 10^{-5}\ \Omega$$

This resistance is 18 orders of magnitude smaller than the radial resistance. Therefore, almost all the current corresponds to charge moving along the length of the cable, with a very small fraction leaking in the radial direction.

WHAT IF? Suppose the coaxial cable is enlarged to twice the overall diameter with two possible choices: (1) the ratio b/a is held fixed, or (2) the difference $b - a$ is held fixed. For which choice does the leakage current between the inner and outer conductors increase when the voltage is applied between them?

Answer For the current to increase, the resistance must decrease. For choice (1), in which b/a is held fixed, Equation (1) shows that the resistance is unaffected. For choice (2), we do not have an equation involving the difference $b - a$ to inspect. Looking at Figure 27.8b, however, we see that increasing b and a while holding the difference constant results in charge flowing through the same thickness of plastic but through a larger area perpendicular to the flow. This larger area results in lower resistance and a higher current.

27.3 A Model for Electrical Conduction

In this section, we describe a structural model of electrical conduction in metals that was first proposed by Paul Drude (1863–1906) in 1900. (See Section 21.1 for a review of structural models.) This model leads to Ohm's law and shows that resistivity can be related to the motion of electrons in metals. Although the Drude model described here has limitations, it introduces concepts that are applied in more elaborate treatments.

Following the outline of structural models from Section 21.1, the Drude model for electrical conduction has the following properties:

1. *Physical components:*
   Consider a conductor as a regular array of atoms plus a collection of free electrons, which are sometimes called *conduction* electrons. We identify the system as the combination of the atoms and the conduction electrons. The conduction electrons, although bound to their respective atoms when the atoms are not part of a solid, become free when the atoms condense into a solid.
2. *Behavior of the components:*
   (a) In the absence of an electric field, the conduction electrons move in random directions through the conductor (Fig. 27.3a). The situation is similar to the motion of gas molecules confined in a vessel. In fact, some scientists refer to conduction electrons in a metal as an *electron gas.*
   (b) When an electric field is applied to the system, the free electrons drift slowly in a direction opposite that of the electric field (Fig. 27.3b), with an average drift speed v_d that is much smaller (typically 10^{-4} m/s) than their average speed v_{avg} between collisions (typically 10^6 m/s).
   (c) The electron's motion after a collision is independent of its motion before the collision. The excess energy acquired by the electrons due to

the work done on them by the electric field is transferred to the atoms of the conductor when the electrons and atoms collide.

With regard to property 2(c) above, the energy transferred to the atoms causes the internal energy of the system and, therefore, the temperature of the conductor to increase.

We are now in a position to derive an expression for the drift velocity, using several of our analysis models. When a free electron of mass m_e and charge $q\ (= -e)$ is subjected to an electric field $\vec{\mathbf{E}}$, it is described by the particle in a field model and experiences a force $\vec{\mathbf{F}} = q\vec{\mathbf{E}}$. The electron is a particle under a net force, and its acceleration can be found from Newton's second law, $\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}}$:

$$\vec{\mathbf{a}} = \frac{\sum \vec{\mathbf{F}}}{m} = \frac{q\vec{\mathbf{E}}}{m_e} \tag{27.11}$$

Because the electric field is uniform, the electron's acceleration is constant, so the electron can be modeled as a particle under constant acceleration. If $\vec{\mathbf{v}}_i$ is the electron's initial velocity the instant after a collision (which occurs at a time defined as $t = 0$), the velocity of the electron at a very short time t later (immediately before the next collision occurs) is, from Equation 4.8,

$$\vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \vec{\mathbf{a}}t = \vec{\mathbf{v}}_i + \frac{q\vec{\mathbf{E}}}{m_e}t \tag{27.12}$$

Let's now take the average value of $\vec{\mathbf{v}}_f$ for all the electrons in the wire over all possible collision times t and all possible values of $\vec{\mathbf{v}}_i$. Assuming the initial velocities are randomly distributed over all possible directions (property 2(a) above), the average value of $\vec{\mathbf{v}}_i$ is zero. The average value of the second term of Equation 27.12 is $(q\vec{\mathbf{E}}/m_e)\tau$, where τ is the *average time interval between successive collisions*. Because the average value of $\vec{\mathbf{v}}_f$ is equal to the drift velocity,

$$\vec{\mathbf{v}}_{f,\text{avg}} = \vec{\mathbf{v}}_d = \frac{q\vec{\mathbf{E}}}{m_e}\tau \tag{27.13}$$

Drift velocity in terms of microscopic quantities

The value of τ depends on the size of the metal atoms and the number of electrons per unit volume. We can relate this expression for drift velocity in Equation 27.13 to the current in the conductor. Substituting the magnitude of the velocity from Equation 27.13 into Equation 27.4, the average current in the conductor is given by

$$I_{\text{avg}} = nq\left(\frac{qE}{m_e}\tau\right)A = \frac{nq^2E}{m_e}\tau A \tag{27.14}$$

Because the current density J is the current divided by the area A,

$$J = \frac{nq^2E}{m_e}\tau$$

Current density in terms of microscopic quantities

where n is the number of electrons per unit volume. Comparing this expression with Ohm's law, $J = \sigma E$, we obtain the following relationships for conductivity and resistivity of a conductor:

$$\sigma = \frac{nq^2\tau}{m_e} \tag{27.15}$$

Conductivity in terms of microscopic quantities

$$\rho = \frac{1}{\sigma} = \frac{m_e}{nq^2\tau} \tag{27.16}$$

Resistivity in terms of microscopic quantities

According to this classical model, conductivity and resistivity do not depend on the strength of the electric field. This feature is characteristic of a conductor obeying Ohm's law.

The model shows that the resistivity can be calculated from a knowledge of the density of the electrons, their charge and mass, and the average time interval τ between collisions. This time interval is related to the average distance between collisions ℓ_{avg} (the *mean free path*) and the average speed v_{avg} through the expression[3]

$$\tau = \frac{\ell_{avg}}{v_{avg}} \tag{27.17}$$

Although this structural model of conduction is consistent with Ohm's law, it does not correctly predict the values of resistivity or the behavior of the resistivity with temperature. For example, the results of classical calculations for v_{avg} using the ideal gas model for the electrons are about a factor of ten smaller than the actual values, which results in incorrect predictions of values of resistivity from Equation 27.16. Furthermore, according to Equations 27.16 and 27.17, the resistivity is predicted to vary with temperature as does v_{avg}, which, according to an ideal-gas model (Chapter 21, Eq. 21.43), is proportional to $\sqrt{T}$. This behavior is in disagreement with the experimentally observed linear dependence of resistivity with temperature for pure metals. (See Section 27.4.) Because of these incorrect predictions, we must modify our structural model. We shall call the model that we have developed so far the *classical* model for electrical conduction. To account for the incorrect predictions of the classical model, we develop it further into a *quantum mechanical* model, which we shall describe briefly.

We discussed two important simplification models in earlier chapters, the particle model and the wave model. Although we discussed these two simplification models separately, quantum physics tells us that this separation is not so clear-cut. As we shall discuss in detail in Chapter 40, particles have wave-like properties. The predictions of some models can only be matched to experimental results if the model includes the wave-like behavior of particles. The structural model for electrical conduction in metals is one of these cases.

Let us imagine that the electrons moving through the metal have wave-like properties. If the array of atoms in a conductor is regularly spaced (that is, periodic), the wave-like character of the electrons makes it possible for them to move freely through the conductor and a collision with an atom is unlikely. For an idealized conductor, no collisions would occur, the mean free path would be infinite, and the resistivity would be zero. Electrons are scattered only if the atomic arrangement is irregular (not periodic), as a result of structural defects or impurities, for example. At low temperatures, the resistivity of metals is dominated by scattering caused by collisions between the electrons and impurities. At high temperatures, the resistivity is dominated by scattering caused by collisions between the electrons and the atoms of the conductor, which are continuously displaced as a result of thermal agitation, destroying the perfect periodicity. The thermal motion of the atoms makes the structure irregular (compared with an atomic array at rest), thereby reducing the electron's mean free path.

Although it is beyond the scope of this text to show this modification in detail, the classical model modified with the wave-like character of the electrons results in predictions of resistivity values that are in agreement with measured values and predicts a linear temperature dependence. Quantum notions had to be introduced in Chapter 21 to understand the temperature behavior of molar specific heats of gases. Here we have another case in which quantum physics is necessary for the model to agree with experiment. Although classical physics can explain a tremendous range of phenomena, we continue to see hints that quantum physics must be incorporated into our models. We shall study quantum physics in detail in Chapters 40 through 46.

[3]Recall that the average speed of a group of particles depends on the temperature of the group (Chapter 21) and is not the same as the drift speed v_d.

27.4 Resistance and Temperature

Over a limited temperature range, the resistivity of a conductor varies approximately linearly with temperature according to the expression

$$\rho = \rho_0[1 + \alpha(T - T_0)] \quad (27.18)$$

◀ **Variation of ρ with temperature**

where ρ is the resistivity at some temperature T (in degrees Celsius), ρ_0 is the resistivity at some reference temperature T_0 (usually taken to be 20°C), and α is the **temperature coefficient of resistivity.** From Equation 27.18, the temperature coefficient of resistivity can be expressed as

$$\alpha = \frac{1}{\rho_0}\frac{\Delta\rho}{\Delta T} \quad (27.19)$$

◀ **Temperature coefficient of resistivity**

where $\Delta\rho = \rho - \rho_0$ is the change in resistivity in the temperature interval $\Delta T = T - T_0$.

The temperature coefficients of resistivity for various materials are given in Table 27.2. Notice that the unit for α is degrees Celsius^{-1} [$(°C)^{-1}$]. Because resistance is proportional to resistivity (Eq. 27.10), the variation of resistance of a sample is

$$R = R_0[1 + \alpha(T - T_0)] \quad (27.20)$$

where R_0 is the resistance at temperature T_0. Use of this property enables precise temperature measurements through careful monitoring of the resistance of a probe made from a particular material.

For some metals such as copper, resistivity is nearly proportional to temperature as shown in Figure 27.9. A nonlinear region always exists at very low temperatures, however, and the resistivity usually reaches some finite value as the temperature approaches absolute zero. This residual resistivity near absolute zero is caused primarily by the collision of electrons with impurities and imperfections in the metal. In contrast, high-temperature resistivity (the linear region) is predominantly characterized by collisions between electrons and metal atoms.

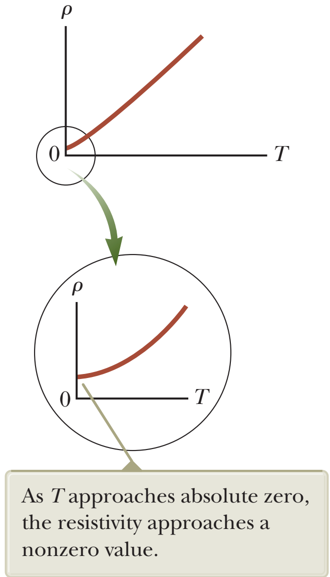


Figure 27.9 Resistivity versus temperature for a metal such as copper. The curve is linear over a wide range of temperatures, and ρ increases with increasing temperature.

Notice that three of the α values in Table 27.2 are negative, indicating that the resistivity of these materials decreases with increasing temperature. This behavior is indicative of a class of materials called *semiconductors,* first introduced in Section 23.2, and is due to an increase in the density of charge carriers at higher temperatures.

Because the charge carriers in a semiconductor are often associated with impurity atoms (as we discuss in more detail in Chapter 43), the resistivity of these materials is very sensitive to the type and concentration of such impurities.

Quick Quiz 27.4 When does an incandescent lightbulb carry more current, **(a)** immediately after it is turned on and the glow of the metal filament is increasing or **(b)** after it has been on for a few milliseconds and the glow is steady?

27.5 Superconductors

There is a class of metals and compounds whose resistance decreases to zero when they are below a certain temperature T_c, known as the **critical temperature.** These materials are known as **superconductors.** The resistance–temperature graph for a superconductor follows that of a normal metal at temperatures above T_c (Fig. 27.10). When the temperature is at or below T_c, the resistivity drops suddenly to zero. This phenomenon was discovered in 1911 by Dutch physicist Heike Kamerlingh-Onnes (1853–1926) as he worked with mercury, which is a superconductor below 4.2 K. Measurements have shown that the resistivities of superconductors below their T_c values are less than $4 \times 10^{-25}\ \Omega \cdot$ m, or approximately 10^{17} times smaller than the resistivity of copper. In practice, these resistivities are considered to be zero.

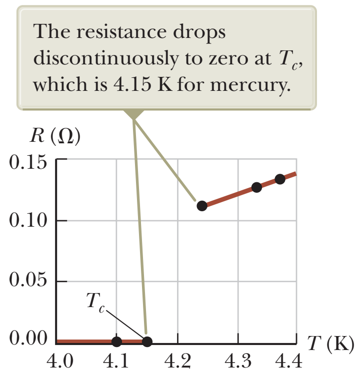


Figure 27.10 Resistance versus temperature for a sample of mercury (Hg). The graph follows that of a normal metal above the critical temperature T_c.

Courtesy of IBM Research Laboratory

A small permanent magnet levitated above a disk of the superconductor $YBa_2Cu_3O_7$, which is in liquid nitrogen at 77 K.

Table 27.3 Critical Temperatures for Various Superconductors

Material	T_c (K)
$HgBa_2Ca_2Cu_3O_8$	134
Tl—Ba—Ca—Cu—O	125
Bi—Sr—Ca—Cu—O	105
$YBa_2Cu_3O_7$	92
Nb_3Ge	23.2
Nb_3Sn	18.05
Nb	9.46
Pb	7.18
Hg	4.15
Sn	3.72
Al	1.19
Zn	0.88

Today, thousands of superconductors are known, and as Table 27.3 illustrates, the critical temperatures of recently discovered superconductors are substantially higher than initially thought possible. Two kinds of superconductors are recognized. The more recently identified ones are essentially ceramics with high critical temperatures, whereas superconducting materials such as those observed by Kamerlingh-Onnes are metals. If a room-temperature superconductor is ever identified, its effect on technology could be tremendous.

The value of T_c is sensitive to chemical composition, pressure, and molecular structure. Copper, silver, and gold, which are excellent conductors, do not exhibit superconductivity.

One truly remarkable feature of superconductors is that once a current is set up in them, it persists *without any applied potential difference* (because $R = 0$). Steady currents have been observed to persist in superconducting loops for several years with no apparent decay!

An important and useful application of superconductivity is in the development of superconducting magnets, in which the magnitudes of the magnetic field are approximately ten times greater than those produced by the best normal electromagnets. Such superconducting magnets are being considered as a means of storing energy. Superconducting magnets are currently used in medical magnetic resonance imaging, or MRI, units, which produce high-quality images of internal organs without the need for excessive exposure of patients to x-rays or other harmful radiation.

27.6 Electrical Power

In typical electric circuits, energy T_{ET} is transferred by electrical transmission from a source such as a battery to some device such as a lightbulb or a radio receiver. Let's determine an expression that will allow us to calculate the rate of this energy transfer. First, consider the simple circuit in Figure 27.11, where energy is delivered to a resistor. (Resistors are designated by the circuit symbol —\/\/\/—.) Because the connecting wires also have resistance, some energy is delivered to the wires and some to the resistor. Unless noted otherwise, we shall assume the resistance of the wires is small compared with the resistance of the circuit element so that the energy delivered to the wires is negligible.

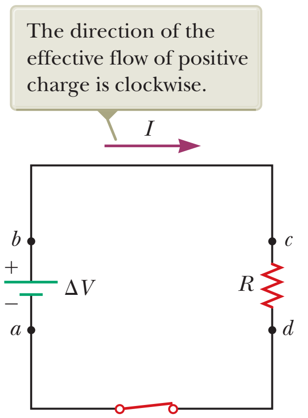


Figure 27.11 A circuit consisting of a resistor of resistance R and a battery having a potential difference ΔV across its terminals.

Imagine following a positive quantity of charge Q moving clockwise around the circuit in Figure 27.11 from point a through the battery and resistor back to point a. We identify the entire circuit as our system. As the charge moves from a to b through the battery, the electric potential energy of the system *increases* by an amount $Q\,\Delta V$

while the chemical potential energy in the battery *decreases* by the same amount. (Recall from Eq. 25.3 that $\Delta U = q\,\Delta V$.) As the charge moves from c to d through the resistor, however, the electric potential energy of the system decreases due to collisions of electrons with atoms in the resistor. In this process, the electric potential energy is transformed to internal energy corresponding to increased vibrational motion of the atoms in the resistor. Because the resistance of the interconnecting wires is neglected, no energy transformation occurs for paths bc and da. When the charge returns to point a, the net result is that some of the chemical potential energy in the battery has been delivered to the resistor and resides in the resistor as internal energy E_{int} associated with molecular vibration.

The resistor is normally in contact with air, so its increased temperature results in a transfer of energy by heat Q into the air. In addition, the resistor emits thermal radiation T_{ER}, representing another means of escape for the energy. After some time interval has passed, the resistor reaches a constant temperature. At this time, the input of energy from the battery is balanced by the output of energy from the resistor by heat and radiation, and the resistor is a nonisolated system in steady state. Some electrical devices include *heat sinks*[4] connected to parts of the circuit to prevent these parts from reaching dangerously high temperatures. Heat sinks are pieces of metal with many fins. Because the metal's high thermal conductivity provides a rapid transfer of energy by heat away from the hot component and the large number of fins provides a large surface area in contact with the air, energy can transfer by radiation and into the air by heat at a high rate.

Let's now investigate the rate at which the electric potential energy of the system decreases as the charge Q passes through the resistor:

$$\frac{dU}{dt} = \frac{d}{dt}(Q\,\Delta V) = \frac{dQ}{dt}\Delta V = I\,\Delta V$$

where I is the current in the circuit. The system regains this potential energy when the charge passes through the battery, at the expense of chemical energy in the battery. The rate at which the potential energy of the system decreases as the charge passes through the resistor is equal to the rate at which the system gains internal energy in the resistor. Therefore, the power P, representing the rate at which energy is delivered to the resistor, is

$$P = I\,\Delta V \qquad \textbf{(27.21)}$$

We derived this result by considering a battery delivering energy to a resistor. Equation 27.21, however, can be used to calculate the power delivered by a voltage source to *any* device carrying a current I and having a potential difference ΔV between its terminals.

Using Equation 27.21 and $\Delta V = IR$ for a resistor, we can express the power delivered to the resistor in the alternative forms

$$P = I^2R = \frac{(\Delta V)^2}{R} \qquad \textbf{(27.22)}$$

When I is expressed in amperes, ΔV in volts, and R in ohms, the SI unit of power is the watt, as it was in Chapter 8 in our discussion of mechanical power. The process by which energy is transformed to internal energy in a conductor of resistance R is often called *joule heating;*[5] this transformation is also often referred to as an I^2R loss.

Pitfall Prevention 27.5

Charges Do Not Move All the Way Around a Circuit in a Short Time In terms of understanding the energy transfer in a circuit, it is useful to *imagine* a charge moving all the way around the circuit even though it would take hours to do so.

Pitfall Prevention 27.6

Misconceptions About Current Several common misconceptions are associated with current in a circuit like that in Figure 27.11. One is that current comes out of one terminal of the battery and is then "used up" as it passes through the resistor, leaving current in only one part of the circuit. The current is actually the same *everywhere* in the circuit. A related misconception has the current coming out of the resistor being smaller than that going in because some of the current is "used up." Yet another misconception has current coming out of both terminals of the battery, in opposite directions, and then "clashing" in the resistor, delivering the energy in this manner. That is not the case; charges flow in the same rotational sense at *all* points in the circuit.

Pitfall Prevention 27.7

Energy Is Not "Dissipated" In some books, you may see Equation 27.22 described as the power "dissipated in" a resistor, suggesting that energy disappears. Instead, we say energy is "delivered to" a resistor.

[4]This usage is another misuse of the word *heat* that is ingrained in our common language.

[5]It is commonly called *joule heating* even though the process of heat does not occur when energy delivered to a resistor appears as internal energy. It is another example of incorrect usage of the word *heat* that has become entrenched in our language.

Figure 27.12 These power lines transfer energy from the electric company to homes and businesses. The energy is transferred at a very high voltage, possibly hundreds of thousands of volts in some cases. Even though it makes power lines very dangerous, the high voltage results in less loss of energy due to resistance in the wires.

When transporting energy by electricity through power lines (Fig. 27.12), you should not assume the lines have zero resistance. Real power lines do indeed have resistance, and power is delivered to the resistance of these wires. Utility companies seek to minimize the energy transformed to internal energy in the lines and maximize the energy delivered to the consumer. Because $P = I\,\Delta V$, the same amount of energy can be transported either at high currents and low potential differences or at low currents and high potential differences. Utility companies choose to transport energy at low currents and high potential differences primarily for economic reasons. Copper wire is very expensive, so it is cheaper to use high-resistance wire (that is, wire having a small cross-sectional area; see Eq. 27.10). Therefore, in the expression for the power delivered to a resistor, $P = I^2R$, the resistance of the wire is fixed at a relatively high value for economic considerations. The I^2R loss can be reduced by keeping the current I as low as possible, which means transferring the energy at a high voltage. In some instances, power is transported at potential differences as great as 765 kV. At the destination of the energy, the potential difference is usually reduced to 4 kV by a device called a *transformer.* Another transformer drops the potential difference to 240 V for use in your home. Of course, each time the potential difference decreases, the current increases by the same factor and the power remains the same. We shall discuss transformers in greater detail in Chapter 33.

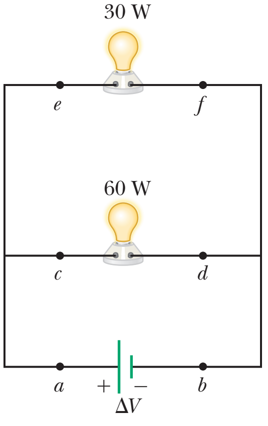


Figure 27.13 (Quick Quiz 27.5) Two lightbulbs connected across the same potential difference.

Quick Quiz 27.5 For the two lightbulbs shown in Figure 27.13, rank the current values at points *a* through *f* from greatest to least.

Example 27.4 Power in an Electric Heater

An electric heater is constructed by applying a potential difference of 120 V across a Nichrome wire that has a total resistance of 8.00 Ω. Find the current carried by the wire and the power rating of the heater.

SOLUTION

Conceptualize As discussed in Example 27.2, Nichrome wire has high resistivity and is often used for heating elements in toasters, irons, and electric heaters. Therefore, we expect the power delivered to the wire to be relatively high.

Categorize We evaluate the power from Equation 27.22, so we categorize this example as a substitution problem.

Use Equation 27.7 to find the current in the wire:

$$I = \frac{\Delta V}{R} = \frac{120\text{ V}}{8.00\ \Omega} = \boxed{15.0\text{ A}}$$

Find the power rating using the expression $P = I^2R$ from Equation 27.22:

$$P = I^2R = (15.0\text{ A})^2(8.00\ \Omega) = 1.80 \times 10^3\text{ W} = \boxed{1.80\text{ kW}}$$

WHAT IF? What if the heater were accidentally connected to a 240-V supply? (That is difficult to do because the shape and orientation of the metal contacts in 240-V plugs are different from those in 120-V plugs.) How would that affect the current carried by the heater and the power rating of the heater, assuming the resistance remains constant?

Answer If the applied potential difference were doubled, Equation 27.7 shows that the current would double. According to Equation 27.22, $P = (\Delta V)^2/R$, the power would be four times larger.

Example 27.5 Linking Electricity and Thermodynamics AM

An immersion heater must increase the temperature of 1.50 kg of water from 10.0°C to 50.0°C in 10.0 min while operating at 110 V.

(A) What is the required resistance of the heater?

SOLUTION

Conceptualize An immersion heater is a resistor that is inserted into a container of water. As energy is delivered to the immersion heater, raising its temperature, energy leaves the surface of the resistor by heat, going into the water. When the immersion heater reaches a constant temperature, the rate of energy delivered to the resistance by electrical transmission (T_{ET}) is equal to the rate of energy delivered by heat (Q) to the water.

Categorize This example allows us to link our new understanding of power in electricity with our experience with specific heat in thermodynamics (Chapter 20). The water is a *nonisolated system.* Its internal energy is rising because of energy transferred into the water by heat from the resistor, so Equation 8.2 reduces to $\Delta E_{int} = Q$. In our model, we assume the energy that enters the water from the heater remains in the water.

Analyze To simplify the analysis, let's ignore the initial period during which the temperature of the resistor increases and also ignore any variation of resistance with temperature. Therefore, we imagine a constant rate of energy transfer for the entire 10.0 min.

Set the rate of energy delivered to the resistor equal to the rate of energy Q entering the water by heat:

$$P = \frac{(\Delta V)^2}{R} = \frac{Q}{\Delta t}$$

Use Equation 20.4, $Q = mc\,\Delta T$, to relate the energy input by heat to the resulting temperature change of the water and solve for the resistance:

$$\frac{(\Delta V)^2}{R} = \frac{mc\,\Delta T}{\Delta t} \quad \rightarrow \quad R = \frac{(\Delta V)^2\,\Delta t}{mc\,\Delta T}$$

Substitute the values given in the statement of the problem:

$$R = \frac{(110\text{ V})^2(600\text{ s})}{(1.50\text{ kg})(4\,186\text{ J/kg}\cdot°\text{C})(50.0°\text{C} - 10.0°\text{C})} = \boxed{28.9\ \Omega}$$

(B) Estimate the cost of heating the water.

SOLUTION

Multiply the power by the time interval to find the amount of energy transferred to the resistor:

$$T_{ET} = P\,\Delta t = \frac{(\Delta V)^2}{R}\Delta t = \frac{(110\text{ V})^2}{28.9\ \Omega}(10.0\text{ min})\left(\frac{1\text{ h}}{60.0\text{ min}}\right)$$

$$= 69.8\text{ Wh} = 0.069\,8\text{ kWh}$$

Find the cost knowing that energy is purchased at an estimated price of 11¢ per kilowatt-hour:

$$\text{Cost} = (0.069\,8\text{ kWh})(\$0.11/\text{kWh}) = \$0.008 = \boxed{0.8¢}$$

Finalize The cost to heat the water is very low, less than one cent. In reality, the cost is higher because some energy is transferred from the water into the surroundings by heat and electromagnetic radiation while its temperature is increasing. If you have electrical devices in your home with power ratings on them, use this power rating and an approximate time interval of use to estimate the cost for one use of the device.

Summary

Definitions

The electric **current** I in a conductor is defined as

$$I \equiv \frac{dQ}{dt} \qquad \textbf{(27.2)}$$

where dQ is the charge that passes through a cross section of the conductor in a time interval dt. The SI unit of current is the **ampere** (A), where 1 A = 1 C/s.

continued

The **current density** J in a conductor is the current per unit area:

$$J \equiv \frac{I}{A} \quad (27.5)$$

The **resistance** R of a conductor is defined as

$$R \equiv \frac{\Delta V}{I} \quad (27.7)$$

where ΔV is the potential difference across the conductor and I is the current it carries. The SI unit of resistance is volts per ampere, which is defined to be 1 **ohm** (Ω); that is, $1\ \Omega = 1$ V/A.

Concepts and Principles

The average current in a conductor is related to the motion of the charge carriers through the relationship

$$I_{avg} = nqv_dA \quad (27.4)$$

where n is the density of charge carriers, q is the charge on each carrier, v_d is the drift speed, and A is the cross-sectional area of the conductor.

The current density in an ohmic conductor is proportional to the electric field according to the expression

$$J = \sigma E \quad (27.6)$$

The proportionality constant σ is called the **conductivity** of the material of which the conductor is made. The inverse of σ is known as **resistivity** ρ (that is, $\rho = 1/\sigma$). Equation 27.6 is known as **Ohm's law,** and a material is said to obey this law if the ratio of its current density to its applied electric field is a constant that is independent of the applied field.

For a uniform block of material of cross-sectional area A and length ℓ, the resistance over the length ℓ is

$$R = \rho\frac{\ell}{A} \quad (27.10)$$

where ρ is the resistivity of the material.

In a classical model of electrical conduction in metals, the electrons are treated as molecules of a gas. In the absence of an electric field, the average velocity of the electrons is zero. When an electric field is applied, the electrons move (on average) with a **drift velocity** $\vec{\mathbf{v}}_d$ that is opposite the electric field. The drift velocity is given by

$$\vec{\mathbf{v}}_d = \frac{q\vec{\mathbf{E}}}{m_e}\tau \quad (27.13)$$

where q is the electron's charge, m_e is the mass of the electron, and τ is the average time interval between electron–atom collisions. According to this model, the resistivity of the metal is

$$\rho = \frac{m_e}{nq^2\tau} \quad (27.16)$$

where n is the number of free electrons per unit volume.

The resistivity of a conductor varies approximately linearly with temperature according to the expression

$$\rho = \rho_0[1 + \alpha(T - T_0)] \quad (27.18)$$

where ρ_0 is the resistivity at some reference temperature T_0 and α is the **temperature coefficient of resistivity.**

If a potential difference ΔV is maintained across a circuit element, the **power,** or rate at which energy is supplied to the element, is

$$P = I\,\Delta V \quad (27.21)$$

Because the potential difference across a resistor is given by $\Delta V = IR$, we can express the power delivered to a resistor as

$$P = I^2R = \frac{(\Delta V)^2}{R} \quad (27.22)$$

The energy delivered to a resistor by electrical transmission T_{ET} appears in the form of internal energy E_{int} in the resistor.

Objective Questions

1. denotes answer available in *Student Solutions Manual/Study Guide*

1. Car batteries are often rated in ampere-hours. Does this information designate the amount of (a) current, (b) power, (c) energy, (d) charge, or (e) potential the battery can supply?

2. Two wires A and B with circular cross sections are made of the same metal and have equal lengths, but the resistance of wire A is three times greater than that of wire B. **(i)** What is the ratio of the cross-sectional area of A to that of B? (a) 3 (b) $\sqrt{3}$ (c) 1 (d) $1/\sqrt{3}$ (e) $\frac{1}{3}$ **(ii)** What is the ratio of the radius of A to that of B? Choose from the same possibilities as in part (i).

3. A cylindrical metal wire at room temperature is carrying electric current between its ends. One end is at potential $V_A = 50$ V, and the other end is at potential $V_B = 0$ V. Rank the following actions in terms of the change that each one separately would produce in the current from the greatest increase to the greatest decrease. In your ranking, note any cases of equality. (a) Make $V_A = 150$ V with $V_B = 0$ V. (b) Adjust V_A to triple the power with which the wire converts electrically transmitted energy into internal energy. (c) Double the radius of the wire. (d) Double the length of the wire. (e) Double the Celsius temperature of the wire.

4. A current-carrying ohmic metal wire has a cross-sectional area that gradually becomes smaller from one end of the wire to the other. The current has the same value for each section of the wire, so charge does not accumulate at any one point. **(i)** How does the drift speed vary along the wire as the area becomes smaller? (a) It increases. (b) It decreases. (c) It remains constant. **(ii)** How does the resistance per unit length vary along the wire as the area becomes smaller? Choose from the same possibilities as in part (i).

5. A potential difference of 1.00 V is maintained across a 10.0-Ω resistor for a period of 20.0 s. What total charge passes by a point in one of the wires connected to the resistor in this time interval? (a) 200 C (b) 20.0 C (c) 2.00 C (d) 0.005 00 C (e) 0.050 0 C

6. Three wires are made of copper having circular cross sections. Wire 1 has a length L and radius r. Wire 2 has a length L and radius $2r$. Wire 3 has a length $2L$ and radius $3r$. Which wire has the smallest resistance? (a) wire 1 (b) wire 2 (c) wire 3 (d) All have the same resistance. (e) Not enough information is given to answer the question.

7. A metal wire of resistance R is cut into three equal pieces that are then placed together side by side to form a new cable with a length equal to one-third the original length. What is the resistance of this new cable? (a) $\frac{1}{9}R$ (b) $\frac{1}{3}R$ (c) R (d) $3R$ (e) $9R$

8. A metal wire has a resistance of 10.0 Ω at a temperature of 20.0°C. If the same wire has a resistance of 10.6 Ω at 90.0°C, what is the resistance of this wire when its temperature is −20.0°C? (a) 0.700 Ω (b) 9.66 Ω (c) 10.3 Ω (d) 13.8 Ω (e) 6.59 Ω

9. The current-versus-voltage behavior of a certain electrical device is shown in Figure OQ27.9. When the potential difference across the device is 2 V, what is its resistance? (a) 1 Ω (b) $\frac{3}{4}$ Ω (c) $\frac{4}{3}$ Ω (d) undefined (e) none of those answers

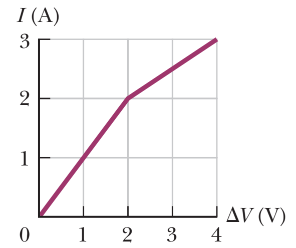


Figure OQ27.9

10. Two conductors made of the same material are connected across the same potential difference. Conductor A has twice the diameter and twice the length of conductor B. What is the ratio of the power delivered to A to the power delivered to B? (a) 8 (b) 4 (c) 2 (d) 1 (e) $\frac{1}{2}$

11. Two conducting wires A and B of the same length and radius are connected across the same potential difference. Conductor A has twice the resistivity of conductor B. What is the ratio of the power delivered to A to the power delivered to B? (a) 2 (b) $\sqrt{2}$ (c) 1 (d) $1/\sqrt{2}$ (e) $\frac{1}{2}$

12. Two lightbulbs both operate on 120 V. One has a power of 25 W and the other 100 W. **(i)** Which lightbulb has higher resistance? (a) The dim 25-W lightbulb does. (b) The bright 100-W lightbulb does. (c) Both are the same. **(ii)** Which lightbulb carries more current? Choose from the same possibilities as in part (i).

13. Wire B has twice the length and twice the radius of wire A. Both wires are made from the same material. If wire A has a resistance R, what is the resistance of wire B? (a) $4R$ (b) $2R$ (c) R (d) $\frac{1}{2}R$ (e) $\frac{1}{4}R$

Conceptual Questions

1. denotes answer available in *Student Solutions Manual/Study Guide*

1. If you were to design an electric heater using Nichrome wire as the heating element, what parameters of the wire could you vary to meet a specific power output such as 1 000 W?

2. What factors affect the resistance of a conductor?

3. When the potential difference across a certain conductor is doubled, the current is observed to increase by a factor of 3. What can you conclude about the conductor?

4. Over the time interval after a difference in potential is applied between the ends of a wire, what would happen to the drift velocity of the electrons in a wire and to the current in the wire if the electrons could move freely without resistance through the wire?

5. How does the resistance for copper and for silicon change with temperature? Why are the behaviors of these two materials different?

6. Use the atomic theory of matter to explain why the resistance of a material should increase as its temperature increases.

7. If charges flow very slowly through a metal, why does it not require several hours for a light to come on when you throw a switch?

8. Newspaper articles often contain statements such as "10 000 volts of electricity surged through the victim's body." What is wrong with this statement?

Problems

WebAssign The problems found in this chapter may be assigned online in Enhanced WebAssign

1. straightforward; **2.** intermediate; **3.** challenging

1. full solution available in the *Student Solutions Manual/Study Guide*

AMT Analysis Model tutorial available in Enhanced WebAssign

GP Guided Problem

M Master It tutorial available in Enhanced WebAssign

W Watch It video solution available in Enhanced WebAssign

Section 27.1 Electric Current

1. AMT M A 200-km-long high-voltage transmission line 2.00 cm in diameter carries a steady current of 1 000 A. If the conductor is copper with a free charge density of 8.50×10^{28} electrons per cubic meter, how many years does it take one electron to travel the full length of the cable?

2. A small sphere that carries a charge q is whirled in a circle at the end of an insulating string. The angular frequency of revolution is ω. What average current does this revolving charge represent?

3. W An aluminum wire having a cross-sectional area equal to 4.00×10^{-6} m^2 carries a current of 5.00 A. The density of aluminum is 2.70 g/cm^3. Assume each aluminum atom supplies one conduction electron per atom. Find the drift speed of the electrons in the wire.

4. AMT In the Bohr model of the hydrogen atom (which will be covered in detail in Chapter 42), an electron in the lowest energy state moves at a speed of 2.19×10^6 m/s in a circular path of radius 5.29×10^{-11} m. What is the effective current associated with this orbiting electron?

5. A proton beam in an accelerator carries a current of 125 μA. If the beam is incident on a target, how many protons strike the target in a period of 23.0 s?

6. A copper wire has a circular cross section with a radius of 1.25 mm. (a) If the wire carries a current of 3.70 A, find the drift speed of the electrons in this wire. (b) All other things being equal, what happens to the drift speed in wires made of metal having a larger number of conduction electrons per atom than copper? Explain.

7. Suppose the current in a conductor decreases exponentially with time according to the equation $I(t) = I_0 e^{-t/\tau}$, where I_0 is the initial current (at $t = 0$) and τ is a constant having dimensions of time. Consider a fixed observation point within the conductor. (a) How much charge passes this point between $t = 0$ and $t = \tau$? (b) How much charge passes this point between $t = 0$ and $t = 10\tau$? (c) **What If?** How much charge passes this point between $t = 0$ and $t = \infty$?

8. W Figure P27.8 represents a section of a conductor of nonuniform diameter carrying a current of $I = 5.00$ A. The radius of cross-section A_1 is $r_1 = 0.400$ cm. (a) What is the magnitude of the current density across A_1? The radius r_2 at A_2 is larger than the radius r_1 at A_1. (b) Is the current at A_2 larger, smaller, or the same? (c) Is the current density at A_2 larger, smaller, or the same? Assume $A_2 = 4A_1$. Specify the (d) radius, (e) current, and (f) current density at A_2.

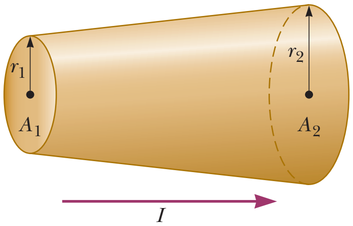


Figure P27.8

9. W The quantity of charge q (in coulombs) that has passed through a surface of area 2.00 cm^2 varies with time according to the equation $q = 4t^3 + 5t + 6$, where t is in seconds. (a) What is the instantaneous current through the surface at $t = 1.00$ s? (b) What is the value of the current density?

10. A Van de Graaff generator produces a beam of 2.00-MeV deuterons, which are heavy hydrogen nuclei containing a proton and a neutron. (a) If the beam current is 10.0 μA, what is the average separation of the deuterons? (b) Is the electrical force of repulsion among them a significant factor in beam stability? Explain.

11. M The electron beam emerging from a certain high-energy electron accelerator has a circular cross section of radius 1.00 mm. (a) The beam current is 8.00 μA. Find the current density in the beam assuming it is uniform throughout. (b) The speed of the electrons is so close to the speed of light that their speed can be taken as 300 Mm/s with negligible error. Find the electron density in the beam. (c) Over what time interval does Avogadro's number of electrons emerge from the accelerator?

12. W An electric current in a conductor varies with time according to the expression $I(t) = 100 \sin(120\pi t)$, where I is in amperes and t is in seconds. What is the total charge passing a given point in the conductor from $t = 0$ to $t = \frac{1}{240}$ s?

13. W A teapot with a surface area of 700 cm^2 is to be plated with silver. It is attached to the negative electrode of an electrolytic cell containing silver nitrate ($Ag^+NO_3^-$). The cell is powered by a 12.0-V battery and has a resistance of 1.80 Ω. If the density of silver is 10.5 × 10^3 kg/m^3, over what time interval does a 0.133-mm layer of silver build up on the teapot?

Section 27.2 Resistance

14. W A lightbulb has a resistance of 240 Ω when operating with a potential difference of 120 V across it. What is the current in the lightbulb?

15. M A wire 50.0 m long and 2.00 mm in diameter is connected to a source with a potential difference of 9.11 V, and the current is found to be 36.0 A. Assume a temperature of 20.0°C and, using Table 27.2, identify the metal out of which the wire is made.

16. A 0.900-V potential difference is maintained across a 1.50-m length of tungsten wire that has a cross-sectional area of 0.600 mm^2. What is the current in the wire?

17. An electric heater carries a current of 13.5 A when operating at a voltage of 120 V. What is the resistance of the heater?

18. Aluminum and copper wires of equal length are found to have the same resistance. What is the ratio of their radii?

19. M Suppose you wish to fabricate a uniform wire from 1.00 g of copper. If the wire is to have a resistance of $R = 0.500\ \Omega$ and all the copper is to be used, what must be (a) the length and (b) the diameter of this wire?

20. Suppose you wish to fabricate a uniform wire from a mass m of a metal with density ρ_m and resistivity ρ. If the wire is to have a resistance of R and all the metal is to be used, what must be (a) the length and (b) the diameter of this wire?

21. A portion of Nichrome wire of radius 2.50 mm is to be used in winding a heating coil. If the coil must draw a current of 9.25 A when a voltage of 120 V is applied across its ends, find (a) the required resistance of the coil and (b) the length of wire you must use to wind the coil.

Section 27.3 A Model for Electrical Conduction

22. If the current carried by a conductor is doubled, what happens to (a) the charge carrier density, (b) the current density, (c) the electron drift velocity, and (d) the average time interval between collisions?

23. A current density of 6.00 × 10^{-13} A/m^2 exists in the atmosphere at a location where the electric field is 100 V/m. Calculate the electrical conductivity of the Earth's atmosphere in this region.

24. GP An iron wire has a cross-sectional area equal to 5.00 × 10^{-6} m^2. Carry out the following steps to determine the drift speed of the conduction electrons in the wire if it carries a current of 30.0 A. (a) How many kilograms are there in 1.00 mole of iron? (b) Starting with the density of iron and the result of part (a), compute the molar density of iron (the number of moles of iron per cubic meter). (c) Calculate the number density of iron atoms using Avogadro's number. (d) Obtain the number density of conduction electrons given that there are two conduction electrons per iron atom. (e) Calculate the drift speed of conduction electrons in this wire.

25. M If the magnitude of the drift velocity of free electrons in a copper wire is 7.84 × 10^{-4} m/s, what is the electric field in the conductor?

Section 27.4 Resistance and Temperature

26. A certain lightbulb has a tungsten filament with a resistance of 19.0 Ω when at 20.0°C and 140 Ω when hot. Assume the resistivity of tungsten varies linearly with temperature even over the large temperature range involved here. Find the temperature of the hot filament.

27. What is the fractional change in the resistance of an iron filament when its temperature changes from 25.0°C to 50.0°C?

28. While taking photographs in Death Valley on a day when the temperature is 58.0°C, Bill Hiker finds that a certain voltage applied to a copper wire produces a current of 1.00 A. Bill then travels to Antarctica and applies the same voltage to the same wire. What current does he register there if the temperature is −88.0°C? Assume that no change occurs in the wire's shape and size.

29. If a certain silver wire has a resistance of 6.00 Ω at 20.0°C, what resistance will it have at 34.0°C?

30. Plethysmographs are devices used for measuring changes in the volume of internal organs or limbs. In one form of this device, a rubber capillary tube with an inside diameter of 1.00 mm is filled with mercury at 20.0°C. The resistance of the mercury is measured with the aid of electrodes sealed into the ends of the tube. If 100 cm of the tube is wound in a helix around a patient's upper arm, the blood flow during a heartbeat causes the arm to expand, stretching the length of the tube by 0.040 0 cm. From this observation and assuming cylindrical symmetry, you can find the change in volume of the arm, which gives an indication of blood flow. Taking the resistivity of mercury to be 9.58 × 10^{-7} Ω · m, calculate (a) the resistance of the mercury and (b) the fractional change in resistance during the heartbeat. *Hint:* The fraction by which the cross-sectional area of the mercury column decreases is the fraction by which the length increases because the volume of mercury is constant.

31. M (a) A 34.5-m length of copper wire at 20.0°C has a radius of 0.25 mm. If a potential difference of 9.00 V is applied across the length of the wire, determine the current in the wire. (b) If the wire is heated to 30.0°C while the 9.00-V potential difference is maintained, what is the resulting current in the wire?

32. An engineer needs a resistor with a zero overall temperature coefficient of resistance at 20.0°C. She designs a pair of circular cylinders, one of carbon and one of Nichrome as shown in Figure P27.32 (page 828). The device must have an overall resistance of $R_1 + R_2 = 10.0\ \Omega$ independent of temperature and a uniform radius of $r = 1.50$ mm. Ignore thermal expansion of the cylinders and assume both are always at the same temperature. (a) Can she meet the design goal with this method? (b) If so, state what you can determine about the lengths ℓ_1 and ℓ_2 of each segment. If not, explain.

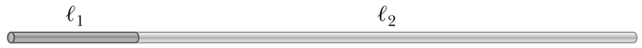


Figure P27.32

33. M An aluminum wire with a diameter of 0.100 mm has a uniform electric field of 0.200 V/m imposed along its entire length. The temperature of the wire is 50.0°C. Assume one free electron per atom. (a) Use the information in Table 27.2 to determine the resistivity of aluminum at this temperature. (b) What is the current density in the wire? (c) What is the total current in the wire? (d) What is the drift speed of the conduction electrons? (e) What potential difference must exist between the ends of a 2.00-m length of the wire to produce the stated electric field?

34. **Review.** An aluminum rod has a resistance of 1.23 Ω at 20.0°C. Calculate the resistance of the rod at 120°C by accounting for the changes in both the resistivity and the dimensions of the rod. The coefficient of linear expansion for aluminum is $2.40 \times 10^{-6}\ (°C)^{-1}$.

35. At what temperature will aluminum have a resistivity that is three times the resistivity copper has at room temperature?

Section 27.6 Electrical Power

36. Assume that global lightning on the Earth constitutes a constant current of 1.00 kA between the ground and an atmospheric layer at potential 300 kV. (a) Find the power of terrestrial lightning. (b) For comparison, find the power of sunlight falling on the Earth. Sunlight has an intensity of 1 370 W/m^2 above the atmosphere. Sunlight falls perpendicularly on the circular projected area that the Earth presents to the Sun.

37. In a hydroelectric installation, a turbine delivers 1 500 hp to a generator, which in turn transfers 80.0% of the mechanical energy out by electrical transmission. Under these conditions, what current does the generator deliver at a terminal potential difference of 2 000 V?

38. A Van de Graaff generator (see Fig. 25.23) is operating so that the potential difference between the high-potential electrode Ⓑ and the charging needles at Ⓐ is 15.0 kV. Calculate the power required to drive the belt against electrical forces at an instant when the effective current delivered to the high-potential electrode is 500 μA.

39. A certain waffle iron is rated at 1.00 kW when connected to a 120-V source. (a) What current does the waffle iron carry? (b) What is its resistance?

40. The potential difference across a resting neuron in the human body is about 75.0 mV and carries a current of about 0.200 mA. How much power does the neuron release?

41. Suppose your portable DVD player draws a current of 350 mA at 6.00 V. How much power does the player require?

42. AMT M **Review.** A well-insulated electric water heater warms 109 kg of water from 20.0°C to 49.0°C in 25.0 min. Find the resistance of its heating element, which is connected across a 240-V potential difference.

43. A 100-W lightbulb connected to a 120-V source experiences a voltage surge that produces 140 V for a moment. By what percentage does its power output increase? Assume its resistance does not change.

44. The cost of energy delivered to residences by electrical transmission varies from \$0.070/kWh to \$0.258/kWh throughout the United States; \$0.110/kWh is the average value. At this average price, calculate the cost of (a) leaving a 40.0-W porch light on for two weeks while you are on vacation, (b) making a piece of dark toast in 3.00 min with a 970-W toaster, and (c) drying a load of clothes in 40.0 min in a 5.20×10^3-W dryer.

45. W Batteries are rated in terms of ampere-hours (A · h). For example, a battery that can produce a current of 2.00 A for 3.00 h is rated at 6.00 A · h. (a) What is the total energy, in kilowatt-hours, stored in a 12.0-V battery rated at 55.0 A · h? (b) At \$0.110 per kilowatt-hour, what is the value of the electricity produced by this battery?

46. W Residential building codes typically require the use of 12-gauge copper wire (diameter 0.205 cm) for wiring receptacles. Such circuits carry currents as large as 20.0 A. If a wire of smaller diameter (with a higher gauge number) carried that much current, the wire could rise to a high temperature and cause a fire. (a) Calculate the rate at which internal energy is produced in 1.00 m of 12-gauge copper wire carrying 20.0 A. (b) **What If?** Repeat the calculation for a 12-gauge aluminum wire. (c) Explain whether a 12-gauge aluminum wire would be as safe as a copper wire.

47. M Assuming the cost of energy from the electric company is \$0.110/kWh, compute the cost per day of operating a lamp that draws a current of 1.70 A from a 110-V line.

48. An 11.0-W energy-efficient fluorescent lightbulb is designed to produce the same illumination as a conventional 40.0-W incandescent lightbulb. Assuming a cost of \$0.110/kWh for energy from the electric company, how much money does the user of the energy-efficient bulb save during 100 h of use?

49. A coil of Nichrome wire is 25.0 m long. The wire has a diameter of 0.400 mm and is at 20.0°C. If it carries a current of 0.500 A, what are (a) the magnitude of the electric field in the wire and (b) the power delivered to it? (c) **What If?** If the temperature is increased to 340°C and the potential difference across the wire remains constant, what is the power delivered?

50. **Review.** A rechargeable battery of mass 15.0 g delivers an average current of 18.0 mA to a portable DVD player at 1.60 V for 2.40 h before the battery must be recharged. The recharger maintains a potential difference of 2.30 V across the battery and delivers a charging current of 13.5 mA for 4.20 h. (a) What is the efficiency of the battery as an energy storage device? (b) How much internal energy is produced in the battery during one charge–discharge cycle? (c) If the battery is surrounded by ideal thermal insulation and has an effective specific heat of 975 J/kg · °C, by how much will its temperature increase during the cycle?

51. A 500-W heating coil designed to operate from 110 V is made of Nichrome wire 0.500 mm in diameter. (a) Assuming the resistivity of the Nichrome remains constant at its 20.0°C value, find the length of wire used. (b) **What If?** Now consider the variation of resistivity with temperature. What power is delivered to the coil of part (a) when it is warmed to 1 200°C?

52. *Why is the following situation impossible?* A politician is decrying wasteful uses of energy and decides to focus on energy used to operate plug-in electric clocks in the United States. He estimates there are 270 million of these clocks, approximately one clock for each person in the population. The clocks transform energy taken in by electrical transmission at the average rate 2.50 W. The politician gives a speech in which he complains that, at today's electrical rates, the nation is losing $100 million every year to operate these clocks.

53. M A certain toaster has a heating element made of Nichrome wire. When the toaster is first connected to a 120-V source (and the wire is at a temperature of 20.0°C), the initial current is 1.80 A. The current decreases as the heating element warms up. When the toaster reaches its final operating temperature, the current is 1.53 A. (a) Find the power delivered to the toaster when it is at its operating temperature. (b) What is the final temperature of the heating element?

54. Make an order-of-magnitude estimate of the cost of one person's routine use of a handheld hair dryer for 1 year. If you do not use a hair dryer yourself, observe or interview someone who does. State the quantities you estimate and their values.

55. M **Review.** The heating element of an electric coffee maker operates at 120 V and carries a current of 2.00 A. Assuming the water absorbs all the energy delivered to the resistor, calculate the time interval during which the temperature of 0.500 kg of water rises from room temperature (23.0°C) to the boiling point.

56. A 120-V motor has mechanical power output of 2.50 hp. It is 90.0% efficient in converting power that it takes in by electrical transmission into mechanical power. (a) Find the current in the motor. (b) Find the energy delivered to the motor by electrical transmission in 3.00 h of operation. (c) If the electric company charges $0.110/kWh, what does it cost to run the motor for 3.00 h?

Additional Problems

57. M A particular wire has a resistivity of $3.0 \times 10^{-8}\ \Omega \cdot \text{m}$ and a cross-sectional area of $4.0 \times 10^{-6}\ \text{m}^2$. A length of this wire is to be used as a resistor that will receive 48 W of power when connected across a 20-V battery. What length of wire is required?

58. Determine the temperature at which the resistance of an aluminum wire will be twice its value at 20.0°C. Assume its coefficient of resistivity remains constant.

59. A car owner forgets to turn off the headlights of his car while it is parked in his garage. If the 12.0-V battery in his car is rated at 90.0 A · h and each headlight requires 36.0 W of power, how long will it take the battery to completely discharge?

60. Lightbulb A is marked "25 W 120 V," and lightbulb B is marked "100 W 120 V." These labels mean that each lightbulb has its respective power delivered to it when it is connected to a constant 120-V source. (a) Find the resistance of each lightbulb. (b) During what time interval does 1.00 C pass into lightbulb A? (c) Is this charge different upon its exit versus its entry into the lightbulb? Explain. (d) In what time interval does 1.00 J pass into lightbulb A? (e) By what mechanisms does this energy enter and exit the lightbulb? Explain. (f) Find the cost of running lightbulb A continuously for 30.0 days, assuming the electric company sells its product at $0.110 per kWh.

61. W One wire in a high-voltage transmission line carries 1 000 A starting at 700 kV for a distance of 100 mi. If the resistance in the wire is 0.500 Ω/mi, what is the power loss due to the resistance of the wire?

62. An experiment is conducted to measure the electrical resistivity of Nichrome in the form of wires with different lengths and cross-sectional areas. For one set of measurements, a student uses 30-gauge wire, which has a cross-sectional area of $7.30 \times 10^{-8}\ \text{m}^2$. The student measures the potential difference across the wire and the current in the wire with a voltmeter and an ammeter, respectively. (a) For each set of measurements given in the table taken on wires of three different lengths, calculate the resistance of the wires and the corresponding values of the resistivity. (b) What is the average value of the resistivity? (c) Explain how this value compares with the value given in Table 27.2.

L (m)	ΔV (V)	I (A)	R (Ω)	ρ (Ω · m)
0.540	5.22	0.72		
1.028	5.82	0.414		
1.543	5.94	0.281		

63. A charge Q is placed on a capacitor of capacitance C. The capacitor is connected into the circuit shown in Figure P27.63, with an open switch, a resistor, and an initially uncharged capacitor of capacitance $3C$. The

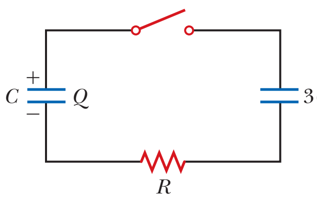


Figure P27.63

switch is then closed, and the circuit comes to equilibrium. In terms of Q and C, find (a) the final potential difference between the plates of each capacitor, (b) the charge on each capacitor, and (c) the final energy stored in each capacitor. (d) Find the internal energy appearing in the resistor.

64. **Review.** An office worker uses an immersion heater to warm 250 g of water in a light, covered, insulated cup from 20.0°C to 100°C in 4.00 min. The heater is a Nichrome resistance wire connected to a 120-V power supply. Assume the wire is at 100°C throughout the 4.00-min time interval. (a) Specify a relationship between a diameter and a length that the wire can have. (b) Can it be made from less than 0.500 cm^3 of Nichrome?

65. An x-ray tube used for cancer therapy operates at 4.00 MV with electrons constituting a beam current of 25.0 mA striking a metal target. Nearly all the power in the beam is transferred to a stream of water flowing through holes drilled in the target. What rate of flow, in kilograms per second, is needed if the rise in temperature of the water is not to exceed 50.0°C?

66. AMT M An all-electric car (not a hybrid) is designed to run from a bank of 12.0-V batteries with total energy storage of 2.00×10^7 J. If the electric motor draws 8.00 kW as the car moves at a steady speed of 20.0 m/s, (a) what is the current delivered to the motor? (b) How far can the car travel before it is "out of juice"?

67. A straight, cylindrical wire lying along the x axis has a length of 0.500 m and a diameter of 0.200 mm. It is made of a material described by Ohm's law with a resistivity of $\rho = 4.00 \times 10^{-8}\ \Omega \cdot$ m. Assume a potential of 4.00 V is maintained at the left end of the wire at $x = 0$. Also assume $V = 0$ at $x = 0.500$ m. Find (a) the magnitude and direction of the electric field in the wire, (b) the resistance of the wire, (c) the magnitude and direction of the electric current in the wire, and (d) the current density in the wire. (e) Show that $E = \rho J$.

68. A straight, cylindrical wire lying along the x axis has a length L and a diameter d. It is made of a material described by Ohm's law with a resistivity ρ. Assume potential V is maintained at the left end of the wire at $x = 0$. Also assume the potential is zero at $x = L$. In terms of L, d, V, ρ, and physical constants, derive expressions for (a) the magnitude and direction of the electric field in the wire, (b) the resistance of the wire, (c) the magnitude and direction of the electric current in the wire, and (d) the current density in the wire. (e) Show that $E = \rho J$.

69. W An electric utility company supplies a customer's house from the main power lines (120 V) with two copper wires, each of which is 50.0 m long and has a resistance of 0.108 Ω per 300 m. (a) Find the potential difference at the customer's house for a load current of 110 A. For this load current, find (b) the power delivered to the customer and (c) the rate at which internal energy is produced in the copper wires.

70. The strain in a wire can be monitored and computed by measuring the resistance of the wire. Let L_i represent the original length of the wire, A_i its original cross-sectional area, $R_i = \rho L_i/A_i$ the original resistance between its ends, and $\delta = \Delta L/L_i = (L - L_i)/L_i$ the strain resulting from the application of tension. Assume the resistivity and the volume of the wire do not change as the wire stretches. (a) Show that the resistance between the ends of the wire under strain is given by $R = R_i(1 + 2\delta + \delta^2)$. (b) If the assumptions are precisely true, is this result exact or approximate? Explain your answer.

71. An oceanographer is studying how the ion concentration in seawater depends on depth. She makes a measurement by lowering into the water a pair of concentric metallic cylinders (Fig. P27.71) at the end of a cable and taking data to determine the resistance between these electrodes as a function of depth. The water between the two cylinders forms a cylindrical shell of inner radius r_a, outer radius r_b, and length L much larger than r_b. The scientist applies a potential difference ΔV between the inner and outer surfaces, producing an outward radial current I. Let ρ represent the resistivity of the water. (a) Find the resistance of the water between the cylinders in terms of L, ρ, r_a, and r_b. (b) Express the resistivity of the water in terms of the measured quantities L, r_a, r_b, ΔV, and I.

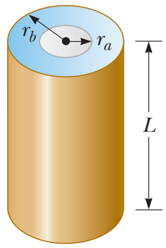


Figure P27.71

72. *Why is the following situation impossible?* An inquisitive physics student takes a 100-W incandescent lightbulb out of its socket and measures its resistance with an ohmmeter. He measures a value of 10.5 Ω. He is able to connect an ammeter to the lightbulb socket to correctly measure the current drawn by the bulb while operating. Inserting the bulb back into the socket and operating the bulb from a 120-V source, he measures the current to be 11.4 A.

73. The temperature coefficients of resistivity α in Table 27.2 are based on a reference temperature T_0 of 20.0°C. Suppose the coefficients were given the symbol α' and were based on a T_0 of 0°C. What would the coefficient α' for silver be? *Note:* The coefficient α satisfies $\rho = \rho_0[1 + \alpha(T - T_0)]$, where ρ_0 is the resistivity of the material at $T_0 = 20.0$°C. The coefficient α' must satisfy the expression $\rho = \rho_0'[1 + \alpha' T]$, where ρ_0' is the resistivity of the material at 0°C.

74. A close analogy exists between the flow of energy by heat because of a temperature difference (see Section 20.7) and the flow of electric charge because of a potential difference. In a metal, energy dQ and electrical charge dq are both transported by free electrons. Consequently, a good electrical conductor is usually a good thermal conductor as well. Consider a thin conducting slab of thickness dx, area A, and electrical conductivity σ, with a potential difference dV between opposite faces. (a) Show that the current $I = dq/dt$ is given by the equation on the left:

Charge conduction	Thermal conduction
$\frac{dq}{dt} = \sigma A \left\lvert \frac{dV}{dx} \right\rvert$	$\frac{dQ}{dt} = kA \left\lvert \frac{dT}{dx} \right\rvert$

In the analogous thermal conduction equation on the right (Eq. 20.15), the rate dQ/dt of energy flow by heat (in SI units of joules per second) is due to a temperature gradient dT/dx in a material of thermal conductivity k. (b) State analogous rules relating the direction of the electric current to the change in potential and relating the direction of energy flow to the change in temperature.

75. **Review.** When a straight wire is warmed, its resistance is given by $R = R_0[1 + \alpha(T - T_0)]$ according to Equation 27.20, where α is the temperature coefficient of resistivity. This expression needs to be modified if we include the change in dimensions of the wire due to thermal expansion. For a copper wire of radius 0.100 0 mm and length 2.000 m, find its resistance at 100.0°C, including the effects of both thermal expansion and temperature variation of resistivity. Assume the coefficients are known to four significant figures.

76. **Review.** When a straight wire is warmed, its resistance is given by $R = R_0[1 + \alpha(T - T_0)]$ according to Equation 27.20, where α is the temperature coefficient of resistivity. This expression needs to be modified if we include the change in dimensions of the wire due to thermal expansion. Find a more precise expression for the resistance, one that includes the effects of changes in the dimensions of the wire when it is warmed. Your final expression should be in terms of R_0, T, T_0, the temperature coefficient of resistivity α, and the coefficient of linear expansion α'.

77. **Review.** A parallel-plate capacitor consists of square plates of edge length ℓ that are separated by a distance d, where $d << \ell$. A potential difference ΔV is maintained between the plates. A material of dielectric constant κ fills half the space between the plates. The dielectric slab is withdrawn from the capacitor as shown in Figure P27.77. (a) Find the capacitance when the left edge of the dielectric is at a distance x from the center of the capacitor. (b) If the dielectric is removed at a constant speed v, what is the current in the circuit as the dielectric is being withdrawn?

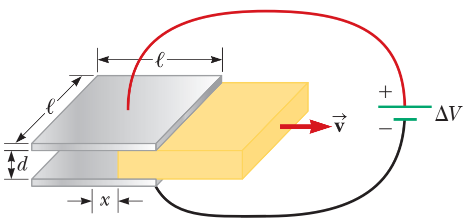


Figure P27.77

78. The dielectric material between the plates of a parallel-plate capacitor always has some nonzero conductivity σ. Let A represent the area of each plate and d the distance between them. Let κ represent the dielectric constant of the material. (a) Show that the resistance R and the capacitance C of the capacitor are related by

$$RC = \frac{\kappa \epsilon_0}{\sigma}$$

(b) Find the resistance between the plates of a 14.0-nF capacitor with a fused quartz dielectric.

79. Gold is the most ductile of all metals. For example, one gram of gold can be drawn into a wire 2.40 km long. The density of gold is 19.3×10^3 kg/m^3, and its resistivity is $2.44 \times 10^{-8}\ \Omega \cdot$ m. What is the resistance of such a wire at 20.0°C?

80. The current–voltage characteristic curve for a semiconductor diode as a function of temperature T is given by

$$I = I_0(e^{e\,\Delta V/k_B T} - 1)$$

Here the first symbol e represents Euler's number, the base of natural logarithms. The second e is the magnitude of the electron charge, the k_B stands for Boltzmann's constant, and T is the absolute temperature. (a) Set up a spreadsheet to calculate I and $R = \Delta V/I$ for $\Delta V = 0.400$ V to 0.600 V in increments of 0.005 V. Assume $I_0 = 1.00$ nA. (b) Plot R versus ΔV for $T = 280$ K, 300 K, and 320 K.

81. The potential difference across the filament of a lightbulb is maintained at a constant value while equilibrium temperature is being reached. The steady-state current in the bulb is only one-tenth of the current drawn by the bulb when it is first turned on. If the temperature coefficient of resistivity for the bulb at 20.0°C is 0.004 50 $(°\text{C})^{-1}$ and the resistance increases linearly with increasing temperature, what is the final operating temperature of the filament?

Challenge Problems

82. A more general definition of the temperature coefficient of resistivity is

$$\alpha = \frac{1}{\rho}\frac{d\rho}{dT}$$

where ρ is the resistivity at temperature T. (a) Assuming α is constant, show that

$$\rho = \rho_0 e^{\alpha(T - T_0)}$$

where ρ_0 is the resistivity at temperature T_0. (b) Using the series expansion $e^x \approx 1 + x$ for $x << 1$, show that the resistivity is given approximately by the expression

$$\rho = \rho_0[1 + \alpha(T - T_0)] \quad \text{for } \alpha(T - T_0) << 1$$

83. A spherical shell with inner radius r_a and outer radius r_b is formed from a material of resistivity ρ. It carries current radially, with uniform density in all directions. Show that its resistance is

$$R = \frac{\rho}{4\pi}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

84. Material with uniform resistivity ρ is formed into a wedge as shown in Figure P27.84. Show that the resistance between face A and face B of this wedge is

$$R = \rho \frac{L}{w(y_2 - y_1)} \ln \frac{y_2}{y_1}$$

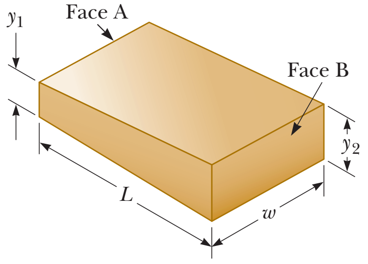


Figure P27.84

85. A material of resistivity ρ is formed into the shape of a truncated cone of height h as shown in Figure P27.85. The bottom end has radius b, and the top end has radius a. Assume the current is distributed uniformly over any circular cross section of the cone so that the current density does not depend on radial position. (The current density does vary with position along the axis of the cone.) Show that the resistance between the two ends is

$$R = \frac{\rho}{\pi}\left(\frac{h}{ab}\right)$$

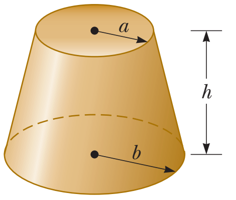


Figure P27.85