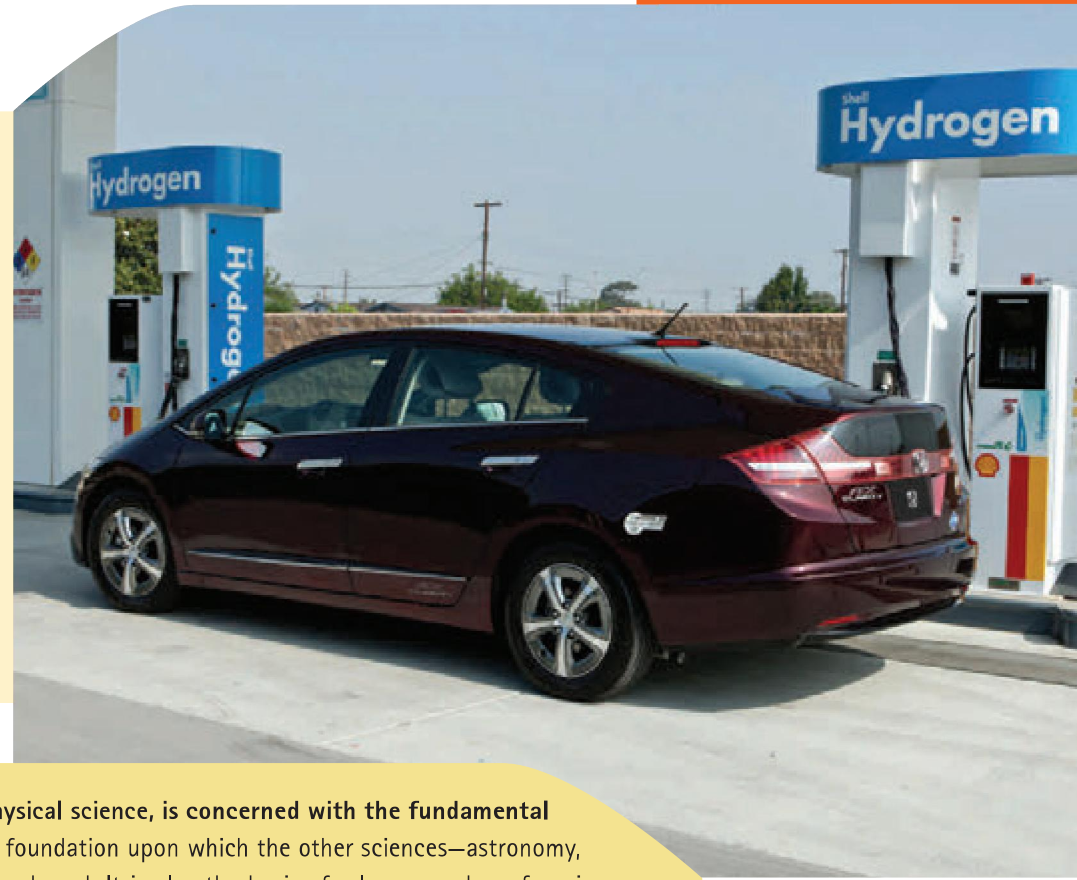
PART 1

# Mechanics




The Honda FCX Clarity, a fuel-cell-powered automobile available to the public, albeit in limited quantities. A fuel cell converts hydrogen fuel into electricity to drive the motor attached to the wheels of the car. Automobiles, whether powered by fuel cells, gasoline engines, or batteries, use many of the concepts and principles of mechanics that we will study in this first part of the book. Quantities that we can use to describe the operation of vehicles include position, velocity, acceleration, force, energy, and momentum. *(PRNewsFoto/American Honda)*

**Physics, the most fundamental physical science, is concerned with the fundamental** principles of the Universe. It is the foundation upon which the other sciences—astronomy, biology, chemistry, and geology—are based. It is also the basis of a large number of engineering applications. The beauty of physics lies in the simplicity of its fundamental principles and in the manner in which just a small number of concepts and models can alter and expand our view of the world around us.

The study of physics can be divided into six main areas:

1. *classical mechanics,* concerning the motion of objects that are large relative to atoms and move at speeds much slower than the speed of light
2. *relativity,* a theory describing objects moving at any speed, even speeds approaching the speed of light
3. *thermodynamics,* dealing with heat, work, temperature, and the statistical behavior of systems with large numbers of particles
4. *electromagnetism,* concerning electricity, magnetism, and electromagnetic fields
5. *optics,* the study of the behavior of light and its interaction with materials
6. *quantum mechanics,* a collection of theories connecting the behavior of matter at the submicroscopic level to macroscopic observations

The disciplines of mechanics and electromagnetism are basic to all other branches of classical physics (developed before 1900) and modern physics (c. 1900–present). The first part of this textbook deals with classical mechanics, sometimes referred to as *Newtonian mechanics* or simply *mechanics.* Many principles and models used to understand mechanical systems retain their importance in the theories of other areas of physics and can later be used to describe many natural phenomena. Therefore, classical mechanics is of vital importance to students from all disciplines. ■

CHAPTER

# 1

# Physics and Measurement



Stonehenge, in southern England, was built thousands of years ago. Various theories have been proposed about its function, including a burial ground, a healing site, and a place for ancestor worship. One of the more intriguing theories suggests that Stonehenge was an observatory, allowing measurements of some of the quantities discussed in this chapter, such as position of objects in space and time intervals between repeating celestial events. *(Stephen Inglis/Shutterstock.com)*

**Like all other sciences, physics is based on experimental observations and quantitative** measurements. The main objectives of physics are to identify a limited number of fundamental laws that govern natural phenomena and use them to develop theories that can predict the results of future experiments. The fundamental laws used in developing theories are expressed in the language of mathematics, the tool that provides a bridge between theory and experiment.

When there is a discrepancy between the prediction of a theory and experimental results, new or modified theories must be formulated to remove the discrepancy. Many times a theory is satisfactory only under limited conditions; a more general theory might be satisfactory without such limitations. For example, the laws of motion discovered by Isaac Newton (1642–1727) accurately describe the motion of objects moving at normal speeds but do not apply to objects moving at speeds comparable to the speed of light. In contrast, the special theory of relativity developed later by Albert Einstein (1879–1955) gives the same results as Newton's laws at low speeds but also correctly describes the motion of objects at speeds approaching the speed of light. Hence, Einstein's special theory of relativity is a more general theory of motion than that formed from Newton's laws.

Classical physics includes the principles of classical mechanics, thermodynamics, optics, and electromagnetism developed before 1900. Important contributions to classical physics were provided by Newton, who was also one of the originators of calculus as a mathematical tool. Major developments in mechanics continued in the 18th century, but the fields of thermodynamics and electromagnetism were not developed until the latter part of the 19th century, principally because before that time the apparatus for controlled experiments in these disciplines was either too crude or unavailable.

A major revolution in physics, usually referred to as *modern physics,* began near the end of the 19th century. Modern physics developed mainly because many physical phenomena could not be explained by classical physics. The two most important developments in this modern era were the theories of relativity and quantum mechanics. Einstein's special theory of relativity not only correctly describes the motion of objects moving at speeds comparable to the speed of light; it also completely modifies the traditional concepts of space, time, and energy. The theory also shows that the speed of light is the upper limit of the speed of an object and that mass and energy are related. Quantum mechanics was formulated by a number of distinguished scientists to provide descriptions of physical phenomena at the atomic level. Many practical devices have been developed using the principles of quantum mechanics.

Scientists continually work at improving our understanding of fundamental laws. Numerous technological advances in recent times are the result of the efforts of many scientists, engineers, and technicians, such as unmanned planetary explorations, a variety of developments and potential applications in nanotechnology, microcircuitry and high-speed computers, sophisticated imaging techniques used in scientific research and medicine, and several remarkable results in genetic engineering. The effects of such developments and discoveries on our society have indeed been great, and it is very likely that future discoveries and developments will be exciting, challenging, and of great benefit to humanity.

## 1.1 Standards of Length, Mass, and Time

To describe natural phenomena, we must make measurements of various aspects of nature. Each measurement is associated with a physical quantity, such as the length of an object. The laws of physics are expressed as mathematical relationships among physical quantities that we will introduce and discuss throughout the book. In mechanics, the three fundamental quantities are length, mass, and time. All other quantities in mechanics can be expressed in terms of these three.

If we are to report the results of a measurement to someone who wishes to reproduce this measurement, a *standard* must be defined. It would be meaningless if a visitor from another planet were to talk to us about a length of 8 "glitches" if we do not know the meaning of the unit glitch. On the other hand, if someone familiar with our system of measurement reports that a wall is 2 meters high and our unit of length is defined to be 1 meter, we know that the height of the wall is twice our basic length unit. Whatever is chosen as a standard must be readily accessible and must possess some property that can be measured reliably. Measurement standards used by different people in different places—throughout the Universe—must yield the same result. In addition, standards used for measurements must not change with time.

In 1960, an international committee established a set of standards for the fundamental quantities of science. It is called the **SI** (Système International), and its fundamental units of length, mass, and time are the *meter, kilogram,* and *second,* respectively. Other standards for SI fundamental units established by the committee are those for temperature (the *kelvin*), electric current (the *ampere*), luminous intensity (the *candela*), and the amount of substance (the *mole*).

## Length

We can identify **length** as the distance between two points in space. In 1120, the king of England decreed that the standard of length in his country would be named the *yard* and would be precisely equal to the distance from the tip of his nose to the end of his outstretched arm. Similarly, the original standard for the foot adopted by the French was the length of the royal foot of King Louis XIV. Neither of these standards is constant in time; when a new king took the throne, length measurements changed! The French standard prevailed until 1799, when the legal standard of length in France became the **meter** (m), defined as one ten-millionth of the distance from the equator to the North Pole along one particular longitudinal line that passes through Paris. Notice that this value is an Earth-based standard that does not satisfy the requirement that it can be used throughout the Universe.

As recently as 1960, the length of the meter was defined as the distance between two lines on a specific platinum–iridium bar stored under controlled conditions in France. Current requirements of science and technology, however, necessitate more accuracy than that with which the separation between the lines on the bar can be determined. In the 1960s and 1970s, the meter was defined as 1 650 763.73 wavelengths[1] of orange-red light emitted from a krypton-86 lamp. In October 1983, however, the meter was redefined as **the distance traveled by light in vacuum during a time of 1/299 792 458 second.** In effect, this latest definition establishes that the speed of light in vacuum is precisely 299 792 458 meters per second. This definition of the meter is valid throughout the Universe based on our assumption that light is the same everywhere.

Table 1.1 lists approximate values of some measured lengths. You should study this table as well as the next two tables and begin to generate an intuition for what is meant by, for example, a length of 20 centimeters, a mass of 100 kilograms, or a time interval of $3.2 \times 10^7$ seconds.

**Pitfall Prevention 1.1**

**Reasonable Values** Generating intuition about typical values of quantities when solving problems is important because you must think about your end result and determine if it seems reasonable. For example, if you are calculating the mass of a housefly and arrive at a value of 100 kg, this answer is *unreasonable* and there is an error somewhere.

## Mass

The SI fundamental unit of **mass,** the **kilogram** (kg), is defined as **the mass of a specific platinum–iridium alloy cylinder kept at the International Bureau of Weights and Measures at Sèvres, France.** This mass standard was established in 1887 and

**Table 1.1 Approximate Values of Some Measured Lengths**

| | Length (m) |
|---|---|
| Distance from the Earth to the most remote known quasar | $1.4 \times 10^{26}$ |
| Distance from the Earth to the most remote normal galaxies | $9 \times 10^{25}$ |
| Distance from the Earth to the nearest large galaxy (Andromeda) | $2 \times 10^{22}$ |
| Distance from the Sun to the nearest star (Proxima Centauri) | $4 \times 10^{16}$ |
| One light-year | $9.46 \times 10^{15}$ |
| Mean orbit radius of the Earth about the Sun | $1.50 \times 10^{11}$ |
| Mean distance from the Earth to the Moon | $3.84 \times 10^{8}$ |
| Distance from the equator to the North Pole | $1.00 \times 10^{7}$ |
| Mean radius of the Earth | $6.37 \times 10^{6}$ |
| Typical altitude (above the surface) of a satellite orbiting the Earth | $2 \times 10^{5}$ |
| Length of a football field | $9.1 \times 10^{1}$ |
| Length of a housefly | $5 \times 10^{-3}$ |
| Size of smallest dust particles | $\sim 10^{-4}$ |
| Size of cells of most living organisms | $\sim 10^{-5}$ |
| Diameter of a hydrogen atom | $\sim 10^{-10}$ |
| Diameter of an atomic nucleus | $\sim 10^{-14}$ |
| Diameter of a proton | $\sim 10^{-15}$ |

[1]We will use the standard international notation for numbers with more than three digits, in which groups of three digits are separated by spaces rather than commas. Therefore, 10 000 is the same as the common American notation of 10,000. Similarly, $\pi = 3.14159265$ is written as 3.141 592 65.

**Table 1.2** **Approximate Masses of Various Objects**

| | Mass (kg) |
|---|---|
| Observable Universe | ~ $10^{52}$ |
| Milky Way galaxy | ~ $10^{42}$ |
| Sun | $1.99 \times 10^{30}$ |
| Earth | $5.98 \times 10^{24}$ |
| Moon | $7.36 \times 10^{22}$ |
| Shark | ~ $10^{3}$ |
| Human | ~ $10^{2}$ |
| Frog | ~ $10^{-1}$ |
| Mosquito | ~ $10^{-5}$ |
| Bacterium | ~ $1 \times 10^{-15}$ |
| Hydrogen atom | $1.67 \times 10^{-27}$ |
| Electron | $9.11 \times 10^{-31}$ |

**Table 1.3** **Approximate Values of Some Time Intervals**

| | Time Interval (s) |
|---|---|
| Age of the Universe | $4 \times 10^{17}$ |
| Age of the Earth | $1.3 \times 10^{17}$ |
| Average age of a college student | $6.3 \times 10^{8}$ |
| One year | $3.2 \times 10^{7}$ |
| One day | $8.6 \times 10^{4}$ |
| One class period | $3.0 \times 10^{3}$ |
| Time interval between normal heartbeats | $8 \times 10^{-1}$ |
| Period of audible sound waves | ~ $10^{-3}$ |
| Period of typical radio waves | ~ $10^{-6}$ |
| Period of vibration of an atom in a solid | ~ $10^{-13}$ |
| Period of visible light waves | ~ $10^{-15}$ |
| Duration of a nuclear collision | ~ $10^{-22}$ |
| Time interval for light to cross a proton | ~ $10^{-24}$ |

has not been changed since that time because platinum–iridium is an unusually stable alloy. A duplicate of the Sèvres cylinder is kept at the National Institute of Standards and Technology (NIST) in Gaithersburg, Maryland (Fig. 1.1a). Table 1.2 lists approximate values of the masses of various objects.

## Time

Before 1967, the standard of **time** was defined in terms of the *mean solar day.* (A solar day is the time interval between successive appearances of the Sun at the highest point it reaches in the sky each day.) The fundamental unit of a **second** (s) was defined as $(\frac{1}{60})(\frac{1}{60})(\frac{1}{24})$ of a mean solar day. This definition is based on the rotation of one planet, the Earth. Therefore, this motion does not provide a time standard that is universal.

In 1967, the second was redefined to take advantage of the high precision attainable in a device known as an *atomic clock* (Fig. 1.1b), which measures vibrations of cesium atoms. One second is now defined as **9 192 631 770 times the period of vibration of radiation from the cesium-133 atom.**[2] Approximate values of time intervals are presented in Table 1.3.

In addition to SI, another system of units, the *U.S. customary system,* is still used in the United States despite acceptance of SI by the rest of the world. In this system, the units of length, mass, and time are the foot (ft), slug, and second, respectively. In this book, we shall use SI units because they are almost universally accepted in science and industry. We shall make some limited use of U.S. customary units in the study of classical mechanics.

In addition to the fundamental SI units of meter, kilogram, and second, we can also use other units, such as millimeters and nanoseconds, where the prefixes *milli-* and *nano-* denote multipliers of the basic units based on various powers of ten. Prefixes for the various powers of ten and their abbreviations are listed in Table 1.4 (page 6). For example, $10^{-3}$ m is equivalent to 1 millimeter (mm), and $10^{3}$ m corresponds to 1 kilometer (km). Likewise, 1 kilogram (kg) is $10^{3}$ grams (g), and 1 mega volt (MV) is $10^{6}$ volts (V).

The variables length, time, and mass are examples of *fundamental quantities.* Most other variables are *derived quantities,* those that can be expressed as a mathematical combination of fundamental quantities. Common examples are *area* (a product of two lengths) and *speed* (a ratio of a length to a time interval).

[2]Period is defined as the time interval needed for one complete vibration.



a




b

**Figure 1.1** (a) The National Standard Kilogram No. 20, an accurate copy of the International Standard Kilogram kept at Sèvres, France, is housed under a double bell jar in a vault at the National Institute of Standards and Technology. (b) A cesium fountain atomic clock. The clock will neither gain nor lose a second in 20 million years.

**Table 1.4 Prefixes for Powers of Ten**

| Power | Prefix | Abbreviation | Power | Prefix | Abbreviation |
|---|---|---|---|---|---|
| $10^{-24}$ | yocto | y | $10^{3}$ | kilo | k |
| $10^{-21}$ | zepto | z | $10^{6}$ | mega | M |
| $10^{-18}$ | atto | a | $10^{9}$ | giga | G |
| $10^{-15}$ | femto | f | $10^{12}$ | tera | T |
| $10^{-12}$ | pico | p | $10^{15}$ | peta | P |
| $10^{-9}$ | nano | n | $10^{18}$ | exa | E |
| $10^{-6}$ | micro | $\mu$ | $10^{21}$ | zetta | Z |
| $10^{-3}$ | milli | m | $10^{24}$ | yotta | Y |
| $10^{-2}$ | centi | c | | | |
| $10^{-1}$ | deci | d | | | |

**A table of the letters in the Greek alphabet is provided on the back endpaper of this book.**

Another example of a derived quantity is **density.** The density $\rho$ (Greek letter rho) of any substance is defined as its *mass per unit volume:*

$$\rho \equiv \frac{m}{V} \tag{1.1}$$

In terms of fundamental quantities, density is a ratio of a mass to a product of three lengths. Aluminum, for example, has a density of $2.70 \times 10^3$ kg/m$^3$, and iron has a density of $7.86 \times 10^3$ kg/m$^3$. An extreme difference in density can be imagined by thinking about holding a 10-centimeter (cm) cube of Styrofoam in one hand and a 10-cm cube of lead in the other. See Table 14.1 in Chapter 14 for densities of several materials.

**Quick Quiz 1.1** In a machine shop, two cams are produced, one of aluminum and one of iron. Both cams have the same mass. Which cam is larger? **(a)** The aluminum cam is larger. **(b)** The iron cam is larger. **(c)** Both cams have the same size.

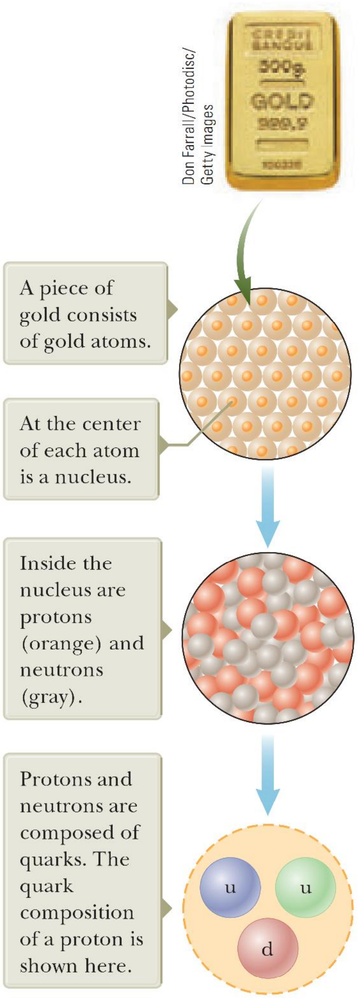



**Figure 1.2** Levels of organization in matter.

## 1.2 Matter and Model Building

If physicists cannot interact with some phenomenon directly, they often imagine a **model** for a physical system that is related to the phenomenon. For example, we cannot interact directly with atoms because they are too small. Therefore, we build a mental model of an atom based on a system of a nucleus and one or more electrons outside the nucleus. Once we have identified the physical components of the model, we make predictions about its behavior based on the interactions among the components of the system or the interaction between the system and the environment outside the system.

As an example, consider the behavior of *matter.* A sample of solid gold is shown at the top of Figure 1.2. Is this sample nothing but wall-to-wall gold, with no empty space? If the sample is cut in half, the two pieces still retain their chemical identity as solid gold. What if the pieces are cut again and again, indefinitely? Will the smaller and smaller pieces always be gold? Such questions can be traced to early Greek philosophers. Two of them—Leucippus and his student Democritus—could not accept the idea that such cuttings could go on forever. They developed a model for matter by speculating that the process ultimately must end when it produces a particle that can no longer be cut. In Greek, *atomos* means "not sliceable." From this Greek term comes our English word *atom.*

The Greek model of the structure of matter was that all ordinary matter consists of atoms, as suggested in the middle of Figure 1.2. Beyond that, no additional structure was specified in the model; atoms acted as small particles that interacted with one another, but internal structure of the atom was not a part of the model.

In 1897, J. J. Thomson identified the electron as a charged particle and as a constituent of the atom. This led to the first atomic model that contained internal structure. We shall discuss this model in Chapter 42.

Following the discovery of the nucleus in 1911, an atomic model was developed in which each atom is made up of electrons surrounding a central nucleus. A nucleus of gold is shown in Figure 1.2. This model leads, however, to a new question: Does the nucleus have structure? That is, is the nucleus a single particle or a collection of particles? By the early 1930s, a model evolved that described two basic entities in the nucleus: protons and neutrons. The proton carries a positive electric charge, and a specific chemical element is identified by the number of protons in its nucleus. This number is called the **atomic number** of the element. For instance, the nucleus of a hydrogen atom contains one proton (so the atomic number of hydrogen is 1), the nucleus of a helium atom contains two protons (atomic number 2), and the nucleus of a uranium atom contains 92 protons (atomic number 92). In addition to atomic number, a second number—**mass number,** defined as the number of protons plus neutrons in a nucleus—characterizes atoms. The atomic number of a specific element never varies (i.e., the number of protons does not vary), but the mass number can vary (i.e., the number of neutrons varies).

Is that, however, where the process of breaking down stops? Protons, neutrons, and a host of other exotic particles are now known to be composed of six different varieties of particles called **quarks,** which have been given the names of *up, down, strange, charmed, bottom,* and *top.* The up, charmed, and top quarks have electric charges of $+\frac{2}{3}$ that of the proton, whereas the down, strange, and bottom quarks have charges of $-\frac{1}{3}$ that of the proton. The proton consists of two up quarks and one down quark as shown at the bottom of Figure 1.2 and labeled u and d. This structure predicts the correct charge for the proton. Likewise, the neutron consists of two down quarks and one up quark, giving a net charge of zero.

You should develop a process of building models as you study physics. In this study, you will be challenged with many mathematical problems to solve. One of the most important problem-solving techniques is to build a model for the problem: identify a system of physical components for the problem and make predictions of the behavior of the system based on the interactions among its components or the interaction between the system and its surrounding environment.

## 1.3 Dimensional Analysis

In physics, the word *dimension* denotes the physical nature of a quantity. The distance between two points, for example, can be measured in feet, meters, or furlongs, which are all different ways of expressing the dimension of length.

The symbols we use in this book to specify the dimensions of length, mass, and time are L, M, and T, respectively.[3] We shall often use brackets [ ] to denote the dimensions of a physical quantity. For example, the symbol we use for speed in this book is $v$, and in our notation, the dimensions of speed are written $[v] = \mathrm{L/T}$. As another example, the dimensions of area $A$ are $[A] = \mathrm{L}^2$. The dimensions and units of area, volume, speed, and acceleration are listed in Table 1.5. The dimensions of other quantities, such as force and energy, will be described as they are introduced in the text.

**Table 1.5 Dimensions and Units of Four Derived Quantities**

| Quantity | Area ($A$) | Volume ($V$) | Speed ($v$) | Acceleration ($a$) |
|---|---|---|---|---|
| Dimensions | $\mathrm{L}^2$ | $\mathrm{L}^3$ | L/T | $\mathrm{L/T}^2$ |
| SI units | $\mathrm{m}^2$ | $\mathrm{m}^3$ | m/s | $\mathrm{m/s}^2$ |
| U.S. customary units | $\mathrm{ft}^2$ | $\mathrm{ft}^3$ | ft/s | $\mathrm{ft/s}^2$ |

[3]The *dimensions* of a quantity will be symbolized by a capitalized, nonitalic letter such as L or T. The *algebraic symbol* for the quantity itself will be an italicized letter such as $L$ for the length of an object or $t$ for time.

In many situations, you may have to check a specific equation to see if it matches your expectations. A useful procedure for doing that, called **dimensional analysis,** can be used because dimensions can be treated as algebraic quantities. For example, quantities can be added or subtracted only if they have the same dimensions. Furthermore, the terms on both sides of an equation must have the same dimensions. By following these simple rules, you can use dimensional analysis to determine whether an expression has the correct form. Any relationship can be correct only if the dimensions on both sides of the equation are the same.

**Pitfall Prevention 1.2**

**Symbols for Quantities** Some quantities have a small number of symbols that represent them. For example, the symbol for time is almost always $t$. Other quantities might have various symbols depending on the usage. Length may be described with symbols such as $x$, $y$, and $z$ (for position); $r$ (for radius); $a$, $b$, and $c$ (for the legs of a right triangle); $\ell$ (for the length of an object); $d$ (for a distance); $h$ (for a height); and so forth.

To illustrate this procedure, suppose you are interested in an equation for the position $x$ of a car at a time $t$ if the car starts from rest at $x = 0$ and moves with constant acceleration $a$. The correct expression for this situation is $x = \frac{1}{2}at^2$ as we show in Chapter 2. The quantity $x$ on the left side has the dimension of length. For the equation to be dimensionally correct, the quantity on the right side must also have the dimension of length. We can perform a dimensional check by substituting the dimensions for acceleration, $L/T^2$ (Table 1.5), and time, T, into the equation. That is, the dimensional form of the equation $x = \frac{1}{2}at^2$ is

$$\mathrm{L} = \frac{\mathrm{L}}{\cancel{\mathrm{T}^2}} \cdot \cancel{\mathrm{T}^2} = \mathrm{L}$$

The dimensions of time cancel as shown, leaving the dimension of length on the right-hand side to match that on the left.

A more general procedure using dimensional analysis is to set up an expression of the form

$$x \propto a^n t^m$$

where $n$ and $m$ are exponents that must be determined and the symbol $\propto$ indicates a proportionality. This relationship is correct only if the dimensions of both sides are the same. Because the dimension of the left side is length, the dimension of the right side must also be length. That is,

$$[a^n t^m] = \mathrm{L} = \mathrm{L}^1\mathrm{T}^0$$

Because the dimensions of acceleration are $L/T^2$ and the dimension of time is T, we have

$$(\mathrm{L}/\mathrm{T}^2)^n\mathrm{T}^m = \mathrm{L}^1\mathrm{T}^0 \quad \rightarrow \quad (\mathrm{L}^n\mathrm{T}^{m-2n}) = \mathrm{L}^1\mathrm{T}^0$$

The exponents of L and T must be the same on both sides of the equation. From the exponents of L, we see immediately that $n = 1$. From the exponents of T, we see that $m - 2n = 0$, which, once we substitute for $n$, gives us $m = 2$. Returning to our original expression $x \propto a^n t^m$, we conclude that $x \propto at^2$ .

**Quick Quiz 1.2** True or False: Dimensional analysis can give you the numerical value of constants of proportionality that may appear in an algebraic expression.

## Example 1.1 Analysis of an Equation

Show that the expression $v = at$, where $v$ represents speed, $a$ acceleration, and $t$ an instant of time, is dimensionally correct.

**SOLUTION**

Identify the dimensions of $v$ from Table 1.5: $[v] = \frac{\mathrm{L}}{\mathrm{T}}$

1.1 continued

Identify the dimensions of $a$ from Table 1.5 and multiply by the dimensions of $t$:

$$[at] = \frac{\text{L}}{\text{T}^{\not 2}}\not\text{T} = \frac{\text{L}}{\text{T}}$$

Therefore, $v = at$ is dimensionally correct because we have the same dimensions on both sides. (If the expression were given as $v = at^2$, it would be dimensionally *incorrect*. Try it and see!)

## Example 1.2 Analysis of a Power Law

Suppose we are told that the acceleration $a$ of a particle moving with uniform speed $v$ in a circle of radius $r$ is proportional to some power of $r$, say $r^n$, and some power of $v$, say $v^m$. Determine the values of $n$ and $m$ and write the simplest form of an equation for the acceleration.

**SOLUTION**

Write an expression for $a$ with a dimensionless constant of proportionality $k$:

$$a = kr^n v^m$$

Substitute the dimensions of $a$, $r$, and $v$:

$$\frac{\text{L}}{\text{T}^2} = \text{L}^n\left(\frac{\text{L}}{\text{T}}\right)^m = \frac{\text{L}^{n+m}}{\text{T}^m}$$

Equate the exponents of L and T so that the dimensional equation is balanced:

$$n + m = 1 \text{ and } m = 2$$

Solve the two equations for $n$:

$$n = -1$$

Write the acceleration expression:

$$a = kr^{-1}v^2 = k\frac{v^2}{r}$$

In Section 4.4 on uniform circular motion, we show that $k = 1$ if a consistent set of units is used. The constant $k$ would not equal 1 if, for example, $v$ were in km/h and you wanted $a$ in m/s$^2$ .

## 1.4 Conversion of Units

Sometimes it is necessary to convert units from one measurement system to another or convert within a system (for example, from kilometers to meters). Conversion factors between SI and U.S. customary units of length are as follows:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 mile | = | 1 609 m | = | 1.609 km | 1 ft | = | 0.304 8 m | = 30.48 cm |
| 1 m | = | 39.37 in. | = | 3.281 ft | 1 in. | = | 0.025 4 m | = 2.54 cm (exactly) |

A more complete list of conversion factors can be found in Appendix A.

Like dimensions, units can be treated as algebraic quantities that can cancel each other. For example, suppose we wish to convert 15.0 in. to centimeters. Because 1 in. is defined as exactly 2.54 cm, we find that

$$15.0 \text{ in.} = (15.0 \not{\text{in.}})\left(\frac{2.54 \text{ cm}}{1 \not{\text{in.}}}\right) = 38.1 \text{ cm}$$

where the ratio in parentheses is equal to 1. We express 1 as 2.54 cm/1 in. (rather than 1 in./2.54 cm) so that the unit "inch" in the denominator cancels with the unit in the original quantity. The remaining unit is the centimeter, our desired result.

**Pitfall Prevention 1.3**

**Always Include Units** When performing calculations with numerical values, include the units for every quantity and carry the units through the entire calculation. Avoid the temptation to drop the units early and then attach the expected units once you have an answer. By including the units in every step, you can detect errors if the units for the answer turn out to be incorrect.

**Quick Quiz 1.3** The distance between two cities is 100 mi. What is the number of kilometers between the two cities? **(a)** smaller than 100 **(b)** larger than 100 **(c)** equal to 100

## Example 1.3 Is He Speeding?

On an interstate highway in a rural region of Wyoming, a car is traveling at a speed of 38.0 m/s. Is the driver exceeding the speed limit of 75.0 mi/h?

**SOLUTION**

Convert meters in the speed to miles:

$$(38.0\ \cancel{\text{m}}/\text{s})\left(\frac{1\ \text{mi}}{1\ 609\ \cancel{\text{m}}}\right) = 2.36 \times 10^{-2}\ \text{mi/s}$$

Convert seconds to hours:

$$(2.36 \times 10^{-2}\ \text{mi}/\cancel{\text{s}})\left(\frac{60\ \cancel{\text{s}}}{1\ \cancel{\text{min}}}\right)\left(\frac{60\ \cancel{\text{min}}}{1\ \text{h}}\right) = 85.0\ \text{mi/h}$$

The driver is indeed exceeding the speed limit and should slow down.

**WHAT IF?** What if the driver were from outside the United States and is familiar with speeds measured in kilometers per hour? What is the speed of the car in km/h?

**Answer** We can convert our final answer to the appropriate units:

$$(85.0\ \cancel{\text{mi}}/\text{h})\left(\frac{1.609\ \text{km}}{1\ \cancel{\text{mi}}}\right) = 137\ \text{km/h}$$

Figure 1.3 shows an automobile speedometer displaying speeds in both mi/h and km/h. Can you check the conversion we just performed using this photograph?




**Figure 1.3** The speedometer of a vehicle that shows speeds in both miles per hour and kilometers per hour.

## 1.5 Estimates and Order-of-Magnitude Calculations

Suppose someone asks you the number of bits of data on a typical musical compact disc. In response, it is not generally expected that you would provide the exact number but rather an estimate, which may be expressed in scientific notation. The estimate may be made even more approximate by expressing it as an *order of magnitude,* which is a power of ten determined as follows:

1. Express the number in scientific notation, with the multiplier of the power of ten between 1 and 10 and a unit.
2. If the multiplier is less than 3.162 (the square root of 10), the order of magnitude of the number is the power of 10 in the scientific notation. If the multiplier is greater than 3.162, the order of magnitude is one larger than the power of 10 in the scientific notation.

We use the symbol $\sim$ for "is on the order of." Use the procedure above to verify the orders of magnitude for the following lengths:

$$0.008\ 6\ \text{m} \sim 10^{-2}\ \text{m} \qquad 0.002\ 1\ \text{m} \sim 10^{-3}\ \text{m} \qquad 720\ \text{m} \sim 10^{3}\ \text{m}$$

Usually, when an order-of-magnitude estimate is made, the results are reliable to within about a factor of 10. If a quantity increases in value by three orders of magnitude, its value increases by a factor of about $10^3 = 1\,000$.

Inaccuracies caused by guessing too low for one number are often canceled by other guesses that are too high. You will find that with practice your guesstimates become better and better. Estimation problems can be fun to work because you freely drop digits, venture reasonable approximations for unknown numbers, make simplifying assumptions, and turn the question around into something you can answer in your head or with minimal mathematical manipulation on paper. Because of the simplicity of these types of calculations, they can be performed on a *small* scrap of paper and are often called "back-of-the-envelope calculations."

### Example 1.4 Breaths in a Lifetime

Estimate the number of breaths taken during an average human lifetime.

**SOLUTION**

We start by guessing that the typical human lifetime is about 70 years. Think about the average number of breaths that a person takes in 1 min. This number varies depending on whether the person is exercising, sleeping, angry, serene, and so forth. To the nearest order of magnitude, we shall choose 10 breaths per minute as our estimate. (This estimate is certainly closer to the true average value than an estimate of 1 breath per minute or 100 breaths per minute.)

Find the approximate number of minutes in a year:

$$1\ \cancel{\text{yr}}\left(\frac{400\ \cancel{\text{days}}}{1\ \cancel{\text{yr}}}\right)\left(\frac{25\ \cancel{\text{h}}}{1\ \cancel{\text{day}}}\right)\left(\frac{60\ \text{min}}{1\ \cancel{\text{h}}}\right) = 6 \times 10^5\ \text{min}$$

Find the approximate number of minutes in a 70-year lifetime:

$$\text{number of minutes} = (70\ \text{yr})(6 \times 10^5\ \text{min/yr}) = 4 \times 10^7\ \text{min}$$

Find the approximate number of breaths in a lifetime:

$$\text{number of breaths} = (10\ \text{breaths/min})(4 \times 10^7\ \text{min}) = 4 \times 10^8\ \text{breaths}$$

Therefore, a person takes on the order of $10^9$ breaths in a lifetime. Notice how much simpler it is in the first calculation above to multiply $400 \times 25$ than it is to work with the more accurate $365 \times 24$.

**WHAT IF?** What if the average lifetime were estimated as 80 years instead of 70? Would that change our final estimate?

**Answer** We could claim that $(80\ \text{yr})(6 \times 10^5\ \text{min/yr}) = 5 \times 10^7$ min, so our final estimate should be $5 \times 10^8$ breaths. This answer is still on the order of $10^9$ breaths, so an order-of-magnitude estimate would be unchanged.

## 1.6 Significant Figures

When certain quantities are measured, the measured values are known only to within the limits of the experimental uncertainty. The value of this uncertainty can depend on various factors, such as the quality of the apparatus, the skill of the experimenter, and the number of measurements performed. The number of **significant figures** in a measurement can be used to express something about the uncertainty. The number of significant figures is related to the number of numerical digits used to express the measurement, as we discuss below.

As an example of significant figures, suppose we are asked to measure the radius of a compact disc using a meterstick as a measuring instrument. Let us assume the accuracy to which we can measure the radius of the disc is ±0.1 cm. Because of the uncertainty of ±0.1 cm, if the radius is measured to be 6.0 cm, we can claim only that its radius lies somewhere between 5.9 cm and 6.1 cm. In this case, we say that the measured value of 6.0 cm has two significant figures. Note that *the* *significant figures include the first estimated digit.* Therefore, we could write the radius as $(6.0 \pm 0.1)$ cm.

Zeros may or may not be significant figures. Those used to position the decimal point in such numbers as 0.03 and 0.007 5 are not significant. Therefore, there are one and two significant figures, respectively, in these two values. When the zeros come after other digits, however, there is the possibility of misinterpretation. For example, suppose the mass of an object is given as 1 500 g. This value is ambiguous because we do not know whether the last two zeros are being used to locate the decimal point or whether they represent significant figures in the measurement. To remove this ambiguity, it is common to use scientific notation to indicate the number of significant figures. In this case, we would express the mass as $1.5 \times 10^3$ g if there are two significant figures in the measured value, $1.50 \times 10^3$ g if there are three significant figures, and $1.500 \times 10^3$ g if there are four. The same rule holds for numbers less than 1, so $2.3 \times 10^{-4}$ has two significant figures (and therefore could be written 0.000 23) and $2.30 \times 10^{-4}$ has three significant figures (also written as 0.000 230).

In problem solving, we often combine quantities mathematically through multiplication, division, addition, subtraction, and so forth. When doing so, you must make sure that the result has the appropriate number of significant figures. A good rule of thumb to use in determining the number of significant figures that can be claimed in a multiplication or a division is as follows:

> When multiplying several quantities, the number of significant figures in the final answer is the same as the number of significant figures in the quantity having the smallest number of significant figures. The same rule applies to division.

Let's apply this rule to find the area of the compact disc whose radius we measured above. Using the equation for the area of a circle,

$$A = \pi r^2 = \pi(6.0 \text{ cm})^2 = 1.1 \times 10^2 \text{ cm}^2$$

If you perform this calculation on your calculator, you will likely see 113.097 335 5. It should be clear that you don't want to keep all of these digits, but you might be tempted to report the result as 113 $\text{cm}^2$. This result is not justified because it has three significant figures, whereas the radius only has two. Therefore, we must report the result with only two significant figures as shown above.

For addition and subtraction, you must consider the number of decimal places when you are determining how many significant figures to report:

> When numbers are added or subtracted, the number of decimal places in the result should equal the smallest number of decimal places of any term in the sum or difference.

**Pitfall Prevention 1.4**

**Read Carefully** Notice that the rule for addition and subtraction is different from that for multiplication and division. For addition and subtraction, the important consideration is the number of *decimal places,* not the number of *significant figures.*

As an example of this rule, consider the sum

$$23.2 + 5.174 = 28.4$$

Notice that we do not report the answer as 28.374 because the lowest number of decimal places is one, for 23.2. Therefore, our answer must have only one decimal place.

The rule for addition and subtraction can often result in answers that have a different number of significant figures than the quantities with which you start. For example, consider these operations that satisfy the rule:

$$1.000\,1 + 0.000\,3 = 1.000\,4$$

$$1.002 - 0.998 = 0.004$$

In the first example, the result has five significant figures even though one of the terms, 0.000 3, has only one significant figure. Similarly, in the second calculation, the result has only one significant figure even though the numbers being subtracted have four and three, respectively.

In this book, most of the numerical examples and end-of-chapter problems will yield answers having three significant figures. When carrying out estimation calculations, we shall typically work with a single significant figure.

◀ **Significant figure guidelines used in this book**

If the number of significant figures in the result of a calculation must be reduced, there is a general rule for rounding numbers: the last digit retained is increased by 1 if the last digit dropped is greater than 5. (For example, 1.346 becomes 1.35.) If the last digit dropped is less than 5, the last digit retained remains as it is. (For example, 1.343 becomes 1.34.) If the last digit dropped is equal to 5, the remaining digit should be rounded to the nearest even number. (This rule helps avoid accumulation of errors in long arithmetic processes.)

A technique for avoiding error accumulation is to delay the rounding of numbers in a long calculation until you have the final result. Wait until you are ready to copy the final answer from your calculator before rounding to the correct number of significant figures. In this book, we display numerical values rounded off to two or three significant figures. This occasionally makes some mathematical manipulations look odd or incorrect. For instance, looking ahead to Example 3.5 on page 69, you will see the operation $-17.7 \text{ km} + 34.6 \text{ km} = 17.0 \text{ km}$. This looks like an incorrect subtraction, but that is only because we have rounded the numbers 17.7 km and 34.6 km for display. If all digits in these two intermediate numbers are retained and the rounding is only performed on the final number, the correct three-digit result of 17.0 km is obtained.

### Pitfall Prevention 1.5

**Symbolic Solutions** When solving problems, it is very useful to perform the solution completely in algebraic form and wait until the very end to enter numerical values into the final symbolic expression. This method will save many calculator keystrokes, especially if some quantities cancel so that you never have to enter their values into your calculator! In addition, you will only need to round once, on the final result.

## Example 1.5 Installing a Carpet

A carpet is to be installed in a rectangular room whose length is measured to be 12.71 m and whose width is measured to be 3.46 m. Find the area of the room.

**SOLUTION**

If you multiply 12.71 m by 3.46 m on your calculator, you will see an answer of 43.976 6 $m^2$. How many of these numbers should you claim? Our rule of thumb for multiplication tells us that you can claim only the number of significant figures in your answer as are present in the measured quantity having the lowest number of significant figures. In this example, the lowest number of significant figures is three in 3.46 m, so we should express our final answer as 44.0 $m^2$.

# Summary

## Definitions

The three fundamental physical quantities of mechanics are **length, mass,** and **time,** which in the SI system have the units **meter** (m), **kilogram** (kg), and **second** (s), respectively. These fundamental quantities cannot be defined in terms of more basic quantities.

The **density** of a substance is defined as its *mass per unit volume:*

$$\rho \equiv \frac{m}{V} \quad \text{(1.1)}$$

*continued*

## Concepts and Principles

The method of **dimensional analysis** is very powerful in solving physics problems. Dimensions can be treated as algebraic quantities. By making estimates and performing order-of-magnitude calculations, you should be able to approximate the answer to a problem when there is not enough information available to specify an exact solution completely.

When you compute a result from several measured numbers, each of which has a certain accuracy, you should give the result with the correct number of **significant figures.**

When **multiplying** several quantities, the number of significant figures in the final answer is the same as the number of significant figures in the quantity having the smallest number of significant figures. The same rule applies to **division.**

When numbers are **added** or **subtracted,** the number of decimal places in the result should equal the smallest number of decimal places of any term in the sum or difference.

## Objective Questions

1. denotes answer available in *Student Solutions Manual/Study Guide*

**1.** One student uses a meterstick to measure the thickness of a textbook and obtains 4.3 cm ± 0.1 cm. Other students measure the thickness with vernier calipers and obtain four different measurements: (a) 4.32 cm ± 0.01 cm, (b) 4.31 cm ± 0.01 cm, (c) 4.24 cm ± 0.01 cm, and (d) 4.43 cm ± 0.01 cm. Which of these four measurements, if any, agree with that obtained by the first student?

**2.** A house is advertised as having 1 420 square feet under its roof. What is its area in square meters? (a) 4 660 $m^2$ (b) 432 $m^2$ (c) 158 $m^2$ (d) 132 $m^2$ (e) 40.2 $m^2$

**3.** Answer each question yes or no. Must two quantities have the same dimensions (a) if you are adding them? (b) If you are multiplying them? (c) If you are subtracting them? (d) If you are dividing them? (e) If you are equating them?

**4.** The price of gasoline at a particular station is 1.5 euros per liter. An American student can use 33 euros to buy gasoline. Knowing that 4 quarts make a gallon and that 1 liter is close to 1 quart, she quickly reasons that she can buy how many gallons of gasoline? (a) less than 1 gallon (b) about 5 gallons (c) about 8 gallons (d) more than 10 gallons

**5.** Rank the following five quantities in order from the largest to the smallest. If two of the quantities are equal, give them equal rank in your list. (a) 0.032 kg (b) 15 g (c) $2.7 \times 10^5$ mg (d) $4.1 \times 10^{-8}$ Gg (e) $2.7 \times 10^8$ $\mu$g

**6.** What is the sum of the measured values 21.4 s + 15 s + 17.17 s + 4.00 3 s? (a) 57.573 s (b) 57.57 s (c) 57.6 s (d) 58 s (e) 60 s

**7.** Which of the following is the best estimate for the mass of all the people living on the Earth? (a) $2 \times 10^8$ kg (b) $1 \times 10^9$ kg (c) $2 \times 10^{10}$ kg (d) $3 \times 10^{11}$ kg (e) $4 \times 10^{12}$ kg

**8.** (a) If an equation is dimensionally correct, does that mean that the equation must be true? (b) If an equation is not dimensionally correct, does that mean that the equation cannot be true?

**9.** Newton's second law of motion (Chapter 5) says that the mass of an object times its acceleration is equal to the net force on the object. Which of the following gives the correct units for force? (a) kg · $m/s^2$ (b) kg · $m^2/s^2$ (c) kg/m · $s^2$ (d) kg · $m^2$/s (e) none of those answers

**10.** A calculator displays a result as $1.365\,248\,0 \times 10^7$ kg. The estimated uncertainty in the result is ±2%. How many digits should be included as significant when the result is written down? (a) zero (b) one (c) two (d) three (e) four

## Conceptual Questions

1. denotes answer available in *Student Solutions Manual/Study Guide*

**1.** Suppose the three fundamental standards of the metric system were length, *density,* and time rather than length, *mass,* and time. The standard of density in this system is to be defined as that of water. What considerations about water would you need to address to make sure that the standard of density is as accurate as possible?

**2.** Why is the metric system of units considered superior to most other systems of units?

**3.** What natural phenomena could serve as alternative time standards?

**4.** Express the following quantities using the prefixes given in Table 1.4. (a) $3 \times 10^{-4}$ m (b) $5 \times 10^{-5}$ s (c) $72 \times 10^2$ g

## Problems

The problems found in this chapter may be assigned online in Enhanced WebAssign

**1.** straightforward; **2.** intermediate; **3.** challenging

**1.** full solution available in the *Student Solutions Manual/Study Guide*

AMT Analysis Model tutorial available in Enhanced WebAssign

GP Guided Problem

M Master It tutorial available in Enhanced WebAssign

W Watch It video solution available in Enhanced WebAssign

### Section 1.1 Standards of Length, Mass, and Time

*Note:* Consult the endpapers, appendices, and tables in the text whenever necessary in solving problems. For this chapter, Table 14.1 and Appendix B.3 may be particularly useful. Answers to odd-numbered problems appear in the back of the book.

**1.** (a) Use information on the endpapers of this book to calculate the average density of the Earth. (b) Where does the value fit among those listed in Table 14.1 in Chapter 14? Look up the density of a typical surface rock like granite in another source and compare it with the density of the Earth.

**2.** W The standard kilogram (Fig. 1.1a) is a platinum–iridium cylinder 39.0 mm in height and 39.0 mm in diameter. What is the density of the material?

**3.** An automobile company displays a die-cast model of its first car, made from 9.35 kg of iron. To celebrate its hundredth year in business, a worker will recast the model in solid gold from the original dies. What mass of gold is needed to make the new model?

**4.** A proton, which is the nucleus of a hydrogen atom, can be modeled as a sphere with a diameter of 2.4 fm and a mass of $1.67 \times 10^{-27}$ kg. (a) Determine the density of the proton. (b) State how your answer to part (a) compares with the density of osmium, given in Table 14.1 in Chapter 14.

**5.** W Two spheres are cut from a certain uniform rock. One has radius 4.50 cm. The mass of the other is five times greater. Find its radius.

**6.** What mass of a material with density $\rho$ is required to make a hollow spherical shell having inner radius $r_1$ and outer radius $r_2$?

### Section 1.2 Matter and Model Building

**7.** A crystalline solid consists of atoms stacked up in a repeating lattice structure. Consider a crystal as shown in Figure P1.7a. The atoms reside at the corners of cubes of side $L = 0.200$ nm. One piece of evidence for the regular arrangement of atoms comes from the flat surfaces along which a crystal separates, or cleaves, when it is broken. Suppose this crystal cleaves along a face diagonal as shown in Figure P1.7b. Calculate the spacing $d$ between two adjacent atomic planes that separate when the crystal cleaves.

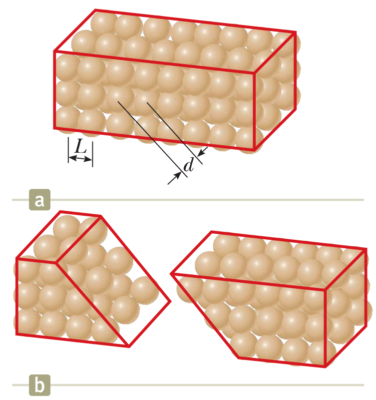



Figure P1.7

**8.** The mass of a copper atom is $1.06 \times 10^{-25}$ kg, and the density of copper is 8 920 kg/m$^3$. (a) Determine the number of atoms in 1 cm$^3$ of copper. (b) Visualize the one cubic centimeter as formed by stacking up identical cubes, with one copper atom at the center of each. Determine the volume of each cube. (c) Find the edge dimension of each cube, which represents an estimate for the spacing between atoms.

### Section 1.3 Dimensional Analysis

**9.** Which of the following equations are dimensionally correct? (a) $v_f = v_i + ax$ (b) $y = (2\text{ m})\cos(kx)$, where $k = 2\text{ m}^{-1}$

**10.** W Figure P1.10 shows a *frustum of a cone.* Match each of the expressions

(a) $\pi(r_1 + r_2)[h^2 + (r_2 - r_1)^2]^{1/2}$,
(b) $2\pi(r_1 + r_2)$, and
(c) $\pi h(r_1^2 + r_1 r_2 + r_2^2)/3$

with the quantity it describes: (d) the total circumference of the flat circular faces, (e) the volume, or (f) the area of the curved surface.

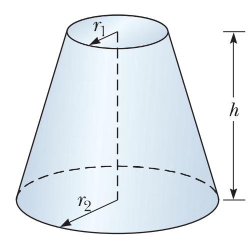



Figure P1.10

**11.** Kinetic energy $K$ (Chapter 7) has dimensions kg · m$^2$/s$^2$. It can be written in terms of the momentum $p$ (Chapter 9) and mass $m$ as

$$K = \frac{p^2}{2m}$$

(a) Determine the proper units for momentum using dimensional analysis. (b) The unit of force is the newton N, where 1 N = 1 kg · m/s$^2$ . What are the units of momentum $p$ in terms of a newton and another fundamental SI unit?

**12.** W Newton's law of universal gravitation is represented by

$$F = \frac{GMm}{r^2}$$

where $F$ is the magnitude of the gravitational force exerted by one small object on another, $M$ and $m$ are the masses of the objects, and $r$ is a distance. Force has the SI units kg · m/s$^2$. What are the SI units of the proportionality constant $G$?

**13.** The position of a particle moving under uniform acceleration is some function of time and the acceleration. Suppose we write this position as $x = ka^m t^n$, where $k$ is a dimensionless constant. Show by dimensional analysis that this expression is satisfied if $m = 1$ and $n = 2$. Can this analysis give the value of $k$?

**14.** (a) Assume the equation $x = At^3 + Bt$ describes the motion of a particular object, with $x$ having the dimension of length and $t$ having the dimension of time. Determine the dimensions of the constants $A$ and $B$. (b) Determine the dimensions of the derivative $dx/dt = 3At^2 + B$.

## Section 1.4 Conversion of Units

**15.** W A solid piece of lead has a mass of 23.94 g and a volume of 2.10 cm$^3$. From these data, calculate the density of lead in SI units (kilograms per cubic meter).

**16.** An ore loader moves 1 200 tons/h from a mine to the surface. Convert this rate to pounds per second, using 1 ton = 2 000 lb.

**17.** A rectangular building lot has a width of 75.0 ft and a length of 125 ft. Determine the area of this lot in square meters.

**18.** W Suppose your hair grows at the rate 1/32 in. per day. Find the rate at which it grows in nanometers per second. Because the distance between atoms in a molecule is on the order of 0.1 nm, your answer suggests how rapidly layers of atoms are assembled in this protein synthesis.

**19.** *Why is the following situation impossible?* A student's dormitory room measures 3.8 m by 3.6 m, and its ceiling is 2.5 m high. After the student completes his physics course, he displays his dedication by completely wallpapering the walls of the room with the pages from his copy of volume 1 (Chapters 1–22) of this textbook. He even covers the door and window.

**20.** W A pyramid has a height of 481 ft, and its base covers an area of 13.0 acres (Fig. P1.20). The volume of a pyramid is given by the expression $V = \frac{1}{3}Bh$, where $B$ is the area of the base and $h$ is the height. Find the volume of this pyramid in cubic meters. (1 acre = 43 560 ft$^2$)

**21.** The pyramid described in Problem 20 contains approximately 2 million stone blocks that average 2.50 tons each. Find the weight of this pyramid in pounds.

Adam Sylvester/Photo Researchers, Inc.

**Figure P1.20** Problems 20 and 21.

**22.** W Assume it takes 7.00 min to fill a 30.0-gal gasoline tank. (a) Calculate the rate at which the tank is filled in gallons per second. (b) Calculate the rate at which the tank is filled in cubic meters per second. (c) Determine the time interval, in hours, required to fill a 1.00-m$^3$ volume at the same rate. (1 U.S. gal = 231 in.$^3$)

**23.** A *section* of land has an area of 1 square mile and contains 640 acres. Determine the number of square meters in 1 acre.

**24.** M A house is 50.0 ft long and 26 ft wide and has 8.0-ft-high ceilings. What is the volume of the interior of the house in cubic meters and in cubic centimeters?

**25.** M One cubic meter (1.00 m$^3$) of aluminum has a mass of 2.70 × 10$^3$ kg, and the same volume of iron has a mass of 7.86 × 10$^3$ kg. Find the radius of a solid aluminum sphere that will balance a solid iron sphere of radius 2.00 cm on an equal-arm balance.

**26.** Let $\rho_{Al}$ represent the density of aluminum and $\rho_{Fe}$ that of iron. Find the radius of a solid aluminum sphere that balances a solid iron sphere of radius $r_{Fe}$ on an equal-arm balance.

**27.** M One gallon of paint (volume = 3.78 × 10$^{-3}$ m$^3$) covers an area of 25.0 m$^2$. What is the thickness of the fresh paint on the wall?

**28.** W An auditorium measures 40.0 m × 20.0 m × 12.0 m. The density of air is 1.20 kg/m$^3$. What are (a) the volume of the room in cubic feet and (b) the weight of air in the room in pounds?

**29.** M (a) At the time of this book's printing, the U.S. national debt is about \$16 trillion. If payments were made at the rate of \$1 000 per second, how many years would it take to pay off the debt, assuming no interest were charged? (b) A dollar bill is about 15.5 cm long. How many dollar bills attached end to end would it take to reach the Moon? The front endpapers give the Earth–Moon distance. *Note:* Before doing these calculations, try to guess at the answers. You may be very surprised.

**30.** A hydrogen atom has a diameter of 1.06 × 10$^{-10}$ m. The nucleus of the hydrogen atom has a diameter of approximately 2.40 × 10$^{-15}$ m. (a) For a scale model, represent the diameter of the hydrogen atom by the playing length of an American football field (100 yards = 300 ft) and determine the diameter of the nucleus in millimeters. (b) Find the ratio of the volume of the hydrogen atom to the volume of its nucleus.

## Section 1.5 Estimates and Order-of-Magnitude Calculations

*Note:* In your solutions to Problems 31 through 34, state the quantities you measure or estimate and the values you take for them.

**31.** Find the order of magnitude of the number of table-tennis balls that would fit into a typical-size room (without being crushed).

**32.** (a) Compute the order of magnitude of the mass of a bathtub half full of water. (b) Compute the order of magnitude of the mass of a bathtub half full of copper coins.

**33.** To an order of magnitude, how many piano tuners reside in New York City? The physicist Enrico Fermi was famous for asking questions like this one on oral Ph.D. qualifying examinations.

**34.** An automobile tire is rated to last for 50 000 miles. To an order of magnitude, through how many revolutions will it turn over its lifetime?

## Section 1.6 Significant Figures

*Note:* Appendix B.8 on propagation of uncertainty may be useful in solving some problems in this section.

**35.** A rectangular plate has a length of $(21.3 \pm 0.2)$ cm and a width of $(9.8 \pm 0.1)$ cm. Calculate the area of the plate, including its uncertainty.

**36.** How many significant figures are in the following numbers? (a) $78.9 \pm 0.2$ (b) $3.788 \times 10^9$ (c) $2.46 \times 10^{-6}$ (d) 0.005 3

**37.** The *tropical year,* the time interval from one vernal equinox to the next vernal equinox, is the basis for our calendar. It contains 365.242 199 days. Find the number of seconds in a tropical year.

**38.** Carry out the arithmetic operations (a) the sum of the measured values 756, 37.2, 0.83, and 2; (b) the product $0.003\,2 \times 356.3$; and (c) the product $5.620 \times \pi$.

*Note:* The next 13 problems call on mathematical skills from your prior education that will be useful throughout this course.

**39. Review.** In a community college parking lot, the number of ordinary cars is larger than the number of sport utility vehicles by 94.7%. The difference between the number of cars and the number of SUVs is 18. Find the number of SUVs in the lot.

**40. Review.** While you are on a trip to Europe, you must purchase hazelnut chocolate bars for your grandmother. Eating just one square each day, she makes each large bar last for one and one-third months. How many bars will constitute a year's supply for her?

**41. Review.** A child is surprised that because of sales tax she must pay \$1.36 for a toy marked \$1.25. What is the effective tax rate on this purchase, expressed as a percentage?

**42. Review.** The average density of the planet Uranus is $1.27 \times 10^3$ kg/m$^3$. The ratio of the mass of Neptune to that of Uranus is 1.19. The ratio of the radius of Neptune to that of Uranus is 0.969. Find the average density of Neptune.

**43. Review.** The ratio of the number of sparrows visiting a bird feeder to the number of more interesting birds is 2.25. On a morning when altogether 91 birds visit the feeder, what is the number of sparrows?

**44. Review.** Find every angle $\theta$ between 0 and 360° for which the ratio of $\sin\theta$ to $\cos\theta$ is $-3.00$.

**45. Review.** For the right triangle shown in Figure P1.45, what are (a) the length of the unknown side, (b) the tangent of $\theta$, and (c) the sine of $\phi$?

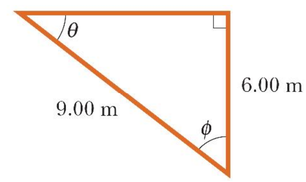



Figure P1.45

**46. Review.** Prove that one solution of the equation

$$2.00x^4 - 3.00x^3 + 5.00x = 70.0$$

is $x = -2.22$.

**47. Review.** A pet lamb grows rapidly, with its mass proportional to the cube of its length. When the lamb's length changes by 15.8%, its mass increases by 17.3 kg. Find the lamb's mass at the end of this process.

**48. Review.** A highway curve forms a section of a circle. A car goes around the curve as shown in the helicopter view of Figure P1.48. Its dashboard compass shows that the car is initially heading due east. After it travels $d$ = 840 m, it is heading $\theta = 35.0°$ south of east. Find the radius of curvature of its path. *Suggestion:* You may find it useful to learn a geometric theorem stated in Appendix B.3.

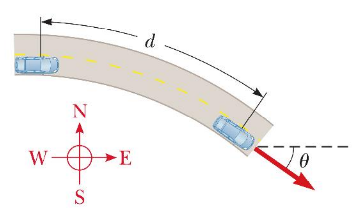



Figure P1.48

**49. Review.** From the set of equations

$$p = 3q$$

$$pr = qs$$

$$\tfrac{1}{2}pr^2 + \tfrac{1}{2}qs^2 = \tfrac{1}{2}qt^2$$

involving the unknowns $p$, $q$, $r$, $s$, and $t$, find the value of the ratio of $t$ to $r$.

**50. Review.** Figure P1.50 on page 18 shows students studying the thermal conduction of energy into cylindrical blocks of ice. As we will see in Chapter 20, this process is described by the equation

$$\frac{Q}{\Delta t} = \frac{k\pi d^2 (T_h - T_c)}{4L}$$

For experimental control, in one set of trials all quantities except $d$ and $\Delta t$ are constant. (a) If $d$ is made three times larger, does the equation predict that $\Delta t$ will get larger or get smaller? By what factor? (b) What pattern of proportionality of $\Delta t$ to $d$ does the equation predict? (c) To display this proportionality as a straight line on a graph, what quantities should you plot on the horizontal and vertical axes? (d) What expression represents the theoretical slope of this graph?

Alexandra Héder

**Figure P1.50**

**51. Review.** A student is supplied with a stack of copy paper, ruler, compass, scissors, and a sensitive balance. He cuts out various shapes in various sizes, calculates their areas, measures their masses, and prepares the graph of Figure P1.51. (a) Consider the fourth experimental point from the top. How far is it from the best-fit straight line? Express your answer as a difference in vertical-axis coordinate. (b) Express your answer as a percentage. (c) Calculate the slope of the line. (d) State what the graph demonstrates, referring to the shape of the graph and the results of parts (b) and (c). (e) Describe whether this result should be expected theoretically. (f) Describe the physical meaning of the slope.

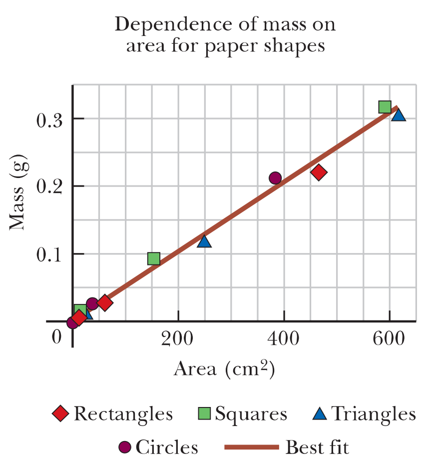



**Figure P1.51**

**52.** The radius of a uniform solid sphere is measured to be (6.50 ± 0.20) cm, and its mass is measured to be (1.85 ± 0.02) kg. Determine the density of the sphere in kilograms per cubic meter and the uncertainty in the density.

**53.** A sidewalk is to be constructed around a swimming pool that measures (10.0 ± 0.1) m by (17.0 ± 0.1) m. If the sidewalk is to measure (1.00 ± 0.01) m wide by (9.0 ± 0.1) cm thick, what volume of concrete is needed and what is the approximate uncertainty of this volume?

### Additional Problems

**54.** Collectible coins are sometimes plated with gold to enhance their beauty and value. Consider a commemorative quarter-dollar advertised for sale at \$4.98. It has a diameter of 24.1 mm and a thickness of 1.78 mm, and it is completely covered with a layer of pure gold 0.180 $\mu$m thick. The volume of the plating is equal to the thickness of the layer multiplied by the area to which it is applied. The patterns on the faces of the coin and the grooves on its edge have a negligible effect on its area. Assume the price of gold is \$25.0 per gram. (a) Find the cost of the gold added to the coin. (b) Does the cost of the gold significantly enhance the value of the coin? Explain your answer.

**55.** In a situation in which data are known to three significant digits, we write 6.379 m = 6.38 m and 6.374 m = 6.37 m. When a number ends in 5, we arbitrarily choose to write 6.375 m = 6.38 m. We could equally well write 6.375 m = 6.37 m, "rounding down" instead of "rounding up," because we would change the number 6.375 by equal increments in both cases. Now consider an order-of-magnitude estimate, in which factors of change rather than increments are important. We write 500 m $\sim 10^3$ m because 500 differs from 100 by a factor of 5 while it differs from 1 000 by only a factor of 2. We write 437 m $\sim 10^3$ m and 305 m $\sim 10^2$ m. What distance differs from 100 m and from 1 000 m by equal factors so that we could equally well choose to represent its order of magnitude as $\sim 10^2$ m or as $\sim 10^3$ m?

**56.** (a) What is the order of magnitude of the number of microorganisms in the human intestinal tract? A typical bacterial length scale is $10^{-6}$ m. Estimate the intestinal volume and assume 1% of it is occupied by bacteria. (b) Does the number of bacteria suggest whether the bacteria are beneficial, dangerous, or neutral for the human body? What functions could they serve?

**57.** The diameter of our disk-shaped galaxy, the Milky Way, is about $1.0 \times 10^5$ light-years (ly). The distance to the Andromeda galaxy (Fig. P1.57), which is the spiral galaxy nearest to the Milky Way, is about 2.0 million ly. If a scale model represents the Milky Way and Andromeda

Robert Gendler/NASA

**Figure P1.57** The Andromeda galaxy.

galaxies as dinner plates 25 cm in diameter, determine the distance between the centers of the two plates.

**58.** *Why is the following situation impossible?* In an effort to boost interest in a television game show, each weekly winner is offered an additional \$1 million bonus prize if he or she can personally count out that exact amount from a supply of one-dollar bills. The winner must do this task under supervision by television show executives and within one 40-hour work week. To the dismay of the show's producers, most contestants succeed at the challenge.

**59.** AMT M A high fountain of water is located at the center of a circular pool as shown in Figure P1.59. A student walks around the pool and measures its circumference to be 15.0 m. Next, the student stands at the edge of the pool and uses a protractor to gauge the angle of elevation of the top of the fountain to be $\phi = 55.0°$. How high is the fountain?


**Figure P1.59** Problems 59 and 60.

**60.** A water fountain is at the center of a circular pool as shown in Figure P1.59. A student walks around the pool and measures its circumference $C$. Next, he stands at the edge of the pool and uses a protractor to measure the angle of elevation $\phi$ of his sightline to the top of the water jet. How high is the fountain?

**61.** The data in the following table represent measurements of the masses and dimensions of solid cylinders of aluminum, copper, brass, tin, and iron. (a) Use these data to calculate the densities of these substances. (b) State how your results compare with those given in Table 14.1.

| Substance | Mass (g) | Diameter (cm) | Length (cm) |
|---|---|---|---|
| Aluminum | 51.5 | 2.52 | 3.75 |
| Copper | 56.3 | 1.23 | 5.06 |
| Brass | 94.4 | 1.54 | 5.69 |
| Tin | 69.1 | 1.75 | 3.74 |
| Iron | 216.1 | 1.89 | 9.77 |

**62.** The distance from the Sun to the nearest star is about $4 \times 10^{16}$ m. The Milky Way galaxy (Fig. P1.62) is roughly a disk of diameter $\sim 10^{21}$ m and thickness $\sim 10^{19}$ m. Find the order of magnitude of the number of stars in the Milky Way. Assume the distance between the Sun and our nearest neighbor is typical.

Richard Payne/NASA

**Figure P1.62** The Milky Way galaxy.

**63.** AMT M Assume there are 100 million passenger cars in the United States and the average fuel efficiency is 20 mi/gal of gasoline. If the average distance traveled by each car is 10 000 mi/yr, how much gasoline would be saved per year if the average fuel efficiency could be increased to 25 mi/gal?

**64.** A spherical shell has an outside radius of 2.60 cm and an inside radius of $a$. The shell wall has uniform thickness and is made of a material with density 4.70 g/cm$^3$. The space inside the shell is filled with a liquid having a density of 1.23 g/cm$^3$. (a) Find the mass $m$ of the sphere, including its contents, as a function of $a$. (b) For what value of the variable $a$ does $m$ have its maximum possible value? (c) What is this maximum mass? (d) Explain whether the value from part (c) agrees with the result of a direct calculation of the mass of a solid sphere of uniform density made of the same material as the shell. (e) **What If?** Would the answer to part (a) change if the inner wall were not concentric with the outer wall?

**65.** Bacteria and other prokaryotes are found deep underground, in water, and in the air. One micron ($10^{-6}$ m) is a typical length scale associated with these microbes. (a) Estimate the total number of bacteria and other prokaryotes on the Earth. (b) Estimate the total mass of all such microbes.

**66.** Air is blown into a spherical balloon so that, when its radius is 6.50 cm, its radius is increasing at the rate 0.900 cm/s. (a) Find the rate at which the volume of the balloon is increasing. (b) If this volume flow rate of air entering the balloon is constant, at what rate will the radius be increasing when the radius is 13.0 cm? (c) Explain physically why the answer to part (b) is larger or smaller than 0.9 cm/s, if it is different.

**67.** A rod extending between $x = 0$ and $x = 14.0$ cm has uniform cross-sectional area $A = 9.00$ cm$^2$. Its density increases steadily between its ends from 2.70 g/cm$^3$ to 19.3 g/cm$^3$. (a) Identify the constants $B$ and $C$ required in the expression $\rho = B + Cx$ to describe the variable density. (b) The mass of the rod is given by

$$m = \int_{\text{all material}} \rho\, dV = \int_{\text{all } x} \rho A\, dx = \int_0^{14.0\text{ cm}} (B + Cx)(9.00\text{ cm}^2)\, dx$$

Carry out the integration to find the mass of the rod.

**68.** In physics, it is important to use mathematical approximations. (a) Demonstrate that for small angles (< 20°)

$$\tan \alpha \approx \sin \alpha \approx \alpha = \frac{\pi \alpha'}{180°}$$

where $\alpha$ is in radians and $\alpha'$ is in degrees. (b) Use a calculator to find the largest angle for which $\tan \alpha$ may be approximated by $\alpha$ with an error less than 10.0%.

**69.** M The consumption of natural gas by a company satisfies the empirical equation $V = 1.50t + 0.008\,00t^2$, where $V$

is the volume of gas in millions of cubic feet and $t$ is the time in months. Express this equation in units of cubic feet and seconds. Assume a month is 30.0 days.

**70.** GP A woman wishing to know the height of a mountain measures the angle of elevation of the mountaintop as 12.0°. After walking 1.00 km closer to the mountain on level ground, she finds the angle to be 14.0°. (a) Draw a picture of the problem, neglecting the height of the woman's eyes above the ground. *Hint:* Use two triangles. (b) Using the symbol $y$ to represent the mountain height and the symbol $x$ to represent the woman's original distance from the mountain, label the picture. (c) Using the labeled picture, write two trigonometric equations relating the two selected variables. (d) Find the height $y$.

**71.** AMT A child loves to watch as you fill a transparent plastic bottle with shampoo (Fig P1.71). Every horizontal cross section of the bottle is circular, but the diameters of the circles have different values. You pour the brightly colored shampoo into the bottle at a constant rate of 16.5 cm$^3$/s. At what rate is its level in the bottle rising (a) at a point where the diameter of the bottle is 6.30 cm and (b) at a point where the diameter is 1.35 cm?

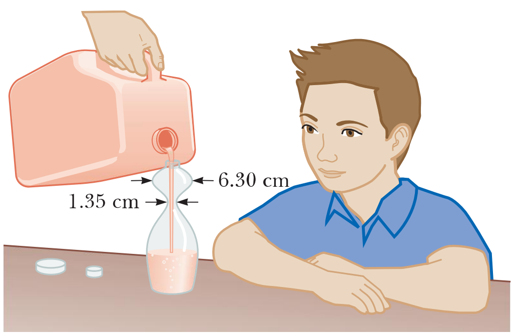



**Figure P1.71**

## Challenge Problems

**72.** A woman stands at a horizontal distance $x$ from a mountain and measures the angle of elevation of the mountaintop above the horizontal as $\theta$. After walking a distance $d$ closer to the mountain on level ground, she finds the angle to be $\phi$. Find a general equation for the height $y$ of the mountain in terms of $d$, $\phi$, and $\theta$, neglecting the height of her eyes above the ground.

**73.** You stand in a flat meadow and observe two cows (Fig. P1.73). Cow A is due north of you and 15.0 m from your position. Cow B is 25.0 m from your position. From your point of view, the angle between cow A and cow B is 20.0°, with cow B appearing to the right of cow A. (a) How far apart are cow A and cow B? (b) Consider the view seen by cow A. According to this cow, what is the angle between you and cow B? (c) Consider the view seen by cow B. According to this cow, what is the angle between you and cow A? *Hint:* What does the situation look like to a hummingbird hovering above the meadow? (d) Two stars in the sky appear to be 20.0° apart. Star A is 15.0 ly from the Earth, and star B, appearing to the right of star A, is 25.0 ly from the Earth. To an inhabitant of a planet orbiting star A, what is the angle in the sky between star B and our Sun?

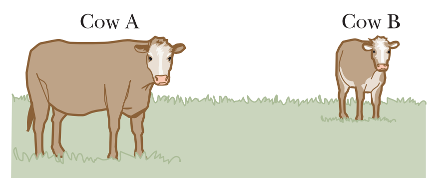



**Figure P1.73** Your view of two cows in a meadow. Cow A is due north of you. You must rotate your eyes through an angle of 20.0° to look from cow A to cow B.